# Cournot equilibria in two-settlement electricity markets with system contingencies

Jian Yao, Shmuel S. Oren* and Ilan Adler

Department of Industrial Engineering and Operations Research
University of California
Berkeley, CA 94720, USA
E-mail: jyao@ieor.berkeley.edu
E-mail: oren@ieor.berkeley.edu
E-mail: adler@ieor.berkeley.edu
*Corresponding author

**Abstract:** We study Nash equilibrium in two-settlement competitive electricity markets with horizontal market power, flow congestion, demand uncertainties and probabilistic system contingencies. The equilibrium is formulated as a stochastic Equilibrium Problem with Equilibrium Constraints (EPEC) in which each firm solves a stochastic Mathematical Programme with Equilibrium Constraints (MPEC). We assume a no-arbitrage relationship between the forward prices and the spot prices. We find that, with two settlements, the generation firms have incentives to commit forward contracts, which increase social surplus and decrease spot energy prices. Furthermore, these effects are amplified when the markets become less concentrated.

**Keywords:** two settlements; Cournot; equilibrium problem with equilibrium constraints.

**Reference** to this paper should be made as follows: Yao, J., Oren, S.S. and Adler, I. (2007) 'Cournot equilibria in two-settlement electricity markets with system contingencies', *Int. J. Critical Infrastructures*, Vol. 3, Nos. 1/2, pp.142–160.

**Biographical notes:** Jian Yao is a PhD candidate in the IEOR department at the University of California at Berkeley. He was Research Assistant on the project Extended Enterprise coalition for integrated Collaborative Manufacturing Systems (EECOMS), funded by NIST Advance Technology Program (ATP), and Software Engineer for Advanced Planning and Scheduling products at Oracle Corporation. He received an MS in Computer Science from the University of North Carolina, and an MS and a BS in Mechanical Engineering from Shanghai Jiao Tong University. Yao is a member of the INFORMS.

Dr. Shmuel S. Oren is Professor of IEOR at the University of California at Berkeley. He is the Berkeley Site Director of PSERC (the Power System Engineering Research Center). He published numerous articles on aspects of electricity market design and has been a consultant to various private and government organisations, including the Brazilian regulatory commission, The Alberta Energy Utility Board, the Public Utility Commission, the Polish system operator and the Public Utility Commission of Texas, where he is currently a Senior Advisor to the Market Oversight Division. He holds a BSc and MSc in Mechanical Engineering and Materials Engineering, respectively, from the Technion in Israel, and he received an MS and PhD in Engineering Economic Systems in 1972 from Stanford. Dr. Oren is a Fellow of INFORMS and the IEEE.

Dr. Ilan Adler is Professor and Chair of the IEOR department at the University of California at Berkeley. His main research activities are in the area of optimisation in general and interior point methods in particular. He has also been a consultant to various companies on large, complex optimisation problems. He holds a BA in Economics and Statistics from the Hebrew University in Israel, an MSc in Operations Research from the Technion in Israel and a PhD in Operations Research from Stanford. Dr. Adler is a member of INFORMS and the Mathematical Association of America.

## 1 Introduction

The last decade has witnessed a fundamental transformation of the electric power industry around the world, from one dominated by regulated vertically integrated monopolies to an industry where electricity is produced and traded as a commodity through competitive markets. In the USA, this transformation was pioneered in the late 1990s by California and the northeastern power pools, including the Pennsylvania-New Jersey-Maryland (PJM) Interchange, New York and New England. A recent arrival is the Electrical Reliability Council of Texas (ERCOT) market.

While there are significant differences among the many implemented and proposed market designs that vary in terms of ownership structure, level of centralisation and authority of the system operator, the primary rationale for electricity restructuring in most markets has been to reap welfare gains by supplanting regulation with competition. Both theory and experience from other formerly regulated industries suggest that these gains will include increased efficiency of short-run production, of resource allocation and of dynamic investment.

A potentially significant obstacle to these welfare gains is market power. Market power exercised by suppliers typically entails the withholding of output and an upward distortion in the market price. Market power is generally associated with various forms of economic inefficiency. Among the many proposed and implemented economic tools for mitigating market power is a multiple-settlement approach wherein forward transactions, day-ahead transactions and real-time balancing transactions are settled at different prices. The crisis in California in 2001 has drawn more attention to the role of forward markets in mitigating market power and in managing price risk in the electricity supply chain.

Theoretical analysis (Allaz, 1992; Allaz and Vila, 1993; Kamat and Oren, 2004) and empirical evidences (von der Fehr and Harbord, 1993; Green, 1999; Newbery, 1998; Powell, 1993) suggest that forward contracting and multisettlement systems reduce the incentives of sellers to manipulate spot market prices, since under a multisettlement approach, the volume of trading that can be affected by an increase in spot prices is reduced substantially. Thus, forward trading is viewed as an effective way of mitigating market power at real time. Allaz (1992) assumes a two-period market and demonstrates that, if all producers have access to a forward market, it leads to a prisoners' dilemma type of game among them. Allaz and Vila (1993) show that, as the number of forward trading period's increases, producers lose their ability to raise energy prices above their marginal cost. Kamat and Oren (2004) analyse two-settlement markets over two- or three-node networks, and extend Allaz's results to a system with uncertain transmission capacities in the spot market. It is also argued that setting prices at commitment time provides incentives for accurate forecasting and provides *ex-ante* price discovery that facilitates trading. Accurate forecasting and advanced scheduling of generation and load also improve system operation and reliability while reducing the cost of reserves to handle unexpected deviations from schedule.

While intuitively the above arguments in favour of forward trading and multisettlement systems are compelling, these studies typically ignore network effects, flow congestion, generator outages and other system contingencies. When flow congestion, system contingencies and demand uncertainties are all present in the spot market, it is not clear to what extent producers are willing to engage in forward transactions, or how their incentives will be affected. Furthermore, it is not well understood whether forward trading may in fact help producers exercise market power in the spot market to lock in or even increase their oligopoly rents. If indeed forward trading can be used to mitigate the exercise of market power but generators have little incentive to engage in such trading, a natural public policy question would be whether forward contracting should be imposed as a regulatory requirement and the market be designed to minimise spot transactions. As a matter of fact, the current market rules in California and in Texas are designed to limit the scope of the real-time balancing markets through penalties or added charges.

In this paper, we formulate the two-settlement competitive electricity markets as a two-period game, and its equilibrium as a subgame-perfect Nash equilibrium (see Fudenberg and Tirole, 1991) expressed as an Equilibrium Problem with Equilibrium Constraints (EPEC), in which each firm faces a Mathematical Programme with Equilibrium Constraints (MPEC, see Luo *et al.*, 1996; Karush, 1939), parametric on other firms' forward commitments. We apply the model to an IEEE 24-bus test network. With the specific data and simplifying assumptions of the example, it is shown that in equilibrium, firms commit certain quantities in forward transactions and adjust their positions in the spot market in responding to contingencies and demand realisation. While this paper covers much of the same ground and employs the same modelling framework as Yao *et al.* (2004), it extends the preliminary work reported in Yao *et al.* (2004) in both content and detail. Furthermore, the preliminary computational tests reported in Yao *et al.* (2004) for a stylised six-bus network have been extended to a more realistic IEEE 24-bus network, which demonstrates the computational feasibility of the proposed model.

The rest of this paper is organised as follows. The next section presents the model assumptions and the mathematical formulation. An example and numerical results are discussed in Section 3.

## 2 The model

### *2.1 Modelling approach*

We shall describe now our model for calculating the equilibrium quantities and prices of electricity over a given network with two settlements. We view the two settlements in the electricity market as a Nash-Cournot game with two periods: a forward market (Period 1), and a spot market (Period 2).

In Period 1, firms enter into forward contracts by competing in a Cournot fashion, anticipating one another's forward commitments and the common knowledge of the expected spot market outcome in Period 2. In the spot market, the uncertain contingencies are realised and the generation firms act as Cournot competitors, choosing their spot production quantities for the generation units. In doing so, they take as given the revealed forward commitments of all other generation firms, the conjectured spot production decisions of all other generators, and the redispatch decisions of the Independent System Operator (ISO) specifying the import/export quantity at each node. Simultaneously with the generators' production decisions, the ISO makes its redispatch decision, determining imports and exports at each node so as to maximise total social welfare based on its conjectured spot production at each node, the transmission constraints and the energy balance constraint.

Our model permits different levels of locational granularity in the forward and spot markets. Specifically, we will assume that in the forward market nodes are clustered into zones and firms enter into forward contracts, which specify forward zonal quantity commitments at agreed-upon zonal prices. Another key assumption underlying our formulation is that the forward market is sufficiently liquid, so that the forward price in each zone is uniform across all firms operating in the zone while the forward commitments are public knowledge in the spot market.

All forward contracts are settled financially in the spot market based on the difference between the forward zonal price and the spot zonal price, which is a weighted average of all spot nodal prices in the zone. The weights used in determining the spot zonal prices are constants that reflect historical load shares but are not endogenously determined based on actual load shares in the spot market. We also assume that risk-neutral speculators take opposite positions to the generation firms and exploit any arbitrage opportunities so that the forward price in a zone equals the corresponding expected spot zonal prices over all possible contingencies (no arbitrage assumption).

The available capacities of generation units and transmission lines in the spot market are unknown in Period 1 and are subject to stochastic variations in Period 2. We model the transmission network constraints in the spot market in terms of a lossless DC approximation of Kirchhoff's laws. Specifically, flows on lines can be calculated using Power Transfer Distribution Factors (PTDFs), which specify the proportion of flow on any particular line resulting from an injection of one unit at a particular node and a corresponding withdrawal at an arbitrary, but fixed, 'slack bus' (Chao and Peck, 1996). Uncertainty regarding the realised network topology in the spot market is characterised by different PTDF matrices with corresponding probabilities.

In order to avoid complications due to discontinuous payoffs in the spot market, we follow the common assumption (see Hobbs, 2001; Smeers and Wei, 1997; Wei and Smeers, 1999) that agents do not game the transmission prices or consider the impact of their production decisions on congestion prices. Assuming that producers can game transmission prices may yield multiple spot market equilibria or may result in no pure strategy equilibrium (see Cardell *et al.*, 1997; Hobbs *et al.*, 2000), which makes the equilibrium computation intractable. Moreover, when gaming the transmissionmarket, generation firms will typically find it optimal to 'barely' congest some lines so as to avoid congestion rent; this will lead to degenerate spot market equilibrium (see Oren, 1997), if any. Empirical evidence suggests that ignoring potential gaming of transmission prices by generators is quite realistic, since it is practically impossible for multiple-generation firms to coordinate their production so as to avoid congestion charges by barely decongesting transmission lines. Indeed, the initial design of the California market, which attempted to control congestion by relying on such coordination in response to advisory congestion charges, proved to be unworkable. For simplicity, we further assume that there is at most one generation facility at a node (this assumption can be easily relaxed by aggregating units owned by a single firm and splitting nodes with multiple firms).

### 2.2 Model notations

Sets:

- $N$ : The set of nodes (or buses)
- $Z$ : The set of zones. Moreover, $z(i)$ represents the zone where node $i$ resides
- $L$ : The set of transmission lines whose congestion in the spot market is under consideration. These lines are called flowgates.
- $C$ : The finite set of states in the spot market
- $G$ : The set of generation firms. $N_g$ denotes the set of nodes where generation facilities of firm $g \in G$ are located

Parameters:

- $\underline{q}_i^c$, $\overline{q}_i^c$ : The lower and upper capacity bounds of generation facility at node $i \in N$ in state $c \in C$.
- $p_i^c(\cdot)$ : The linear Inverse Demand Function (IDF) at node $i$ in state $c$:

  $$p_i^c(q) = \overline{p}^c - b_i q \quad i \in N,\ c \in C$$

  We assume that in each state $c$ the price intercepts of the inverse demand curves are uniform across all nodes, and that, for each node $i$, the nodal demand shifts inward and outward in different states, but the slope remains unchanged.
- $C_i(\cdot)$ : The cost function at node $i$. In this model, the cost functions are assumed linear.

  $$C_i(q) = d_i q \quad i \in N$$
- $K_l^c$ : The flow capacity of line $l \in L$ in state $c \in C$.
- $D_{l,i}^c$ : The power transfer distribution factor in state $c \in C$ on line $l \in L$ with respect to node $i \in N$.
- $Pr(c)$ : The probability of state $c \in C$ of the spot market
- $\delta_i$ : The weights used to settle the spot zonal prices $\left(\delta_i \geq 0,\ \sum_{i:z(i)=z} \delta_i = 1\right)$.

Decision variables:

- $x_{g,z}$ : Forward quantity committed by firm $g \in G$ to zone $z \in Z$.
- $q_i^c$ : Generation level at node $i \in N$ in state $c \in C$ of the spot market
- $r_i^c$ : Import/export quantity at node $i \in N$ by the ISO in state $c \in C$ of the spot market

## 2.3 *The formulation*

The zonal forward market, which is conducted at zonal trading hubs (*e.g.*, PJMs Western hub), ignores intrazonal transmission congestion (although such congestion is accounted for implicitly through the rational expectation of the spot zonal price, which is based on a weighted nodal price average). The spot market, on the other hand, is organised at a nodal level with all transmission constraints recognised in the ISO redispatches.

In each state $c \in C$, the spot nodal price at each node $i \in N$ is given by the nodal inverse demand function $p_i^c(q_i^c + r_i^c)$ applied to the net local consumption that results from the local production decision by the generating firms and the redispatch decision by the ISO.

The spot zonal (settlement) price $u_z^c$ at a zone $z$ in each state $c$ is defined as the weighted average of the nodal prices in that zone with predetermined weights $\delta_i$. Mathematically, the zonal spot settlement prices are given by:

$$u_z^c = \sum_{i:z(i)=z} \delta_i p_i^c(r_i^c + q_i^c), \quad z \in Z.$$

The forward zonal prices $h_z$ are the prices at which forward commitments are traded in the respective zones. The no-arbitrage assumption implies that the forward zonal prices are equal to the expected spot zonal settlement prices:

$$h_z = \sum_{c \in C} Pr(c)\, u_z^c, \quad z \in Z. \tag{1}$$

In each state $c$ of the spot market, the firms choose the production levels. Each firm $g$'s revenue in each state $c$ is the sum of its forward commitment settlement at the spot zonal settlement prices, and the payment for its production quantities at the spot nodal prices. So its profit is:

$$\pi_g^c = \sum_{i \in N_g} p_i^c(r_i^c + q_i^c) q_i^c - \sum_{z \in Z} u_z^c x_{g,z} - \sum_{i \in N_g} C_i(q_i^c)$$

Each firm $g$'s objective in the spot market is to maximise its profit $\pi_g^c$. It solves the following profit maximisation problem parametric on its forward commitments $x_{g,z}$ and the ISO's redispatch quantities $r_i^c$:

$$G_g^c : \max_{q_i^c : i \in N_g} \quad \pi_g^c$$

subject to:

$$q_i^c \geq \underline{q}_i^c, \quad i \in N_g \tag{2}$$

$$q_i^c \leq \overline{q}_i^c, \quad i \in N_g. \tag{3}$$

In this programme, constraints (2) and (3) ensure that the production levels $q_i^c$ fall between the capacity bounds of the generation facilities in each state *c*.

The ISO determines import/export quantities $r_i^c$ at the nodes. Its objective is to maximise the social surplus defined by the consumers' willingness to pay minus the total generation cost. It solves a social-welfare-maximisation problem:

$$S^c : \max_{r_i^c} \sum_{i \in N} \left( \int_0^{r_i^c + q_i^c} p_i^c(\tau_i) d\tau_i - c_i(q_i^c) \right)$$

subject to:

$$\sum_{i \in N} r_i^c = 0 \tag{4}$$

$$\sum_{i \in N} D_{l,i}^c r_i^c \geq -K_l^c, \quad l \in L \tag{5}$$

$$\sum_{i \in N} D_{l,i}^c r_i^c \leq K_l^c, \quad l \in L \tag{6}$$

Here, constraint (4) represents energy balance (assuming no losses), whereas constraints (5) and (6) enforce the network feasibility, *i.e.*, the power flows resulting from the ISO redispatch must not exceed the thermal limits.

Since the nodal inverse demand functions as well as the cost functions are assumed linear, problems $G_g^c$ and $S^c$ are both strictly concave-maximisation programmes, which implies that their first-order necessary conditions (commonly referred to as the Karush-Kuhn-Tucker, or KKT conditions; see Karush, 1939; Kuhn and Tucker, 1951) are also sufficient. The spot market outcomes can thus be characterised by the KKT conditions of the firms and the ISO's problems. Let $\alpha^c$, $\lambda_{l-}^c$ and $\lambda_{l+}^c$ be the Lagrange multipliers corresponding to (4)–(6), then the KKT conditions derived from problem $S^c$ are:

$$\sum_{j \in N} r_j^c = 0 \tag{7}$$

$$\overline{p}^c - (q_i^c + r_i^c) b_i - \alpha^c + \sum_{t \in L} (\lambda_{t-}^c D_{t,i}^c - \lambda_{t+}^c D_{t,i}^c) = 0 \quad i \in N \tag{8}$$

$$0 \leq \lambda_{l-}^c \perp \sum_{j \in N} D_{l,i}^c r_j^c + K_l^c \geq 0 \quad l \in L \tag{9}$$

$$0 \leq \lambda_{l+}^c \perp K_l^c - \sum_{j \in N} D_{l,j}^c r_j^c \geq 0 \quad l \in L \tag{10}$$

Here and henceforth, we use the conventional notation $x \perp y$ to represent the complementarity condition $x^T y = 0$. Similarly, let $\rho_{i-}^c$ and $\rho_{i+}^c$ be the Lagrange multipliers corresponding to (2) and (3), then the KKT conditions for problem $G_g^c$ are:

$$\overline{p}^c - 2b_i q_i^c - b_i r_i^c - d_i + \delta_i b_i x_{g,z(i)} + \rho_{i-}^c - \rho_{i+}^c = 0 \quad i \in N_g \tag{11}$$

$$0 \leq \rho_{i-}^c \perp q_i^c - \underline{q}_i^c \geq 0 \quad i \in N_g \tag{12}$$

$$0 \leq \rho_{i+}^c \perp \overline{q}_i^c - q_i^c \geq 0 \quad i \in N_g \tag{13}$$

If we restrict $\left\{x_{g,z}=0\right\}_{z\in Z, g\in G}$, *i.e.*, no firm commits to forward contracts, the solutions to KKT conditions (7)–(13) characterise the outcomes of the single-settlement market, *i.e.*, there is no forward market, and all firms act only in the spot market.

If there is no flow congestion in some state $c$ of the spot market, the shadow prices corresponding to the transmission capacities will all be zero, *i.e.*:

$$\lambda_{l-}^c=\lambda_{l+}^c=0, \quad l\in L$$

This leads condition (8) to:

$$\overline{p}^c-(q_i^c+r_i^c)b_i-\alpha^c=0,\ i\in N,$$

Thus:

$$\alpha^c=\overline{p}^c-\frac{\sum_{i\in N}q_i^c}{\sum_{i\in N}1/b_i}.$$

That is, all the spot nodal prices are identical and equal to $\alpha^c$.

The forward market is organised as a transparent financial market at zonal trading hubs and is settled in real time at zonal settlement prices computed as weighted averages of nodal prices. The expected congestion costs due to transmission constraints are hence propagated to the forward market through rational expectations of the zonal settlement prices. In the forward market, each firm $g$ conjectures the other firms' forward quantities and determines its own forward quantities. In general, the firms' objectives are to maximise their respective expected utility function over total profit from spot productions and forward settlements. For simplicity, the firms are assumed here to be risk neutral, so their forward objectives are to maximise the expected joint profits in both the forward and the spot markets, subject to the no-arbitrage condition and the preceding KKT conditions. Each firm $g$ solves the following stochastic MPEC programme in the forward market:

$$\max_{x_{g,z}} \quad \sum_{z\in Z}h_z x_{g,z}+\sum_{c\in C}Pr(c)\pi_g^c$$

subject to:

$$\pi_g^c=\sum_{i\in N_g}p_i^c(r_i^c+q_i^c)q_i^c-\sum_{z\in Z}u_z^c x_{g,z}-\sum_{i\in N_g}C_i(q_i^c)$$

$$h_z=\sum_{c\in C}Pr(c)u_z^c, \quad z\in Z$$

$$\sum_{j\in N}r_j^c=0, \quad c\in C$$

$$\overline{p}^c-(q_i^c+r_i^c)b_i-\alpha^c+\sum_{t\in L}(\lambda_{t-}^c D_{t,i}^c-\lambda_{t+}^c D_{t,i}^c)=0, \quad i\in N,\ c\in C$$

$$0\le\lambda_{l-}^c\perp\sum_{j\in N}D_{l,i}^c r_j^c+K_l^c\ge 0, \quad l\in L,\ c\in C$$

$$0 \le \lambda_{l+}^c \perp K_l^c - \sum_{j \in N} D_{l,j}^c r_j^c \ge 0, \quad l \in L,\ c \in C$$

$$\overline{p}^c - 2b_i q_i^c - b_i r_i^c - d_i + \delta_i b_i x_{m,z(i)} + \rho_{i-}^c - \rho_{i+}^c = 0, \quad m \in G,\ i \in N_m,\ c \in C$$

$$0 \le \rho_{i-}^c \perp q_i^c - \underline{q}_i^c \ge 0, \quad i \in N,\ c \in C$$

$$0 \le \rho_{i+}^c \perp \overline{q}_i^c - q_i^c \ge 0, \quad i \in N,\ c \in C$$

Note that the forward settlement terms in the objective function are cancelled owing to constraint (1), so that the MPEC programme for each firm $g$ is reduced to:

$$\max_{x_{g,z}} \quad \sum_{c \in C} Pr(c) \left( \sum_{i \in N_g} p_i^c (r_i^c + q_i^c) q_i^c - \sum_{i \in N_g} C_i(q_i^c) \right)$$

subject to:

$$\sum_{j \in N} r_j^c = 0, \quad c \in C$$

$$\overline{p}^c - (q_i^c + r_i^c) b_i - \alpha^c + \sum_{t \in L} (\lambda_{t-}^c D_{t,i}^c - \lambda_{t+}^c D_{t,i}^c) = 0, \quad i \in N,\ c \in C$$

$$0 \le \lambda_{l-}^c \perp \sum_{j \in N} D_{l,i}^c r_j^c + K_l^c \ge 0, \quad l \in L,\ c \in C$$

$$0 \le \lambda_{l+}^c \perp K_l^c - \sum_{j \in N} D_{l,j}^c r_j^c \ge 0, \quad l \in L,\ c \in C$$

$$\overline{p}^c - 2b_i q_i^c - b_i r_i^c - d_i + \delta_i b_i x_{m,z(i)} + \rho_{i-}^c - \rho_{i+}^c = 0, \quad m \in G,\ i \in N_m,\ c \in C$$

$$0 \le \rho_{i-}^c \perp q_i^c - \underline{q}_i^c \ge 0, \quad i \in N,\ c \in C$$

$$0 \le \rho_{i+}^c \perp \overline{q}_i^c - q_i^c \ge 0, \quad i \in N,\ c \in C$$

The general structure of each firm's MPEC problem (after rearranging and relabelling the variables) is of the form:

$$\underset{x_g, y, w}{mix} \quad f_g(x_g,\ x_{-g},\ y,\ w)$$

subject to:

$$w = a + A^{-g} x_{-g} + A^g x_g + My$$

$$0 \le w \perp y \ge 0$$

In this programme, $x_g$ represents decision variables that are the firm's forward variables and $x_{-g}$ are the corresponding decision variables controlled by all other firms, whereas $w$ and $y$ are the shared state variables. Likewise, $a$, $A_g$, $A_{-g}$, and $M$ represent suitable vectors and matrices implied by the system's parameters. Owing to the linearity of the demand functions and cost functions, the objective functions in these MPEC problems are quadratic and the KKT constraints (7)–(13) are reduced to a Linear Complementarity Problem (LCP, see Cottle *et al.*, 1992). Combining all firms' MPEC programmes, the equilibrium problem in the forward market is an EPEC, which involves simultaneous solutions of the individual firms' MPECs. In the following numerical example, we have employed a special-purpose algorithm for such problems that exploits their special structure. This MPEC algorithm treats $y$ and $w$ as piecewise linear functions of $x_g$. Thus, each firm $g$'s MPEC problem is reduced to a programme with respect only to $x_g$. The algorithm partitions the space of $x_g$ into a set of polyhedra according to the feasible complementary bases of the LCP constraint. Such partitioning allows the MPEC algorithm to search for a stationary point of the MPEC problem via parametric LCP pivoting and finitely many quadratic programmes. A detailed description of the algorithm is out of the scope of this paper and will be reported elsewhere.

## 3 The 24-bus system

In this section, we apply our model to the IEEE 24-bus test network with different, fictitious generator-ownership structures, and observe the economic results of two settlements. The 24-bus network is composed of 24 nodes and 38 lines (see Figure 1). There are ten generators in this system, each located at one node. The resource ownerships, zonal structures, demand functions and contingency states are hypothetical. (When applying this model to a real system, one can generate the demand functions and contingency states by sampling historical data.)

Table 1 lists the nodal information, including inverse demand function slopes, marginal generation costs and full capacities of generation plants. We assume two zones in the system with node 1 through 13 in zone 1 and the rest of the nodes in zone 2. As to the thermal limits, we ignore the intrazonal flows and focus only on flowgates 3–24, 11–14, 12–23 and 13–23.

We assume seven states in the spot market (see Table 2). In the first state, the demands are at peak, all generation plants operate at their full capacities and all transmission lines are rated at their full thermal limits. The second state is the same as the first state except that it has shoulder demands. States 3 through 6 also have shoulder demands, but represent the contingencies of unavailability of the four flowgates, respectively. Off-peak state 7 differs from states 1 and 2 in having very low demand levels. Table 2 also illustrates the price intercepts of IDFs as well as the probabilities of the states.

We run tests on the systems with single settlement and two settlements, and observe the likelihood of congestion, generator output changes, social welfare changes and the behaviours of the spot nodal and zonal prices due to forward contracting. For the case of two settlements, we test different generator ownership structures with two, three, four or five firms. The details of the ownerships are listed in Table 3.

We observe that with two settlements, the firms have strategic incentives for committing to forward contracts. Furthermore, the incentives for forward contracting are strengthened by decreased market concentration. Figure 2 compares the total forward contracting quantities with different numbers of firms. It shows that the total forward contract quantity increases from 60 MW with two firms to 640 MW with five firms.

**Figure 1** The 24-bus network

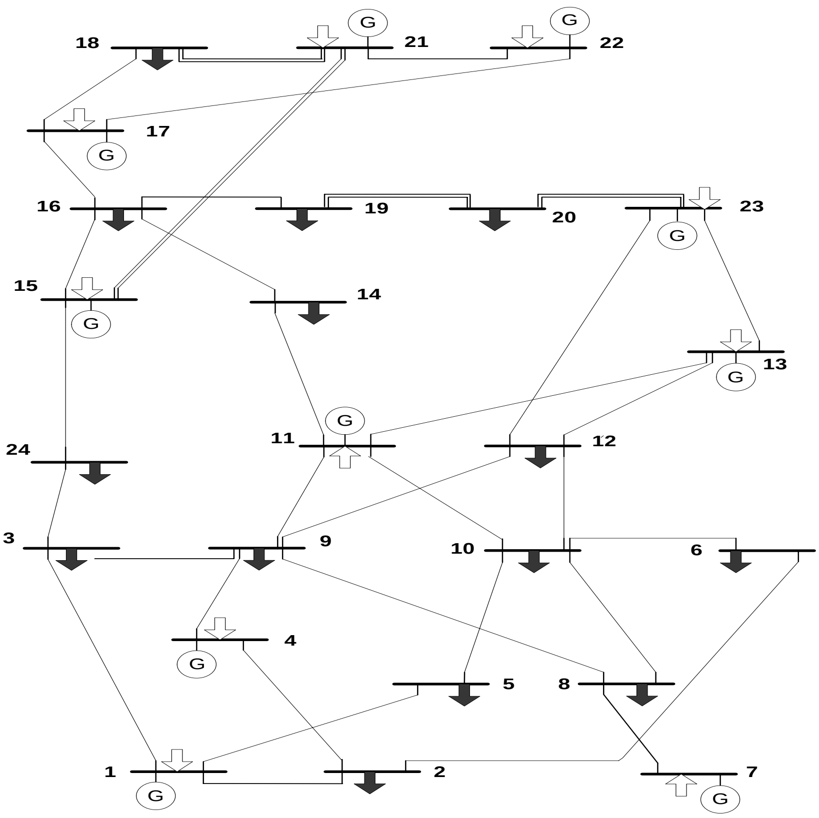



**Table 1** Nodal information

| Node | IDF slope | Marginal cost ($/MWh) | Capacity (MW) | Node | IDF slope | Marginal cost ($/MWh) | Capacity (MW) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 30 | 70 | 13 | 1 | 30 | 70 |
| 2 | 0.82 | – | 0 | 14 | 0.85 | – | 0 |
| 3 | 1.13 | – | 0 | 15 | 1 | 25 | 70 |
| 4 | 1.1 | 30 | 70 | 16 | 1.15 | – | 0 |
| 5 | 0.93 | – | 0 | 17 | 1 | 20 | 70 |
| 6 | 0.85 | – | 0 | 18 | 0.79 | – | 0 |
| 7 | 1 | 30 | 70 | 19 | 0.68 | – | 0 |
| 8 | 1 | – | 0 | 20 | 1.03 | – | 0 |
| 9 | 0.88 | – | 0 | 21 | 1 | 25 | 70 |
| 10 | 0.5 | – | 0 | 22 | 1.05 | 30 | 70 |
| 11 | 1 | 20 | 70 | 23 | 1 | 20 | 70 |
| 12 | 0.73 | – | 0 | 24 | 0.73 | – | 0 |

**Table 2** States of the spot market

| *State* | *Prob.* | *IDF intercept ($/MWh)* | *Description* |
| --- | --- | --- | --- |
| 1 | 0.15 | 100 | On-peak state: The demands are on the peak. |
| 2 | 0.6 | 50 | Shoulder state: The demands are at shoulder. |
| 3 | 0.025 | 50 | Shoulder demands with line breakdown: Line 3–24 goes down. |
| 4 | 0.025 | 50 | Shoulder demands with line breakdown: Line 11–14 goes down. |
| 5 | 0.025 | 50 | Shoulder demands with line breakdown: Line 12–23 goes down. |
| 6 | 0.025 | 50 | Shoulder demands with line breakdown: Line 13–23 goes down. |
| 7 | 0.15 | 25 | Off-peak state: The demands are off peak. |

**Table 3** Generator ownership structure

| *Node* | *Number of firms* | | | |
| --- | --- | --- | --- | --- |
| | *Two* | *Three* | *Four* | *Five* |
| 1 | #1 | #1 | #1 | #1 |
| 4 | #1 | #2 | #2 | #2 |
| 7 | #1 | #2 | #3 | #3 |
| 11 | #2 | #3 | #4 | #4 |
| 13 | #2 | #3 | #4 | #5 |
| 15 | #2 | #1 | #1 | #1 |
| 17 | #2 | #2 | #2 | #2 |
| 21 | #2 | #2 | #3 | #3 |
| 22 | #2 | #3 | #3 | #4 |
| 23 | #1 | #3 | #4 | #5 |

In all states of the spot market, the aggregated spot outputs increase under two settlements; moreover, the more firms in the markets, the greater is this effect (see Figure 3). Despite this phenomenon, some generators still decrease the outputs in some states. This is because, when facing intensive competition, some firms have to reduce production of the generators located in the nodes with lower spot prices, so as to sustain their profits by increasing their outputs from other plants. For example, when there are five firms in the markets, generators at nodes 15 and 21 increase their production levels only in the peak state, but reduce them in the other six states (see Table 4). Consequently, the expected spot outputs from these generators might be lower under two settlements than those under a single settlement. The expected generation quantities are shown in Figure 4, with the dark bars denoting the outputs for a single settlement and the grey and white bars for two settlements with two firms and five firms respectively. It is shown that, under two settlements with five firms, the generators in both nodes 15 and 21 operate at expected levels that are lower than those under a single settlement.

**Figure 2** Total forward contracting

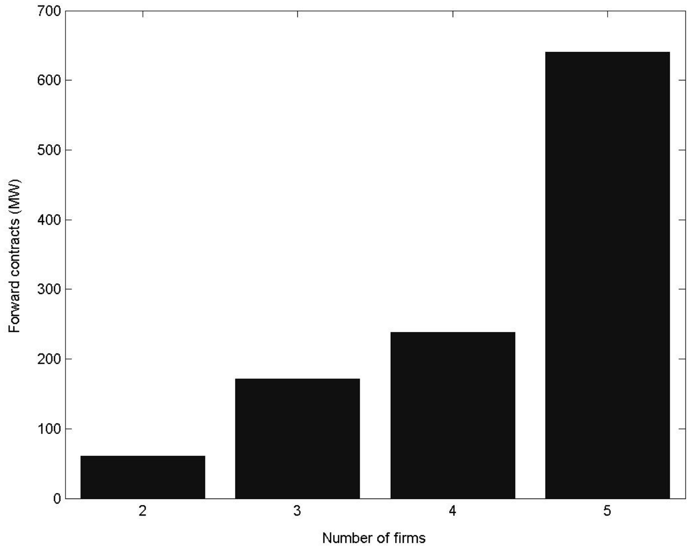



**Figure 3** Total spot generation

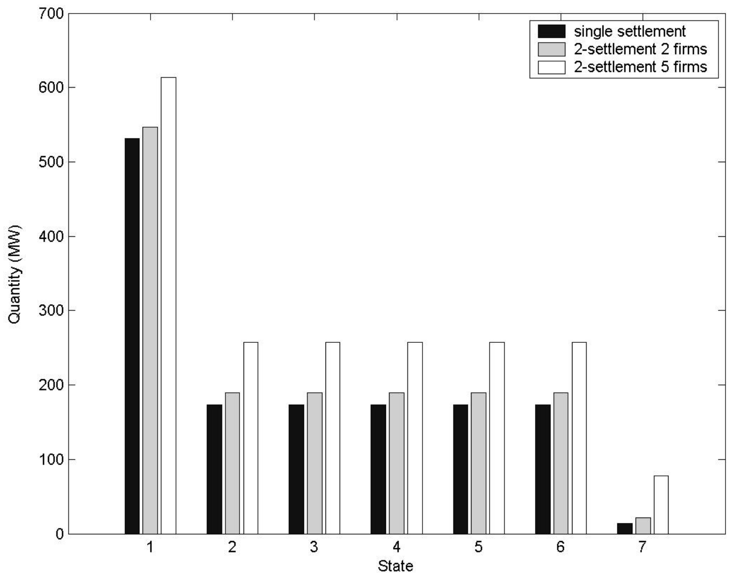
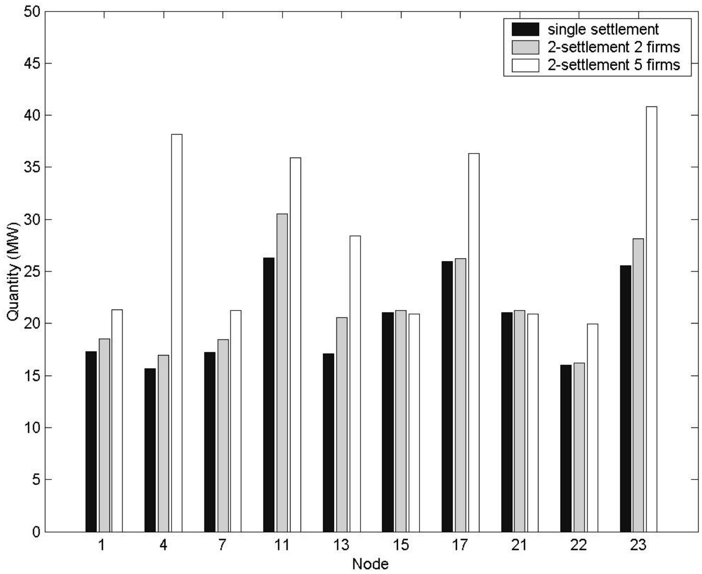


**Table 4** Output level changes (MW)

| *State* | *Node* 1 | 4 | 7 | 11 | 13 | 15 | 17 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4.6704 | 22.9899 | 4.4798 | 8.5305 | 11.2008 | 0.7479 | 11.1849 | 0.7684 | 5.5010 | 12.1438 |
| 2 | 4.7752 | 23.1898 | 4.7924 | 9.6474 | 12.2285 | –0.3026 | 10.0909 | –0.3045 | 4.4713 | 15.6964 |
| 3 | 4.7138 | 23.1340 | 4.7310 | 9.5054 | 12.1053 | –0.1455 | 10.2480 | –0.1474 | 4.6209 | 15.3486 |
| 4 | 4.7119 | 23.1146 | 4.6940 | 9.5360 | 12.0476 | –0.2067 | 10.1942 | –0.2048 | 4.5675 | 15.8469 |
| 5 | 4.7139 | 23.1192 | 4.7015 | 9.5613 | 12.0691 | –0.2300 | 10.1697 | –0.2287 | 4.5446 | 15.9265 |
| 6 | 4.8146 | 23.2347 | 4.8498 | 9.6961 | 12.2878 | –0.3449 | 10.0448 | –0.3487 | 4.4285 | 15.5619 |
| 7 | 0 | 18.8401 | 0 | 10.3492 | 7.4455 | 0 | 10.8999 | 0 | 0 | 16.4569 |

**Figure 4** Expected spot nodal generation




Spot nodal and zonal prices under two settlements decrease in all states. This follows directly from the fact that the aggregate output is increased in the spot market under two settlements. Figure 5 compares the expected spot nodal prices under a single settlement to those corresponding to two settlements for either two and for five firms. The prices under a single settlement are drawn dark, while the prices under two settlements with two firms are grey, and with five firms, white. In Table 5, we report the spot zonal prices under a single settlement in columns 2 and 3, and the spot zonal prices under two settlements in columns 4 through 7. The last row of this table lists the forward zonal prices. It is also shown in Figure 5 and Table 5 that as more firms compete in the two-settlement system, the lower are the spot nodal and zonal prices.

**Figure 5** Expected spot nodal prices

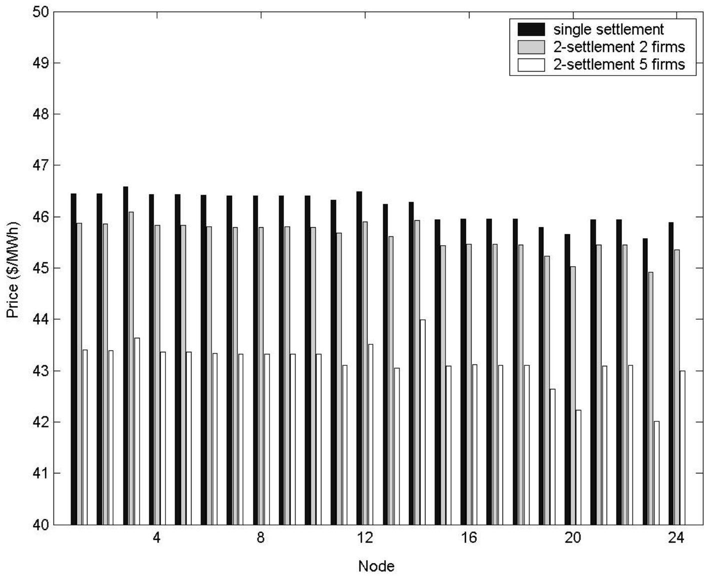



**Table 5** Zonal prices

| Market | Single settlement | | Two settlements: Two firms | | Two settlements: Five firms | |
|---|---|---|---|---|---|---|
| | *Zone 1* | *Zone 2* | *Zone 1* | *Zone 2* | *Zone 1* | *Zone 2* |
| Spot: state 1 | 81.6964 | 77.7208 | 80.4398 | 77.2604 | 79.1841 | 75.2275 |
| Spot: state 2 | 43.8607 | 43.1124 | 42.7772 | 42.5626 | 41.4331 | 39.1074 |
| Spot: state 3 | 44.0055 | 42.9175 | 42.9319 | 42.3577 | 41.5875 | 38.9027 |
| Spot: state 4 | 43.9174 | 43.0761 | 42.7188 | 42.6434 | 41.4948 | 39.1408 |
| Spot: state 5 | 43.8697 | 43.1084 | 42.8144 | 42.5160 | 41.4314 | 39.2197 |
| Spot: state 6 | 44.0106 | 42.9529 | 42.8768 | 42.4506 | 41.5921 | 39.0052 |
| Spot: state 7 | 24.4973 | 24.4973 | 24.2061 | 24.2061 | 22.3382 | 21.8949 |
| Forward | – | – | 45.6468 | 45.0068 | 44.2409 | 41.9395 |

Social surplus increases under two settlements. Moreover, the social welfare of the two-settlement system increases as the number of firms increases. The expected social welfare of a single settlement is $7796/h, which, under two settlements, is increased to $8133/h with two firms and $9383/h with five firms. Figure 6 shows that the same trend applies to the consumer surplus. These results are qualitatively consistent with those of Allaz and Vila (1993).

Lines not congested in the single-settlement system might be congested in the spot market of the two-settlement system, or vice versa. This follows from the fact that the firms adjust their outputs, which alters the flows on the transmission lines. For example, in state 3, the single-settlement market has only line 11–14 congested; however, the only congested line under two settlements with five firms is line 12–23 (see Table 6).

**Figure 6** Social surplus

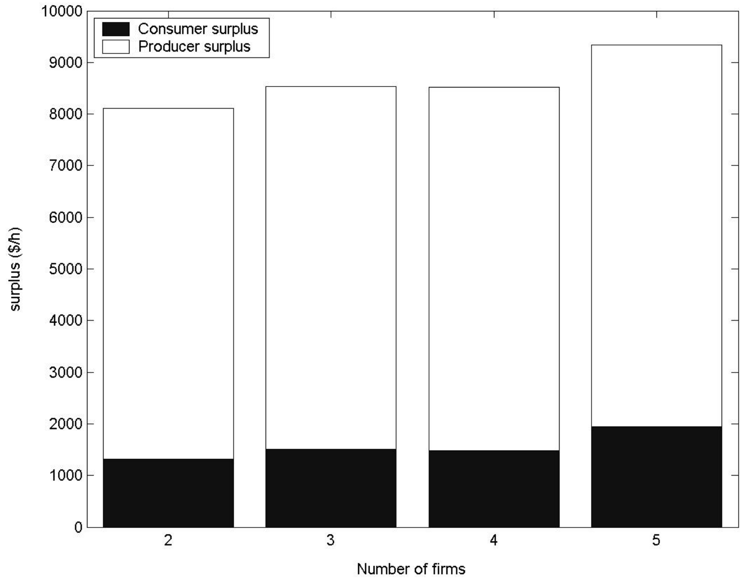
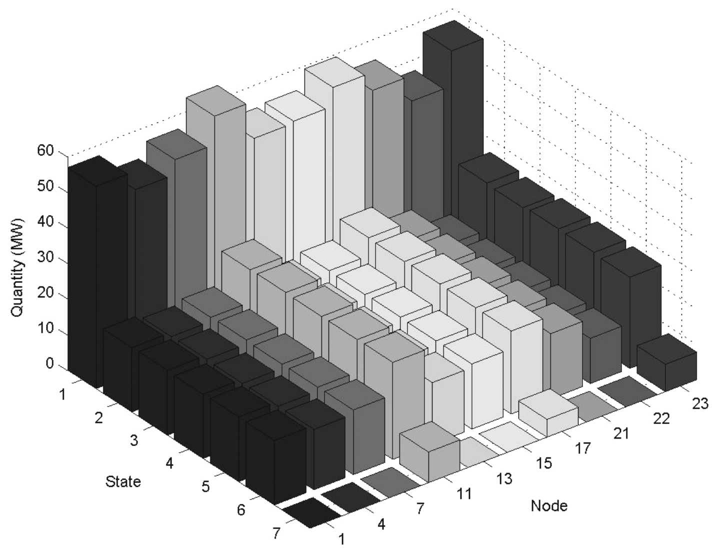



**Table 6** Flow congestion

| *Line* | *State* | *Single settlement* | *Two settlements* | *Line* | *State* | *Single settlement* | *Two settlements* |
|---|---|---|---|---|---|---|---|
| 3–24 | 1 | Congested | Congested | 12–23 | 1 | Congested | Congested |
| | 2 | Uncongested | Uncongested | | 2 | Uncongested | Uncongested |
| | 3 | Breakdown | Breakdown | | 3 | Uncongested | Congested |
| | 4 | Uncongested | Uncongested | | 4 | Uncongested | Uncongested |
| | 5 | Uncongested | Uncongested | | 5 | Breakdown | Breakdown |
| | 6 | Uncongested | Uncongested | | 6 | Congested | Congested |
| | 7 | Uncongested | Uncongested | | 7 | Uncongested | Uncongested |
| 11–14 | 1 | Congested | Congested | 13–23 | 1 | Uncongested | Uncongested |
| | 2 | Uncongested | Uncongested | | 2 | Uncongested | Uncongested |
| | 3 | Congested | Uncongested | | 3 | Uncongested | Uncongested |
| | 4 | Breakdown | Breakdown | | 4 | Congested | Uncongested |
| | 5 | Uncongested | Uncongested | | 5 | Congested | Congested |
| | 6 | Uncongested | Uncongested | | 6 | Breakdown | Breakdown |
| | 7 | Uncongested | Congested | | 7 | Uncongested | Uncongested |

Finally, Figure 7 illustrates generator outputs for the seven states under two settlements with two firms. We note that the generators produce at levels between 40 MW and 70 MW in the peak state, and that only three generators operate in the off-peak state. Compared to their forward contracts, the firms are in fact net buyers in the off-peak state, and net suppliers in the peak state.

**Figure 7** Statewise spot generations




## 4 Concluding remarks

In this paper, we model the two-settlement electricity system as a two-period game with multiple states of the world in the second period. The Cournot equilibrium is a subgame-perfect Nash equilibrium represented in the format of an EPEC. We assume linear demand functions and constant marginal generation costs, so the spot market equilibrium can be computed as a linear complementarity problem. In the forward market, firms solve MPECs subject to the no-arbitrage relationship between the forward prices and the expected spot zonal prices, and the spot market equilibrium conditions.

We apply our model to the 24-bus network, and observe from it the strategic incentives of the firms for forward contracting, the likelihood of congestion, increased generation quantities, increased social surplus and decreased spot prices with the introduction of a forward market. We also find that these effects are amplified when there are more firms in the network.

Finally, it should be pointed out that our numerical tests are limited, and that our goal is not to reach conclusive economic results, which require far more extensive simulations, but to validate our modelling methodology. Moreover, one should expect quantitatively dissimilar results if the model is applied to a different setting of resource ownership structures, demand function distributions or contingency states, or to different networks.

We plan to relax the no-arbitrage assumption between the forward and spot prices with a market-clearing condition that sets the forward prices based on the expected demands in the spot market. Such an analysis will attempt to capture how lack of liquidity (or high-risk aversion) on the buyers' side might be reflected in a high-risk premium embedded in the forward prices. We expect that such a condition enhances firms' market power and enables them to raise forward prices above the expected spot prices while increasing their profits.

## Acknowledgements

This research is supported by the National Science Foundation Grant ECS-0224779, by the Power System Engineering Research Center (PSERC), by the University of California Energy Institute (UCEI) and by the Department of Energy through the Consortium for Electric Reliability Technology Solutions (CERTS).

## References

Allaz, B. (1992) 'Oligopoly, uncertainty and strategic forward transactions', *Internal Journal of Industrial Organization*, Vol. 10, pp.297–308.

Allaz, B. and Vila, J-L. (1993) 'Cournot competition, forward markets and efficiency', *Journal of Economic Theory*, Vol. 59, pp.1–16.

Cardell, J., Hitt, C. and Hogan, W.W. (1997) 'Market power and strategic interaction in electricity networks', *Resource and Energy Economics*, Vol. 19, pp.109–137.

Chao, H-P. and Peck, S.C. (1996) 'A market mechanism for electric power transmission', *Journal of Regulatory Economics*, Vol. 10, No. 1, pp.25–60.

Cottle, R.W., Pang, J.S. and Stone, R.E. (1992) *The Linear Complementarity Problem*, Boston, MA: Academic Press.

Fudenberg, D. and Tirole, J. (1991) *Game Theory*, Cambridge, MA: The MIT Press.

Green, R.J. (1999) 'The electricity contract market in England and Wales', *Journal of Industrial Economics*, Vol. 47, No. 1, pp.107–124.

Hobbs, B.F., Metzler, C.B. and Pang, J.S. (2000) 'Strategic gaming analysis for electric power systems: an MPEC approach', *IEEE Transactions on Power Systems*, Vol. 15, No. 2, pp.638–645.

Hobbs, B.F. (2001) 'Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets', *IEEE Transactions on Power Systems*, Vol. 16, No. 2, pp.194–202.

Kamat, R. and Oren, S.S. (2004) 'Multi-settlement systems for electricity markets: zonal aggregation under network uncertainty and market power', *Journal of Regulatory Economics*, Vol. 25, No. 1, pp.5–37.

Karush, W. (1939) 'Minima of functions of several variables with inequalities as side constraints', *MS Thesis*, Department of Mathematics, University of Chicago, Chicago, IL.

Kuhn, H.W. and Tucker, A.W. (1951) 'Nonlinear programming', J. Neyman (Ed.) *Proceedings of the Second Berkeley Symposium*, Berkeley, CA: University of California Press.

Luo, Z.Q., Pang, J.S. and Ralph, D. (1996) *Mathematical Programmes with Equilibrium Constraints*, Cambridge: Cambridge University Press.

Newbery, D.M. (1998) 'Competition, contracts, and entry in the electricity spot market', *Rand Journal of Economics*, Vol. 29, No. 4, pp.726–749.

Oren, S.S. (1997) 'Economic inefficiency of passive transmission rights in congested electricity systems with competitive generation', *The Energy Journal*, Vol. 18, pp.63–83.

Powell, A. (1993) 'Trading forward in an imperfect market: the case of electricity in Britain', *The Economic Journal*, Vol. 103, pp.444–453.

Smeers, Y. and Wei, J-Y. (1997) *Spatial Oligopolistic Electricity Models with Cournot Firms and Opportunity Cost Transmission Prices*, Center for Operations Research and Econometrics, Universite Catholique de Louvain, Louvain-la-newve, Belgium.

von der Fehr, N-H.M. and Harbord, D. (1993) 'Long-term contracts and imperfectly competitive spot markets: a study of UK electricity industry', *Memorandum No. 14,* Department of Economics, University of Oslo, Oslo, Sweden.

Wei, J-Y. and Smeers, Y. (1999) 'Spatial oligopolistic electricity models with Cournot firms and regulated transmission prices', *Operations Research,* Vol. 47, No. 1, pp.102–112.

Yao, J., Oren, S.S. and Adler, I. (2004) 'Computing Cournot equlibria in two-settlement electricity markets with transmission constraints', *Proceeding of the 37th Hawaii International Conference on Systems Sciences (HICCS 37),* Big Island, Hawaii, 5–8 January.