

IEOR 250  
Assignment #3  
Due 10/16/07

1. The buyer for Needless Markup must decide on the quantity of a high-priced woman's handbag to procure in Italy for the following Christmas season. The unit cost of the handbag to the store is \$28.50 and the handbag will sell for \$150.00. Any handbags not sold by the end of the season are purchased by a discount firm for \$20.00. In addition, the store accountants estimate that there is a cost of \$0.40 for each dollar tied up in inventory, as this dollar invested elsewhere could have yielded a gross profit. Assume that this cost is attached to unsold bags only.
  - (a) Suppose the sales of the bags are equally likely to be anywhere from 50 to 250 bags during the season. Based on this, how many bags should the buyer purchase?
  - (b) A detailed analysis of past data shows that the number of bags sold is better described by a normal distribution, with mean 150 and standard deviation 20. Now what is the optimal number of bags to purchase?
  - (c) The expected demand was the same in the previous parts, but the optimal order quantities should have been different. What accounted for this difference?
2. Weiss's paint store uses a (Q,R) inventory system to control its stock levels. For a particularly popular white paint, historical data shows that the distribution of monthly demand is approximately normal, with mean 28 and standard deviation 8. Replenishment lead time for this paint is about 14 weeks. Each can of paint costs the store \$6. Although excess demands are back-ordered, the store owner estimates that unfilled demands cost about \$10 each in bookkeeping and loss-of-goodwill costs. Fixed costs of replenishment are \$15 per order, and holding costs are based on a 30 percent annual rate of interest.
  - (a) What are the optimal lot sizes and reorder points for this brand of paint?
  - (b) What is the optimal safety stock for this paint?
3. After taking a production seminar, Al Weiss, the owner of the paint store mentioned in the previous question, decides that his stock-out cost of \$10 may not be very accurate, and switches to a service level model. He decides to set his lot size by the EOQ formula, and determines his reorder point so that there is no stock-out in 90 percent of the order cycles.
  - (a) Find the resulting (Q,R) values.
  - (b) Suppose that, unfortunately, he really wanted to satisfy 90 percent of his demands (that is, achieve a 90 percent fill rate). What fill rate did he actually achieve from the policy determined in part (a)?
4. Prove that in the single item capacitated lot sizing model with time varying demands (the Florian and Klein model), an optimal solution satisfies the property  $y_t I_{t-1} (C_t - y_t) = 0$  for every period  $t$ , where  $y_t$  is the order quantity in period  $t$ ,  $C_t$  is the production capacity in period  $t$ , and  $I_{t-1}$  is the inventory at the end of period  $t - 1$ .

5. Consider a firm that manufactures a product  $A$  at cost  $c_A$ , and faces a single period random demand  $D$  with density function  $f(D)$  and CDF  $F(D)$  for the product. The firm sells the product for price  $p_1 > c_A$  per unit, and if the firm runs out of product demand is lost. If the firm has extra product, the product is disposed of at no additional cost. The firm can also (or in addition) produce another version of the product  $B$  at cost  $c_B > c_A$  to meet the same demand at the same price  $p_1$ . The advantage of this product is that the firm has entered into an agreement with an overseas distributor in which the distributor is willing to pay the firm  $p_2$ ,  $p_1 > p_2 > c_B$  per unit for up to  $K$  units of the product that the firm might have left over at the end of the selling period.
- (a) Write an expression for the firm's total expected profit in this situation.
  - (b) Derive an expression for the optimal amount of product  $B$  the firm should manufacture.
  - (c) Write the necessary condition for the optimal amount of product  $A$  that the firm should manufacture.