1. Young MBA Erica Cudahy may invest up to $1,000. She can invest her money in stocks and loans. Each dollar invested in stocks yields 10 cent profit, and each dollar invested in a loan yields 5 cent profit. At least 30% of all money invested must be in stocks, and at least $400 must be in loans. Formulate an LP that can be used to maximize total profit earned from Erica's investment. Then graphically solve the LP.

2. Daisy drugs manufactures two drugs: 1 and 2. The drugs are produced by blending two chemicals: 1 and 2. By weight, drug 1 must contain at least 65% chemical 1, and drug 2 must contain at least 55% chemical 1. Drug 1 sells for $6 per ounce and drug 2 sells for $4 per ounce. Chemicals 1 and 2 can be produced using either of two production processes. Running process 1 for an hour requires 3 oz of raw material and 2 hours skilled labor and yields 3 oz of each chemical. Running process 2 for an hour requires 2 oz of raw material and 3 hours of skilled labor and yields 3 oz of chemical 1 and 1 oz of chemical 2. A total of 120 hours of skilled labor and 100 oz of raw material are available. Formulate an LP that can be used to maximize Daisy's sales revenues.

3. Consider the following Optimal Paper Recycling problem.

   A company produces 5 types of paper products:

   1   Newsprint.
   2   Wall paper.
   3   Coated paper.
   4   Uncoated book paper.
   5   Writing and related paper.

   For each type, j, of paper, the demand, d_j is given in thousands of tons (see table below). The company meets the demand for its products in two ways. It can manufacture new paper from fresh wood and it can also recycle old paper. A certain percentage, m_j, of the demand for paper type j must be satisfied with paper produced from fresh wood, but the rest can come from recycling. For example, all of the newsprint can come from recycled paper, but at least 47% of the wall paper must come from fresh wood.

   When paper is recycled, there is some loss of material. As a result, x tons of recycled paper produce fewer than x tons of new paper. The actual amount, $t_j x$, depends on the type of new paper being produced. For example, 100 tons of old wall paper can be recycled into 85 tons of newsprint.

   In the table below, the s_j column indicates the amount of old paper (in thousands of tons) of type j that is available for recycling. Note that a given type of recycled paper can only be used to produce certain types of new paper. This information is given in the last column of the table.

j	d_j	m_j	s_j	t_j	Can be used to make
1	3,475	0%	4000	0.85	1,2
2	1,223	47%	1600	0.90	1,2,3,4
3	2,260	50%	1000	0.85	2,3,4,5
4	2,700	40%	990	0.85	2,3,5
5	2,950	30%	2800	0.90	5

For ecological and economic reasons, the company wishes to use as little fresh wood as possible. Formulate a linear program to solve the problem of producing enough of each product to meet the demand in such a way as to minimize the use of fresh wood.

Define all variables, state clearly all assumptions, and briefly justify each of the constraints and the objective function.

4. A company produces two kinds of products. A product of the first type requires 1/4 hours of assembly labor, 1/8 hours of testing, and \$1.20 worth of raw materials. A product of the second type requires 1/3 hours of assembly, 1/3 hours of testing, and \$0.90 worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing, each day. Products of the first and second type have a market value of \$9 and \$8, respectively. You may assume demand for both products is unlimited.

   (a) Formulate a linear programming problem that can be used to maximize the daily profit of the company.

   (b) Consider the following modifications of the problem:

       (1) Suppose that up to 50 hours of overtime assembly labor can be scheduled, at a cost of \$7 per hour.

       (2) Suppose that the raw material supplier provides a 10% discount if the daily bill is above \$300.

       For each of the two modifications, specify if it can be done with a linear program, or if it requires integer variables. Specifically, show how you would add these modifications to the formulation in both cases. Please specify all new variables and all new constraints in your answer.

5. You are interested in solving a **continuous** knapsack problem, **where you need to select which items out of four (fraction of any item is allowed) to pack your bag. Specifically, you** have 4 different types of products you can put in your bag, each of which has a related value, weight and volume. You also are required to carry a minimum of amount of water. The optimization model is:

$$\max 1x_1 + 4x_2 + 3x_3 + 4x_4$$

$$\text{s.t. } 3x_1 + 2x_2 + x_3 + 2x_4 \leq 100 \text{ (Volume of the bag)}$$

$$2x_1 + 3x_2 + 2x_3 + 3x_4 \leq 50 \text{ (Weight of the bag)}$$
$$1x_1 + 0x_2 + 0x_3 + 0x_4 \geq 3 \text{ (Minimum amount of water)}$$
$$x_1, \quad x_2, \quad x_3, \quad x_4 \geq 0$$

You're going to present to a class the results using CPLEX in sensitivity form, but you realize that the ink that you use was really bad and the document is not completely readable. In fact you were only able to recover all the data that does not have an '?' sign[*] (the parameters names have been change to be readable for anyone even if they haven't used the sensitivity report of AMPL using CPLEX). Here is the output:

CPLEX 12.6.1.0: optimal solution; objective function value ? e)

Variables Sensitivity

Variable Number	x_j^*	Reduced Cost	Obj. Fn. Coeff.	Lower Bound	Upper Bound
1	? e)	0	1	-1e+20	3
2	0	? f)	4	-1e+20	4.5
3	22	0	3	2.66667	? b)
4	? c)	-0.5	4	-1e+20	4.5

Constraints Sensitivity

:	Constraint	Shadow Price	Slack	RHS	Lower Bound	Upper Bound
1	volume	0	69	100	? a)	? a)
2	weight	1.5	0	50	6	188
3	water	?2 d)	0	3	0	0

In the Constraint sensitivity table 'RHS' stands for "right hand side". Answer **and provide justification for** the following questions:

(a) What is the Upper and Lower Bound for the Volume constraint?

(b) What is the Upper bound for the objective function coefficient related to the third variable?

(c) What is the value of x_4^*?

(d) Is the shadow price (optimal dual variable) related to the water constraint positive or negative (is it $+2$ or -2)?

(e) What's the optimal function value and what's the value of x_1^*?

(f) What is the reduced cost of the second variable?

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[*]Next to the '?' there is a reference to the appropriate question below.

(g) Does the optimal solution or optimal objective function change if you find a bag with 50 units less of volume and the same weight capacity?

(h) How does the optimal objective function change if we are allowed to carry 100 more units of weight?

6. You're choosing decorations and candies for your house to make it a cool place for Halloween. You visited the Awazon webpage and made a detailed list of five products and a table specifying the minimum and maximum amount of each that you can purchase, the cost of the product, its spookiness quality (decoration quality) and its sweetness quality (sugar level). The table is shown below:

Product	MinPurchase	MaxPurchase	Price	Spooky Quality	Sweetness Quality
Lights A	0	2	$15	14	0
Lights B	0	2	$20	25	0
Spiderwebs	0	1	$30	30	0
Skittles (Candy)	0	100	$1	0	1
Skull Candy	15	100	$1.5	0.5	1

You have set the following constraints:

- You can spend at most $200.

- Your house is required to have a minimum spookiness level of 60 and a minimum sweetness level of 70.

- You should buy at least two sets of lights but no more than three. To make the decoration coherent, if you buy two sets of lights, the two of them should be of the same type, and if you buy three sets of lights then the three of them can not all be of the same type.

- If you buy three sets of lights then you won't have space for the spiderwebs.

- You can assume that the amount of skittles and skull candies you buy can be be expressed by continuous variables.

- Of the total amount of candies you purchase, at most 70% of them can be of the same type.

- You had a family conversation and decided that each unit of spookiness gives the house 1.5 units of utility, each unit of sweetness gives a 1.0 unit of utility, and that you don't care if you spend all the budget or not.

Write an MILP (Mixed Integer Linear Program) formulation that could solve the problem of maximizing the total utility units while satisfying all the constraints above. Clearly define any variables and parameters you use and explain the purpose of each constraint.

7. You are going to the racetrack to bet on horses! There are six horses available, and you may choose as many or as few as you wish to bet on. If you bet on a horse to win the race, your bet amount must be exactly $1. Additionally, you have a heart condition, and the doctor has told you that you may not tolerate too much excitement. The horses available to be bet on, their expected payoffs for a $1 bet, and the excitement that comes with them, are as follows:

Horse	Expected Payoff	Excitement Level
Magnificent Miler	3.25	17
Screaming Yellow Zonkers	2.5	20
Zippity Doo Dah	1.5	12
Pokey	0.75	1
The Grim Streaker	1	9
Sir Kicks A Lot	1.9	15

Additionally, you must obey the following rules:

(1) You may only bet on the Magnificent Miler if you bet on Pokey too.

(2) You may bet on either Zippity Doo Dah or the Grim Streaker, but not both. You are allowed to bet on neither of them.

(3) You must bet on at least two of the following three: Pokey, The Grim Streaker, and Sir Kicks A Lot.

(4) You may not exceed your budget of $6.00.

(5) Finally, your doctor has told you not to exceed an excitement threshold of 40.

You want to maximize your expected payoff from bets you place on the horses, while still obeying the rules above. Formulate this as an integer programming problem. Be sure to define clearly all the decision variables used.

8. For each of the following statements, determine whether it is true or false. Justify your answer by providing either a short proof or an example as appropriate.
(**No explanation = no credit**)

(a) If the dual of the linear problem (P) has an optimal solution, it is possible for (P) to be be unbounded.

(b) A linear program can be both infeasible and unbounded.

(c) Suppose that linear programs (P1) and (P2) have identical objectives and identical sets of constraints but different right-hand-side values. Then, it is possible that (P1) has an optimal solution and (P2) is unbounded.

(d) All linear programs have either no optimal solutions, exactly one optimal solution, or infinitely many optimal solutions.

(e) Given a minimization integer linear program, the optimal objective function value of the integer solution will always be less than or equal to the optimal objective function value of the relaxed problem (that is, when the integer requirement is removed).

(f) Suppose that the shadow price (optimal dual variable) associated with a certain constraint is 0. Then, regardless of how much we increase the right hand side value of this constraint, the optimal objective function value remains unchanged.