Errata for the paper entitled

"Second-order stagewise backpropagation for Hessian-matrix analyses and investigation of negative curvature"

by Eiji Mizutani and Stuart E. Dreyfus.

Appeared in *Neural Networks*, pages 193–203, Vol.21, 2008.
— 2008 Special Issue: Advances in Neural Networks Research: IJCNN'07.

As of **July 27, 2008**.

(1) **Page 194**: In Eq.(4), $\boldsymbol{\theta}^s$ should be $\boldsymbol{\theta}^{s,s+1}$, and add "=" (notation for being equal to) between $\mathbf{G}^s$ and $\mathbf{y}^s_+ \boldsymbol{\delta}^{s+1^T}$.

(2) **Page 195**: Eq.(5) should be expressed for recursion as

$$\underbrace{\mathbf{Z}^s}_{P_s \times P_s} = \underbrace{\mathbf{N}^{s,s+1^T}}_{P_s \times P_{s+1}} \underbrace{\mathbf{Z}^{s+1}}_{P_{s+1} \times P_{s+1}} \underbrace{\mathbf{N}^{s,s+1}}_{P_{s+1} \times P_s} + \underbrace{\left\langle \left[\frac{\partial^2 \mathbf{y}^s}{\partial \mathbf{x}^s \partial \mathbf{x}^s} \right], \boldsymbol{\xi}^s \right\rangle}_{P_s \times P_s},$$

where all the notations are defined in the paper except a matrix $\mathbf{N}$ below

$$\underbrace{\mathbf{N}^{s,s+1}}_{P_{s+1} \times P_s} \stackrel{\text{def}}{=} \left[\frac{\partial \mathbf{x}^{s+1}}{\partial \mathbf{x}^s} \right] = \boldsymbol{\Theta}^{s,s+1}_{\text{void}} \left[\frac{\partial \mathbf{y}^s}{\partial \mathbf{x}^s} \right].$$

The above recurrence relation is the same as Eq.(13) in Mizutani, Dreyfus, & Demmel (2005); see also Eq.(27) in Mizutani & Dreyfus, 2006.

(3) **Page 195**: In Eqs.(10) and (11), $\boldsymbol{\theta}^s$ should be $\boldsymbol{\theta}^{s,s+1}$; likewise, $\boldsymbol{\theta}^r$ should be $\boldsymbol{\theta}^{r,r+1}$.

(4) **Page 197**: Just above Eq.(14), the sentence

> In general, the residual Hessian matrix $\mathbf{S}$ in $\mathbf{H}$ can separate into two types of blocks

should read

> In general, the residual Hessian matrix $\mathbf{S}$ in $\mathbf{H}$ always includes two types of blocks

(5) **Page 197**: Eq.(16) should have included an additional matrix $\mathbf{T}$ for a general expression as

$$\mathbf{S} = \begin{bmatrix} \underbrace{\mathbf{V}^3}_{n_3 \times n_3} & & \\ & \underbrace{\mathbf{V}^2}_{n_2 \times n_2} & \\ & & \underbrace{\mathbf{V}^1}_{n_1 \times n_1} \end{bmatrix} + \begin{bmatrix} & \boldsymbol{\Gamma}^{2,3^T} & \boldsymbol{\Gamma}^{1,3^T} \\ \underbrace{\boldsymbol{\Gamma}^{2,3}}_{n_2 \times n_3} & & \boldsymbol{\Gamma}^{1,2^T} \\ \underbrace{\boldsymbol{\Gamma}^{1,3}}_{n_1 \times n_3} & \underbrace{\boldsymbol{\Gamma}^{1,2}}_{n_1 \times n_2} & \end{bmatrix} + \mathbf{T},$$

where $\mathbf{T}$ denotes all the remaining terms; see Eqs.(22) and (23) in Mizutani, Dreyfus, & Demmel (2005) for such terms that construct $\mathbf{T}$. See also Mizutani 2008.

(6) **Page 201**: In Section 5, the second sentence

> In MLP-learning, special sparsity structure inevitably arises in $\mathbf{S}$,

should read

> In single-hidden-layer linear-output MLP-learning, special sparsity structure inevitably arises in $\mathbf{S}$ (e.g., see $\mathbf{S}$ in Proof of Lemma 2),

References:

Mizutani, E., Dreyfus, S. E., & Demmel, J. W. (2005). Second-order backpropagation algorithms for a stagewise-partitioned separable Hessian matrix. In *Proceedings of the INNS-IEEE International Joint Conference on Neural Networks* (pages 1027–1032) Vol.2.

Mizutani, E., & Dreyfus, S.E. (2006). On derivation of stagewise second-order backpropagation by invariant imbedding for multi-stage neural-network learning. In *Proceedings of the IEEE-INNS International Joint Conf. on Neural Networks (IJCNN'06)* (pages 4762–4769).

Mizutani, E. (2008). A tutorial on stagewise backpropagation for efficient gradient and Hessian evaluations. To appear in *Proceedings of the 2008 Joint 4th International Conference on Soft Computing and Intelligent Systems and 9th International Symposium on advanced Intelligent Systems (SCIS & ISIS 2008)*, (invited paper), Nagoaya, JAPAN, 2008.

Author information:

- **Eiji Mizutani** (e-mail: eiji@mail.ntust.edu.tw) is with Department of Industrial Management, National Taiwan University of Science & Technology, Taipei, TAIWAN.
- **Stuart Dreyfus** (e-mail: dreyfus@ieor.berkeley.edu) is with Department of Industrial Engineering & Operations Research, University of California, Berkeley, CA USA.