

IEOR 250
Assignment #5
Due 12/4/07

1. Prove that if $g_1(y)$ and $g_2(y)$ are K_1 -convex and K_2 -convex respectively, then for $\alpha, \beta > 0$, $\alpha g_1(y) + \beta g_2(y)$ is $(\alpha K_1 + \beta K_2)$ -convex.
2. Consider the P-Median model we discussed in class. In many cases, we not only want to minimize the sum over all demand nodes of the demand-weighted average distance between a demand node and its nearest facility, but we also want to be sure that each demand node has a facility located within at most D_f distance units of it.
 - (a) Define any additional notation that you need and show how this can be handled by adding a constraint to the formulation shown above.
 - (b) Explain how instead of adding a constraint, you can deal with this problem by changing certain data inputs to the problem.
 - (c) Briefly discuss how instead you can solve the problem by eliminating certain variables from the formulation before you begin to solve the problem.
 - (d) Now suppose that we also want to be sure that each demand node has a second alternative facility within at most $D_s > D_f$ distance units of it. This substitute facility will serve the demand node in the case that the primary facility of that demand node goes out of commission. Note that, obviously, a facility cannot be both the primary and the substitute facility of the same demand node. Also assume that a facility can serve as a primary facility to at most F nodes and can be assigned as a substitute facility to at most S nodes. Define any additional notation that you need and add the necessary constraints to the P-median formulation to model this situation.
3. The set covering location model we discussed in class is listed below. Recall that variables $x_j, j = 1, 2, \dots, J$ are set to one if a facility is built at site j , and to zero otherwise. Each potential site has a fixed cost f_j that is incurred if a facility is built at that site. Furthermore, there are I customers that need to be “covered” by built facilities. The parameter a_{ij} equals one if customer i can be covered by a facility built at site j , and zero otherwise. The model follows:

$$\begin{aligned} \text{(SC)} \quad & \min \sum_{i=1}^I \sum_{j=1}^J f_j x_j \\ & \text{s.t.} \\ & \sum_{j=1}^J a_{ij} x_j \geq 1 \quad i = 1, 2, \dots, I \quad (1) \\ & x_j \in \{0, 1\} \quad j = 1, 2, \dots, J \quad (2) \end{aligned}$$

This model is often extended to secondary objectives. There are often many alternate optima for a given set covering location model, so these extensions often serve to select “the best” alternate optimal solution according to some secondary criteria.

Suppose that in each of the newly built facilities must be supplied by one of a set of K plants. Each week, a truck goes from one of the plants to each of the newly built facilities, and returns to the plant (facilities are visited one at a time). You are given a distance matrix $d_{jk}, j = 1, 2, \dots, J, k = 1, 2, \dots, K$ and the per unit distance cost of operating the truck α .

- (a) Extend the set covering location model given above so that **among those solutions that minimize the cost covering all of the customers**, the optimal solution that minimizes the weekly cost of supplying the newly built facilities is selected. Clearly define additional notation used (if any), and explain the constraints and objective.
 - (b) For the model in this question, relax the appropriate set of constraints, and formulate the Lagrangian problem related to this model. (Choose a set of constraints to relax so that the sub-problem is easy to solve.)
 - (c) State how you would solve this Lagrangian problem for a given set of multipliers.
 - (d) For a given set of multipliers, will the set of locations in the optimal solution to the Lagrangian problem always be feasible for the original problem? Why or why not?
4. Consider the n -city TSP in which the triangle inequality holds. Let $c^* > 0$ be the length of an optimal tour, and let c' be the length of the second best tour. Prove $(c' - c^*)/c^* \leq \frac{2}{n}$.