IEOR 250
Assignment #2
Due 10/2/07

1. A manufacturer is considering getting into the sausage making business. If the manufacturer in fact enters the business, he anticipates facing constant deterministic demand for sausages at rate D. For the purpose of this analysis, sausage making has two steps: a continuous extruding step, in which raw sausage is produced at rate $\alpha > 0$, and a batch cooking step, in which up to C units of sausage are cooked simultaneously for time G. The manufacturer already owns the sausage cooker, and knows that each time it is used (independent of how full it is), C_c dollars worth of energy are consumed. The manufacturer does not own the extruder, and intends to lease an extruder if he goes into business. The total leasing and operating costs of the extruder are a function of the production rate: the manufacturer can have any production rate he wants, but the total operating cost per unit time will be $C_\alpha \alpha$. Once a sausage has been manufactured (is done cooking), the manufacturer will pay a holding cost of h per unit per unit time. Until the cooking is complete, no holding cost is paid. The sausage cooker and the extruder can operate at the same time. What rate of extruder should the manufacturer lease, and what should his production strategy be?

2. Consider a multi-item inventory model in which a company has n different types of items stored in a warehouse. For each item i, we assume constant demand D_i items per unit of time; a known and constant lead time; order cost that is a linear function of the number Q_i of items ordered, $K_i + c_i Q_i$; holding cost h_i\$ per item per unit of time; shortage not allowed; and an infinite horizon. The company wants to rent space for the warehouse, and it is known that the annual rent cost is λ dollars per square foot. Assume that each unit of item i stored in the warehouse uses α_i square feet. Our objective is to find an inventory policy for each item so as to minimize annual inventory holding costs plus the annual rent cost. The company decides to use a policy in which all orders for a specific item i, are of equal size Q_i units, and to not be concerned with the timing of orders for different products. Thus, the number of units ordered for the different types is the one minimizing total annual cost:

$$\sum_i \left[\frac{K_i D_i}{Q_i} + \frac{h_i Q_i}{2} \right] + \lambda \sum_i \alpha_i Q_i.$$

   Prove that the cost associated with this solution is at most $\sqrt{2}$ times the optimal cost.

3. Recall the Economic Lot Scheduling Problem (ELSP) we discussed in class. There are n types of products, each with demand per unit time D_i, production rate per unit time P_i, setup cost K_i, and holding cost per unit time h_i. We considered a variety of heuristics in class, including one in which we found the optimal common cycle, although I intuitively argued that a common cycle, even the optimal one, could be a very bad approach for particular problem instances.

   Find a specific example instance for which you can come up with a solution whose objective is at least 10% better than that of the optimal common cycle solution for the instance.

4. Consider a general version of the single item lot sizing model with time varying demands (the Wagner-Whiten model), where order costs are general concave and time-dependent functions

of the number of items produced, and holding costs are general concave and time-dependent functions of the number of items held in inventory. Prove that the main Wagner-Whiten theorem (the zero inventory ordering policy) holds in this general setting.

5. Consider the Wagner-Whiten model with holding cost h and fixed order cost K. Although the Wagner-Whiten problem is quite easy to solve to optimality, in practice, heuristics are often used for this problem. For example, in the Part Period Balancing heuristic, given that it is time to produce, production levels are set so that the holding cost over the number of covered periods equals the setup cost during that time. For example, let $D_1, D_2, ..., D_T$ be the demands in a T period planning horizon. Let $H(n)$ be the total holding cost under the condition that production covers demand in the next n periods. Then $H(1) = 0$, $H(2) = hD_2$, $H(3) = hD_2 + 2hD_3$, etc. In the Part Period Balancing heuristic, we calculate these until $H(i) > K$, and then we stop and produce in period 1 to meet demand in the first i or $i - 1$ periods, depending on whether $H(i)$ or $H(i-1)$ is closest to K. We then restart this procedure in period i or $i + 1$ as appropriate.

   Construct an example in which the Part Period Balancing heuristic does not find the optimal solution.