Minimal Work: A Grasp Quality Metric for Deformable Hollow Objects

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***Abstract*— Robot grasping of deformable hollow objects such as plastic bottles and cups is challenging, as the grasp should resist disturbances while minimally deforming the object so as not to damage it or dislodge liquids. We propose minimal work as a novel grasp quality metric that combines wrench resistance and object deformation. We introduce an efficient algorithm to compute the work required to resist an external wrench for a manipulation task by solving a linear program. The algorithm first computes the minimum required grasp force and an estimation of the gripper jaw displacements based on the object's empirical stiffness at different locations. The work done by the jaws is the product of the grasp force and the displacements. Grasps requiring minimal work are considered to be of high quality. We collect 460 physical grasps with a UR5 robot and a Robotiq gripper. We consider a grasp to be successful if it completes the task without damaging the object or dislodging the content. Physical experiments suggest that the minimal work quality metric reaches 74.2% balanced accuracy, a metric that is the raw accuracy normalized by the number of successful and failed real-world grasps, and is up to 24.2% higher than classical wrench-based quality metrics.**

Fig. 1: Plastic cup example. Left: stiffness of cup, blue indicates high stiffness and red indicates low stiffness. Intuitively, the cup is stiffer near the rim and bottom, where the shape provides reinforcement. Middle: three planned grasps, shown as cylinders representing the grasp axis for a parallel-jaw gripper, where green indicates high quality and red indicates low quality according to the minimal work metric. Right: execution of the highest quality grasp according to the minimal work grasp metric with a Robotiq gripper.

I. Introduction

For rigid objects, wrench-based quality metrics [8, 16] are widely used to optimize grasp placements and to estimate grasp success [11, 25], since they quantify grasps and are suitable for both general and task-oriented grasps. However, grasping deformable objects is more challenging. In addition to resisting external disturbances, grasps should also minimize the deformation of the object to avoid damage or dislodging liquids, for instance, when grasping plastic cups and bottles. Existing grasp planning for deformable objects focuses on either holding planar deformable objects [9, 12, 26] or lifting 3D objects with a pre-selected grasp placement [18, 34]. The former papers do not consider gravity, as they operate in the plane, while the latter do not incorporate a quality metric for comparison to other grasp placements and cannot be applied to other tasks besides lifting the object.

We propose the *minimal work* quality metric, a novel quality metric that considers both task-specific wrench resistance and object deformation. Figure 1 shows an example of planned grasps for a plastic cup with the proposed metric. To compute the quality of a grasp, we decouple the wrench analysis and object deformation computation so that the grasp quality can be efficiently computed without using Finite Element Method (FEM) simulation or repeatedly determining wrench resistance. We first estimate the minimum grasp force required to resist a specified external wrench without considering deformation by assuming the object is rigid. We use the Robust Efficient Area Contact Hypothesis (REACH) model [6] to estimate the contact area and the pressure distribution and model the friction wrenches of the non-planar area contacts with a 6D ellipsoidal limit surface [33]. We formulate an optimization problem to solve for the minimum grasp force subject to the contact friction wrench constraints. The jaws' displacements are approximated as the object's deformation when using the computed minimal grasp force. The product of these displacements and the minimum grasp force is the minimal work necessary to resist the external wrench.

This paper provides the following contributions:

1) A novel minimal work grasp quality metric for 3D deformable hollow objects that considers both grasp wrench resistance and object deformation.
2) An efficient algorithm to compute the minimal work quality metric for a task-specific 6D external wrench by solving a linear program.
3) Physical experiments that suggest predicting grasp success with the minimal work quality metric leads to 74.2% balanced accuracy, 24.2% and 12.7 % higher than when predicting with the grasp wrench resistance metric and the minimal force metric, respectively.

II. Related Work

We summarize related work in wrench-based grasp quality metrics for rigid objects and grasp planning for deformable objects. Excellent surveys for contact modeling can be found

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in [2, 13, 27, 31], for grasp quality metrics in [28], and for deformable object manipulation in [29].

A. Grasping Rigid Objects

A common grasp quality metric, force closure [24], evaluates a grasp by whether it can resist any external 6D disturbance wrench with an arbitrarily large grasp force. The volume of the grasp wrench space (GWS) [16] and the ϵ-metric [8] are also widely used to quantify grasp quality. While the volume reflects the quality of the entire grasp, the ϵ-metric identifies the weakest point of a grasp, as ϵ is the shortest distance between the origin and any facet of the GWS. The GWS used to evaluate grasps is typically constructed with a bounded sum-magnitude of grasp forces for computational efficiency. Krug et al. [14] suggest that such a construction is over-conservative for fully actuated grippers, as the force of each jaw is limited independently.

Task-oriented grasp quality metrics are well-suited for specific tasks with known external disturbances. A task wrench space (TWS) describes expected disturbance wrenches during the manipulation and is typically modeled as the set of all possible wrenches that will be imposed on an object during a task [3, 25], or a 6D ellipsoid [16]. The quality of a grasp is the maximum scale of the TWS such that it remains within the GWS [16]. In contrast, the grasp quality can also be measured with the minimal force required for a task [10] or minimal coefficient of friction [11]. Boyd and Wegbreit [4] efficiently computed the minimal grasp force by formulating a semidefinite programming problem. Lin and Yu [19] observed that some disturbance wrenches happen more often than others during task execution and selected the grasp whose corresponding GWS covers most frequent disturbances. They extend their analysis to select the grasp which additionally minimizes the required motion effort of the end effector to fulfill a certain task [20].

B. Grasping Deformable Objects

Manipulating deformable objects has been an active area, with applications such as food handling [21], fabric manipulation [15, 30], and elastic rod manipulation [5]. When a frictionless grasp immobilizes a rigid object, it is defined as form closure. Gopalakrishnan and Goldberg [9] generalized this concept to holding deformable objects with frictionless contacts, where a grasp is defined as deform closure when positive work is required to release the object. Wakamatsu et al. [32] introduced the bounded force closure metric to grasp deformable objects, which guarantees a force closure grasp under a maximal allowable external force. Delgado et al. [7] reduced object deformation for a holding task by computing the maximum allowed force to be exerted on an object. Jia et al. [12] proposed a grasping strategy to squeeze deformable planar objects based on work performed by the jaws. When two jaws squeeze and immobilize an object, and a third jaw tries to break the grasp by pushing the object, the strategy selects the translations of the two pushing jaws that minimize the required work to balance the object. Since the metric targets planar objects, the 3D geometry or the gravity is not considered and a point contact model is used for friction analysis. Lin et al. [17, 18] addressed the problem of lifting a deformable object based on an object mesh model and jaw positions. An FEM formulation computes the object deformation based on the jaw displacements. The object will be lifted if the majority of the contact points are sticking. Similarly, Zaidi et al. [34] used FEM simulations to manipulate objects with large deformations, such as objects made of foam or rubber. Alt et al. [1] also used FEM simulations and heuristics to plan grasps for deformable hollow objects.

Inspired by [9] and [12], the proposed minimal work quality metric optimizes grasp placements to manipulate 3D deformable hollow objects. Furthermore, the metric is suitable for tasks that can be modeled as target wrenches to be resisted.

III. Problem Statement

We consider the problem of grasp planning and grasp success prediction for 3D deformable hollow objects with compliant jaw pads based on the ability of a grasp to resist target wrenches and the deformability of the object at the grasp location.

A. Assumptions

We make the following assumptions:

1) Quasi-static analysis and Coulomb friction with a known coefficient of friction.
2) The geometry and the stiffness are known for the objects to be grasped.
3) A linear elastic model (linear stiffness) of soft jaw pads and objects.
4) Object's local deformation is small such that the contact profile remains unchanged during the grasp.

B. Notation

- $\boldsymbol{w} \in \mathbb{R}^6$: a contact wrench, consisting of a 3D force and a 3D torque.
- $\mathcal{C}_i$: the constraint set that limits the maximum possible friction wrench and the wrench impressed by the normal pressure of the i-th contact.
- W: the work performed by the gripper jaws
- $\boldsymbol{t} \in \mathbb{R}^6$: a target wrench to be resisted by a grasp

C. Metrics

We consider a grasp to be successful if it completes the manipulation task without damaging the object or dislodging contents and to be failed otherwise. The predicted grasp success is binary and evaluates to 1 if the metric is higher than a threshold. We use balanced accuracy to evaluate the predictions made by a given metric by comparing them with real-world grasp success labels. Balanced accuracy is suitable for imbalanced datasets and is computed by weighting each sample with the inverse prevalence of its true class when finding accuracy.

IV. Minimal Work Grasp Quality Metric

To evaluate a grasp candidate, we compute the minimal work of the gripper jaws required to complete a manipulation task. We first model the frictional contacts and compute the minimal grasp force by solving a linear program (LP). We then estimate the object deformation based on the force and object stiffness at the contact locations. The work of each gripper jaw is the product of the grasp force and the jaw displacement. The sum of the work of each jaw forms the work of the grasp.

The proposed algorithm to compute the minimal work can use different contact models and object stiffness acquisition methods. We use the REACH model [6] to obtain the contact profile and a 6D ellipsoidal limit surface [33] to model the maximal friction wrench that can be applied at each contact. The stiffness is collected empirically by grasping each object at a set of locations with a physical robot. Details can be found in Sections V and VI-A.

For a grasp with N contacts, we denote $\mathbf{G} \in \mathbb{R}^{6\times 6N}$ as the grasp matrix, $\boldsymbol{w}_i \in \mathbb{R}^6$ as the wrench applied at the i-th contact, and $\boldsymbol{F} = [F_1, \dots, F_N]^T$ as a vector of grasp forces, where F_i is the grasp force at the i-th contact. The minimal required grasp force to resist a target wrench $\boldsymbol{t}$ is:

$$\begin{aligned} \min_{\boldsymbol{F}, \boldsymbol{w}_1, \dots, \boldsymbol{w}_N} \quad & \boldsymbol{F} \cdot \mathbf{1}_N \\ \text{subject to} \quad & \mathbf{G} \begin{bmatrix} \boldsymbol{w}_1 \\ \vdots \\ \boldsymbol{w}_N \end{bmatrix} = -\boldsymbol{t}, \\ & \boldsymbol{w}_i \in \mathcal{C}_i, \forall i \in \{1, \dots, N\}. \end{aligned} \tag{1}$$

Denoting d_i as the displacement of the i-th jaw and s_i as the object stiffness at the i-th contact, the work W is computed based on Hooke's law:

$$W = \sum_{i=1}^{N} F_i \cdot d_i, \text{ with } d_i = \frac{F_i}{s_i} + \epsilon, \tag{2}$$

where ϵ is a small positive number, which allows the minimal work quality metric to also apply to rigid objects or objects containing a rigid part. In this case, the displacement d_i is equal to ϵ and the minimal work grasp quality metric reduces to the minimal force metric.

Denoting $W_{\max}$ as the maximal work, we compute the minimal work grasp quality q_w with:

$$q_w = 1 - \frac{W}{W_{\max}}. \tag{3}$$

$W_{\max}$ is for normalization and is selected based on the collected data.

V. Algorithm

To compute the minimal required grasp force, classical wrench-based grasp analysis algorithms first model the possible friction and normal wrenches for each contact and then estimate the total wrench that a grasp can exert on an object [4]. This wrench estimation highly depends on the contact profiles. We first describe the method used in this work to estimate contact area and pressure distribution.

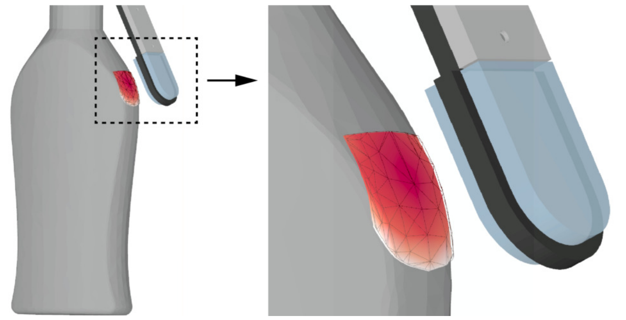

Fig. 2: Contact profile with an enlarged view obtained by the Robust Efficient Area Contact Hypothesis (REACH) model [6]. The contact area consists of triangles and the redder colors represent higher pressure due to larger deformation of the soft jaw pad at that point.

A. REACH: Contact Profile

Danielczuk et al. [6] proposed the Robust Efficient Area Contact Hypothesis (REACH) model for contact profile estimation between soft jaw pads and rigid objects. Given an object's geometry modeled as a triangular mesh, the contact area is computed as the constructive solid geometry intersection of the extruded polygon of the jaw with the object. The intersection estimates the deformation of the soft pad around the object at each point on the contact and the pressure distribution linearly scales with the gripper pad deformation. The REACH model thus provides the contact area, consisting of a triangular mesh, and the pressure distribution over each triangle of the contact area mesh, as illustrated in Figure 2.

We use the REACH model to estimate the profile for contact between compliant jaw pads and deformable hollow objects due to its computational efficiency compared to the Finite Element Method. We note that the obtained contact profiles may not be accurate for objects with low stiffness since the model assumes that the deformation is small such that the contact profile remains constant during the grasp.

B. 6D Ellipsoidal Limit Surface

Grasps with compliant jaws may result in a non-planar contact area, where the friction wrench applied at the contact is six-dimensional (6D). This work uses the 6D ellipsoid proposed in [33] as the limit surface model to represent the constraints on the friction wrench that can be applied at each contact. Note that other friction models are applicable to the proposed work computation algorithm as well.

We briefly summarize the algorithm to determine the 6D ellipsoid and the friction constraints. For a given contact area and pressure distribution, possible friction wrenches are obtained by sampling the instantaneous relative motion, defined as body twist in screw theory [23]. The direction of frictional force of each triangle is obtained by projecting the velocity onto the triangle plane, while the magnitude is the product of the coefficient of friction and the normal force applied at the triangle. The frictional torque is computed with respect to the friction-weighted center of pressure. By summing up the friction contributions of each triangle, we obtain a friction wrench of the contact for each sampled twist.

We then fit the sampled friction wrenches to a 6D ellipsoid by solving a convex optimization problem. Given the ellipsoid matrix $\mathbf{A} \in \mathbb{R}^{6\times 6}$, possible friction wrenches $\boldsymbol{w} \in \mathbb{R}^6$ of the contact are constrained by:

$$\boldsymbol{w}^T \mathbf{A} \boldsymbol{w} \leq 1. \tag{4}$$

For computational efficiency, we determine the linearized constraints $\mathcal{C}_i$ of the i-th contact by evenly sampling the surface of the corresponding ellipsoid at M points $\boldsymbol{p} \in \mathbb{R}^{6\times M}$. Each point and its outward normal form a hyperplane. Denoting $\boldsymbol{w}_i^{\perp} \in \mathbb{R}^6$ as the wrench impressed by the normal pressure and $\boldsymbol{n}_i$ as the normals of the ellipsoid represented by $\mathbf{A}_i$ at the points $\boldsymbol{p}_i$, the friction wrench of the i-th contact is constrained to the interior of the M hyperplanes:

$$\begin{gathered}\mathcal{C}_i = \{\boldsymbol{w}_i \in \mathbb{R}^6 \mid \boldsymbol{n}_i \cdot (\boldsymbol{w}_i - \boldsymbol{w}_i^{\perp}) \leq \boldsymbol{n}_i \cdot \boldsymbol{p}_i\}, \\ \text{with } \boldsymbol{n}_i = \mathbf{A}_i \boldsymbol{p}_i.\end{gathered} \tag{5}$$

We compute the minimal grasp force required to resist the target wrench $\boldsymbol{t}$ by substituting Equation (5) into (1).

VI. Experiments and Results

We describe the object's stiffness acquisition required to compute work. We further show planned grasps in simulation and physical grasp prediction results with the proposed minimal work quality metric compared to two baseline metrics.

A. Acquisition of Object Stiffness

We estimate the object's stiffness using its 3D mesh and physical experiments. One can use the Finite Element Analysis to compute the object's deformation with a closing force of the gripper. However, the stiffness of hollow objects such as plastic bottles and cups highly depends on the wall thickness, the geometry, and material of the object, which are non-trivial to simulate. Therefore, we use a physical robot to collect object's stiffness at different locations in this work.

We estimate the object's stiffness based on 1) a known gripper closing force F_c, 2) the gripper opening L_s when it first makes contact with the object, and 3) the gripper opening L_e when F_c is reached. We first plan antipodal grasps in simulation for each object. The minimal distance between the 3D object mesh and the gripper jaw along the grasp axis provides an estimation of L_s.

At each planned grasp location, the Robotiq 2F-85 gripper closes with a minimal possible force $F_c = 20N$. The object's stiffness s_i at the location i, the intersection point of the grasp axis and the object surface is $s_i = F_c/(L_{s,i} - L_{e,i})$. We repeat each grasp 5 times and select the median of the collected gripper opening after reaching F_c as $L_{e,i}$.

We note that the actual grasp force of the Robotiq gripper depends on the object's material and the gripper closing speed, so we measure the repeatability of the deformation measurements by varying the gripper closing speed for a set of grasps.

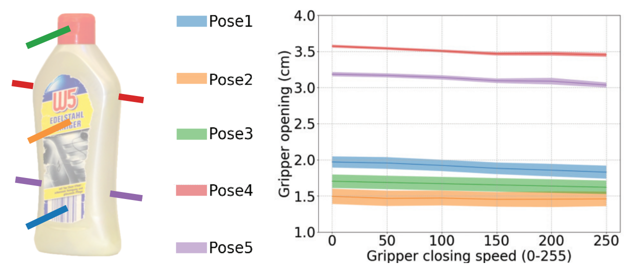

Fig. 3: Robotiq gripper repeatability test. The repeatability is high at poses 4 and 5, where the object is rigid and wide, but lower at poses 1-3, where the object is more deformable and narrower.

An object with an open lid is grasped at five locations, each with different stiffness. Figure 3 shows the mean and standard deviation of the gripper opening at each grasp location with different closing speeds ranging from 0 to 255, where 0 is the lowest speed. Although the absolute gripper opening and the collected object's stiffness measurements may not be highly accurate, the relative stiffness is reasonable, allowing for accurate comparison between grasp qualities when computing optimal grasp placements.

B. Baseline Metrics

We compare the proposed minimal work grasp quality metric with two baseline metrics.

1) Grasp reliability metric q_r: We define a binary reward R that describes the predicted grasp success. $R = 1$ if the grasp is able to resist the target wrench $\boldsymbol{t} \in \mathbb{R}^6$ without exceeding the maximum closing force and $R = 0$ otherwise. The metric addresses uncertainties in actuation by using Monte-Carlo sampling over the grasp pose similar to Dex-Net 1.0 [22]. More specifically, we add isotropic Gaussian uncertainty in gripper translation and rotation, resulting in K perturbations for each grasp pose. The grasp quality q_r of the pose is the average reward over the K perturbations:

$$q_r = \frac{1}{K} \sum_{k=1}^{K} R_k.$$

2) Minimal force metric q_f: The grasp quality is the averaged minimal required grasp force F to resist $\boldsymbol{t}$ given the maximal force limit $F_{\max}$:

$$q_f = 1 - \frac{F}{F_{\max}},$$

where $F_{\max}$ is determined experimentally.

C. Grasp Planning in Simulation

We plan grasps on five objects based on their 3D meshes and the interpolated stiffness maps, as shown in Figure 4(a) and (b), where two objects are 3D printed using NinjaFlex TPU flexible filament to provide diversity in object shape and material. Note that some inaccuracies of the measured stiffness exist, such as the rigid cap of the object 1 and the neck of object 2.

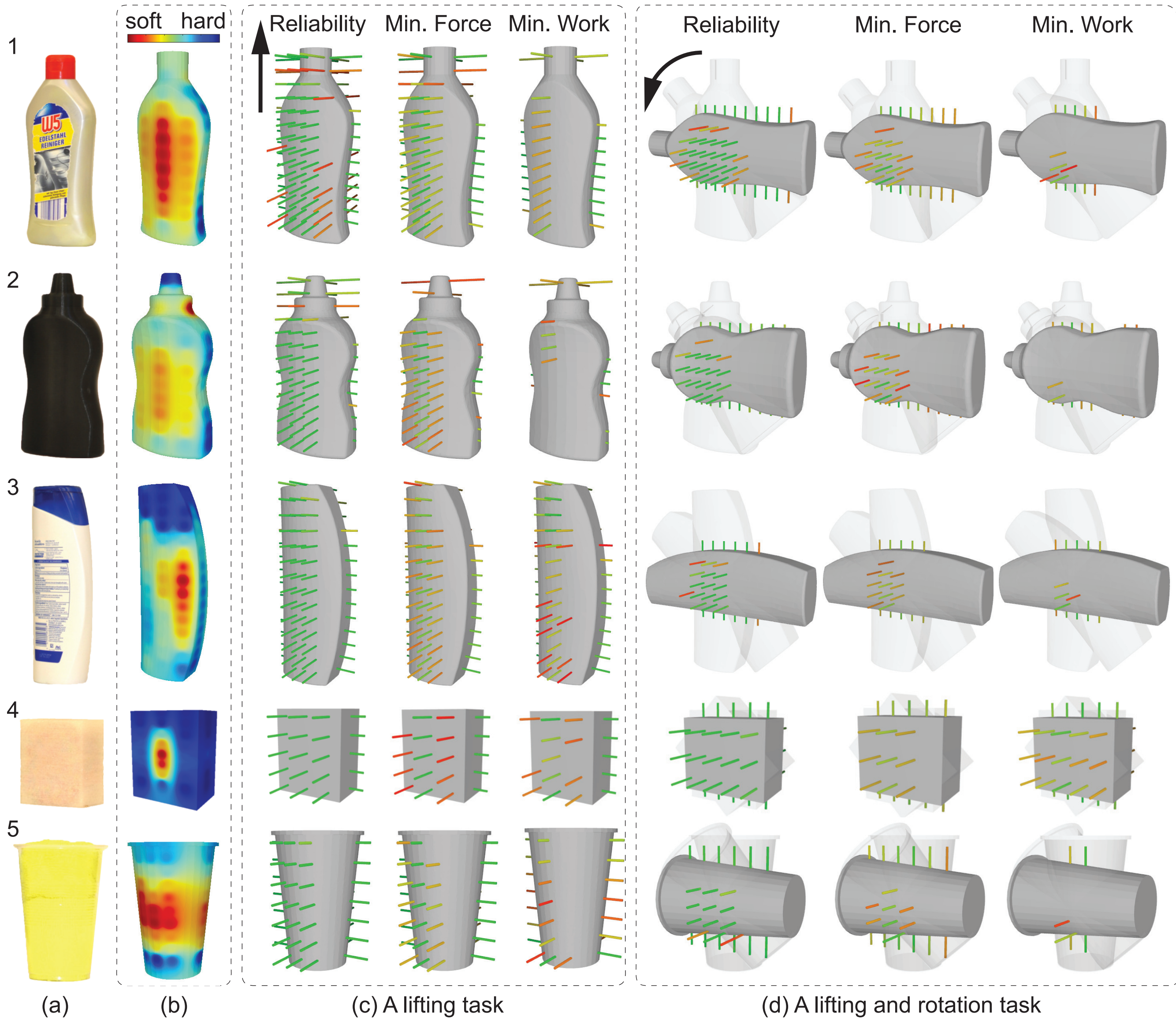

Fig. 4: Planned grasps for five physical objects with three metrics: grasp reliability, minimal force, and the proposed minimal work. Objects 2 and 4 are 3D printed using NinjaFlex TPU flexible filament. (a) The five objects used in simulated and physical experiments. (b) The interpolated stiffness map for each object, where red is low stiffness and blue is high stiffness. (c-d) The planned grasps for a lifting task and a lifting and 90° rotation task, respectively. Each colored line represents a grasp axis for the parallel-jaw gripper, and the color indicates quality according to the given metric. Red indicates low quality, while green is high quality. While the grasp reliability and the minimal force metrics only consider wrench resistance, the proposed minimal work metric computes grasps that resist gravitational disturbances without causing large deformations.

We sample antipodal grasp candidates and compute grasp quality for 1) vertical lifting and 2) lifting and 90° rotation tasks. Three quality metrics are compared: grasp reliability, minimal force, and the proposed minimal work, as shown in Figure 4(c) and (d). We model the two tasks with a 6D gravity wrench to be resisted under one and three object poses obtained by discretizing the manipulation trajectory, respectively, since the gravity wrench remains the same for the vertical lifting task. The lowest quality value of a grasp among all object poses is selected as the value for each metric. The colored lines represent the grasp axes; green indicates high quality under the given metric, while red indicates low quality. Figure 4 suggests that the planned grasps using the proposed minimal work quality metric avoid causing large deformations of the object while resisting the gravitational disturbances of the manipulation tasks.

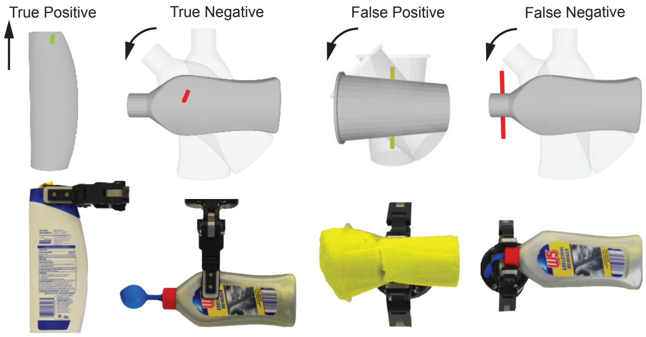

Fig. 5: Examples of grasp quality predictions with the minimal work quality metric. A grasp is considered successful if the manipulation task succeeded while the content is not dislodged and the object returns to its original shape after grasping. An inflated balloon suggests liquids in the container might have been dislodged.

D. *Physical Experiments*

We evaluate the planned grasps for three representative objects (objects 1, 3, and 5) with physical experiments for the two manipulation tasks. We select 46 grasp poses in total for the three objects that cover different regions of each object. Each grasp is repeated five times for each task, resulting in 460 total grasps. We consider a grasp to be successful if 1) the task is completed, 2) the object returns to its original shape when the grasp force is released, and 3) the content is not dislodged during the grasp.

We filled the objects with wet towels to simulate the mass of the object filled with liquid without changing the object's stiffness or damaging the electrical devices. Object 1 and 3 are sealed with a balloon to infer the content spillage. By measuring the balloon's inflation before and after the grasp, the content is considered spilled if the inflation difference is larger than a threshold.

We use balanced accuracy, or the accuracy weighted by the number of successful and failed grasps in the collected data, to evaluate the prediction accuracy. Each metric's grasp quality prediction is binarized by thresholding the quality at a threshold $\sigma = 0.5$. Table I shows the balanced accuracy of the three grasp quality metrics for the two manipulation tasks and Figure 5 shows examples of correct and incorrect predictions for the planned minimal work grasps. The proposed metric reaches 74.2% and 71.3% balanced accuracy, respectively, for the two tasks, up to 24.2% higher than the grasp reliability and the minimal force metrics. However, we note that the balanced accuracy for the minimal work quality metric is relatively low for object 5 (a plastic cup) compared to other objects. This suggests that the proposed algorithm for minimal work computation may not perform well for objects with large deformations. Furthermore, the minimum grasp force with the Robotiq Gripper can be much larger than the planned force and can cause large object deformations and false positives.

TABLE I: Balanced accuracy of three quality metrics: grasp reliability q_r, minimal force q_f, and the proposed minimal work q_w.

Object	Vertical lifting			Lifting and 90° rotation		
	q_r	q_f	q_w	q_r	q_f	q_w
1	0.500	0.767	**0.833**	0.575	0.785	**0.875**
3	0.500	0.687	**0.851**	0.516	0.714	**0.813**
5	0.500	0.392	**0.542**	**0.680**	0.630	0.450
All	0.500	0.615	**0.742**	0.590	0.710	**0.713**

VII. Discussion and Future Work

This paper proposes a minimal work quality metric to plan grasps for 3D deformable hollow objects. We evaluate the proposed metric with real-world grasps for a vertical lifting and for a lifting and 90° rotation task. Physical experiments suggest that 74.2% and 71.3% balanced accuracy can be achieved for the two tasks, respectively, up to 24.2% higher than classical wrench-based quality metrics.

A. *Limitations*

We note that the proposed method may not perform well for objects having large deformations due to the linear stiffness assumption and the simplified model to acquire contact profiles. To address this, one can simulate the contact with the Finite Element Method and fit a strain-stress curve for each grasp location by applying different loads. The algorithms then use the obtained pressure distribution and the deformed object shape to compute the grasp quality.

Furthermore, one reason for predicted false positives is that the actual minimal grasp force of the Robotiq gripper is higher than specified in the planned grasps. This issue can be addressed by mounting force sensors on the gripper to execute grasps with the desired grasp force.

B. *Future Work*

We note that the minimal work grasp quality metric is also applicable to grasps on rigid objects with compliant gripper jaws. We intend to further investigate this duality and plan grasps for both rigid and deformable objects.

VIII. Acknowledgments

This research was performed at the AUTOLAB at UC Berkeley in affiliation with the Berkeley AI Research (BAIR) Lab. The authors were supported in part by the NSF National Robotics Initiative Award 1734633, National Science Foundation Graduate Research Fellowship Program under Grant No. 1752814, and by donations from Google, Siemens, Amazon Robotics, Toyota Research Institute, Autodesk, ABB, Samsung, Knapp, Loccioni, Honda, Intel, Comcast, Cisco, Hewlett-Packard and by equipment grants from PhotoNeo and NVIDIA. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Sponsors. We thank our colleagues Prof. Grace Gu and Kahraman Demir for 3D printing, and who helped with experiments and provided helpful feedback and suggestions, especially William Wong, Priya Sundaresan, Harry Zhang, Jackson Chui, Kate Sanders.

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