

IEOR 250  
Assignment #1  
Due 9/18/07

1. A local outdoor vegetable stand has exactly 1,000 square feet of space to display three vegetables: tomatoes, lettuce, and zucchini. Annual demand for these three vegetables in pounds is 800, 1350, and 670, respectively, and the cost per pound is \$0.29, \$0.45, and \$0.25, respectively. The setup cost for replenishment of vegetables is \$90 in each case, and the space consumed by each vegetable is proportional to its cost. The annual holding cost is 18 percent of the optimal cost.
  - (a) What are the optimal order quantities of each vegetable if all three are purchased at the same time?
  - (b) What if the vegetable stand operator is not restricted to purchasing these items at the same time. Can you find a lower cost solution?
2. Prove that at the optimal EOQ quantity, the holding cost per unit time equals the fixed cost per unit time.
3. The supplier of a product wants to discourage large quantity purchases. Suppose that all of the standard EOQ assumptions apply, except that a reverse quantity discount is given, so that if the purchase volume is less than  $q$ , the variable purchase cost is  $c_1$ , and if the purchase volume is greater than or equal to  $q$ , the variable purchase cost is  $c_2$ , where  $c_1 < c_2$ .
  - (a) Write an expression or expressions for the total cost in a year as a function of order quantity.
  - (b) Using the types of sketches we saw in class, illustrate possible positions for the optimal order quantity.
  - (c) What is the best order quantity for the following instance:  $K = 10, D = 50000, c_1 = 1, c_2 = 1.005, h = 0.2, q = 1500$ .
4. A firm buys a product from a supplier using EOQ to determine order quantity. The supplier recently called and asked the buyer to purchase in larger lots so that the supplier's production process can be made more efficient. Specifically, assume that the buyer will buy  $k$  times EOQ units, where  $k > 1$ . The per unit cost must be reduced to make this attractive to the buyer. What is the fractional discount at which the buyer will be indifferent between the old EOQ at the old price and  $k$ EOQ at the new price?
5. In the real world, it is often difficult to precisely estimate the value of various problem parameters. In class, we observed that for the standard EOQ setting, total cost is not particularly sensitive to order quantity. In this question, we will extend that analysis. Consider the Economic Lot Size Model with setup cost  $K$ , holding cost  $h$  per item per unit of time, and demand rate  $D$ , with no shortage allowed. Now, assume that estimates of all of the parameters are correct, except the estimate of the value of  $K$ . Instead, assume that this parameter has inaccurately been estimated as  $K'$ . Let TC represent the optimal average cost per unit

time if the correct  $K$  is used, and  $TC'$  represent the cost per unit time with the order quantity determined using  $K'$ .

- (a) Develop an expression relating  $\frac{TC'}{TC}$  to  $\frac{K'}{K}$ .
  - (b) Is it better to have an estimate for  $K$  that is too high or too low?
6. The Japanese concept of JIT (Just In Time) advocates reducing setup cost as much as possible. To analyze this concept, consider the Economic Lot Size model with holding cost  $h$  per item per unit of time, demand rate  $D$ , no shortage allowed, and current setup cost  $K_0$ . Suppose you can lease a new technology that allows you to reduce the setup cost from  $K_0$  to  $K$  at an annual leasing cost of  $A - B \ln K$  dollars. That is, reducing the setup cost from  $K_0$  to  $K$  will annually cost  $A - B \ln K$  dollars. We assume  $A - B \ln K_0 = 0$ , implying that using the current setup cost requires no leasing. What is the optimal setup cost? What is the optimal order quantity in this case?
7. Consider the Economic Lot Size Model we discussed in class, with setup cost  $K$ , holding cost  $h$  per item per unit of time, and demand rate  $D$ , with no shortage allowed. As we discussed, the objective is to minimize the long-run average cost:

$$TC(Q) = \frac{KD}{Q} + \frac{hQ}{2}$$

where  $Q$  is the order quantity. We proved that the optimal solution to this problem is to order the economic order quantity, which for this problem we will call  $Q^e$ :

$$Q^e = \sqrt{\frac{2KD}{h}}.$$

Now, suppose that the warehouse can order only integer multiples of  $q$  units, so for example the warehouse can order  $q$ , or  $2q$ , or  $3q$ , etc.

- (a) Prove that the optimal order quantity  $Q^*$  has the following property: There exists an integer  $m$  such that  $Q^* = mq$ , and

$$\sqrt{\frac{m-1}{m}} \leq \frac{Q^e}{Q^*} \leq \sqrt{\frac{m+1}{m}}.$$

- (b) Suppose that  $m \geq 2$ . Show that  $TC(Q^*) \leq 1.06TC(Q^e)$ .