Cournot Equilibrium in Price-capped Two-Settlement Electricity Markets

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Abstract—We compare two alternative mechanisms for capping prices in two-settlement electricity markets. With sufficient lead time and competitive entry opportunities, forward market prices are implicitly capped by competitive pressure of potential entry that will occur when forward prices rise above a certain level. Another more direct approach is to cap spot prices through regulatory intervention. In this paper we explore the implications of the two alternative mechanisms in a two settlement Cournot equilibrium framework. We formulate the market equilibrium as a stochastic equilibrium problem with equilibrium constraints (EPEC) capturing congestion effects, probabilistic contingencies and market power. As an illustrative test case we use the 53-bus Belgian electricity network with representative generator cost but hypothetical demand and ownership assumptions. When compared to two-settlement systems without price caps we find that either of the price capping alternatives results in reduced forward contracting. Furthermore the reduction in spot prices due to forward contracting is smaller.

I. Introduction

It is generally believed that forward contracting mitigates generators' horizontal market power in the spot markets and protect marketers against spot price volatility resulting from system contingencies and demand uncertainty. Competitive entry in the forward market and regulatory caps on spot prices are further means of mitigating price spikes and market power abuse. Previous works by [1], [2], [13], [20], focus on the impact of forward markets on spot prices and social welfare under alternative assumptions regarding the relationship between forward and spot prices. It was shown that generators have incentives to contract in the forward markets whereas forward contracting reduces spot prices and increases consumption levels and social welfare. These models, however, assume a fixed generation stock which is an appropriate assumption for two-settlement system over short time intervals (e.g., day ahead and real time markets). For long term forward contracts, potential competitive entries impose an implicit price cap on forward contract prices since new investment in generation capacity will occur when forward prices rise above a certain level. Alternatively, regulators in many restructured electricity markets have imposed price or offer caps in the spot markets

in an attempt to rectify market imperfections such as demand inelasticity, barriers to entry, imperfect capital markets and locational market power. In this paper we extend our earlier model in [20] by considering, separately, the effects of these two cap types on the spot and forward prices.

In particular, we address the following questions: To what extent do the generators commit forward contracts under price caps? How do caps on forward prices affect the spot market and how do caps on spot prices affect the forward market? We study these questions via a two-settlement Cournot equilibrium model where generation firms have horizontal market power. The system is subject to contingencies due to transmission and generation outages as well as to demand uncertainties. Our model also accounts for network congestion which is represented through a capacity constrained DC load flow approximation of the electricity grid.

We convert our formulation into a stochastic equilibrium problem with equilibrium constraints (EPEC) where each generation firm faces a stochastic mathematical program with equilibrium constraints (MPEC, see [14]). These MPEC problems have quadratic objective functions and share identical lower-level constraints in the form of a parametric linear complementarity problem (LCP, see [7]).

The remainder of this paper is organized as follows. Related electricity market models and MPEC algorithms are reviewed in the next section, section III presents the formulation for the two-settlement markets. In section IV, we run a variety of experiments on our MPEC and EPEC algorithms. Finally, we investigate the implications of hypothetical price caps in a network describing a reallized model of the Belgian electricity market.

II. Related Research

In this section, we review models for electricity markets with forward contracts and Cournot competition, as well as MPEC algorithms.

Wei and Smeers [19] consider a Cournot model with regulated transmission prices. They solve the variational inequalities to determine unique long-run equilibria in their models. In subsequent work, Smeers and Wei [18] consider a separated energy and transmission market, where the system operator

conducts a transmission capacity auction with power marketers purchasing transmission contracts to support bilateral transactions. They conclude that such a market converges to the optimal dispatch for a large number of marketers.

Hobbs [12] calculates a Cournot equilibrium under the assumptions of linear demand and cost functions, which leads to a mixed linear complementarity problem (mixed LCP). In a market without arbitrageurs, non-cost based price differences can arise because the bilateral nature of the transactions gives firms more degrees of freedom to discriminate between electricity demand at various nodes. This is equivalent to a separated market as in Smeers and Wei [18]. In the market with arbitrageurs, any non-cost differences are arbitraged by traders who buy and sell electricity at nodal prices. This equilibrium is shown to be equivalent to a Nash-Cournot equilibrium in a POOLCO-type market. Hobbs, Metzler and Pang [11] present an oligopolistic market where each firm submits a linear supply function to the Independent System Operator (ISO). They assume that firms can only manipulate the intercepts of the supply functions, but not the slopes, while power flows and pricing strategies are constrained by the ISO's linearized optimal power flow (OPF). Each firm in this model faces a MPEC problem with spatial price equilibrium as the inner problem.

Work in forward markets has focused on the welfare enhancing properties of forward markets and the commitment value of forward contracts. The basic model in [1] assumes that producers meet in a two period market where there is some demand uncertainty in the second period. Allaz shows that generators have a strategic incentive to contract forward if other producers do not. This result can be understood using the concepts of strategic substitutes and complements of Bulow, Geneakoplos and Klemperer [4]. In these terms, the availability of the forward market makes a particular producer more aggressive in the spot market. Due to the strategic substitutes effect, this produces a negative effect on its competitors' production. The producer with access to the forward market can therefore use its forward commitment to improve its profitability to the detriment of its competitors. Allaz shows, however, that if all producers have access to the forward market, it lead to a prisoners' dilemma type of effect, reducing profits for all producers. Allaz and Vila [2] extend this result to the case where there is more than one time period where forward trading takes place. For a case without uncertainty, they establish that as the number of periods when forward trading takes place tends to infinity, producers lose their ability to raise market prices above marginal cost due to the competitive solution.

von der Fehr and Harbord [8] and Powell [17] study contracts and their impact on an imperfectly competitive electricity spot market: the UK pool. von der Fehr and Harbord [8] focus on price competition in the spot market with capacity constraints and multiple demand scenarios. They find that contracts tend to put downward pressure on spot prices. Although, this provides disincentive to generators to offer such contracts, there is a countervailing force in that selling a large number of contracts commits a firm to be more aggressive in the spot market, and ensures that it is dispatched into its

full capacity in more demand scenarios. Powell [17] models explicitly re-contracting by Regional Electricity Companies (RECs) after the maturation of the initial portfolio of contracts set up after deregulation. He adds risk aversion on the part of RECs to earlier models. Generators act as price setters in the contract market. He shows that the degree of coordination has an impact on the hedge cover demanded by the RECs, and points to a "free rider" problem which leads to a lower hedge cover chosen by the RECs.

Newbery [15] analyzes the role of contracts as a barrier to entry in the England and Wales electricity market. He extends earlier work by modelling equilibria of supply functions in the spot market. He further shows that if entrants can sign base load contracts and incumbents have enough capacity, the incumbent can sell enough contacts to drive down the spot price below the entry deterring level, resulting in more volatile spot prices if producers coordinate on the highest profit supply function equilibrium (SFE). Capacity limit however may imply that incumbents cannot play a low enough SFE in the spot market and hence cannot deter entry. Green [10] extends Newbery's model showing that when generators compete in SFEs in the spot market, together with the assumption of Cournot conjectural variations in the forward market, imply that no contracting will take place unless buyers are risk averse and willing to provide a hedge premium in the forward market. He shows that forward sales can deter excess entry, and increase economic efficiency and long-run profits of a large incumbent firm faced with potential entrants.

Yao, Oren and Adler [20] study the Nash Equilibrium in the two-settlement competitive electricity markets with uncertainty of transmission, generation and demand in the spot market. The Cournot generators' and the social-welfare-maximizing system operator's behaviors are modelled in both forward and spot markets. The equilibrium is modelled as an EPEC in which each generation firm solves a MPEC problem. The model is applied to a six-bus illustrative example, and it is found that the generators have incentives for committing forward contracts under spot market uncertainties and congestions, whereas two settlements increase social welfare, decrease spot price magnitudes and volatilities.

III. The Model

In this section, we introduce a generic model of the two-settlement electricity system with both spot and forward price caps. The system with either spot or forward market is a special case of this generic model by setting the forward or the spot price caps, or both, to infinity.

In this generic model, we formulate the two-settlement electricity markets as a complete-information two-period game with the forward market being settled in the first period, and the spot market being settled in the second period. The equilibrium in either market is a sub-game perfect Nash equilibrium (SPNE, see [9]).

In the forward market, the generation firms determine their forward commitments while anticipating other firms' forward quantities and the spot market outcomes. The spot market is a subgame with three stages: in stage one, Nature picks the

state of the world so as to reveal the actual capacities of the generation facilities and the transmission lines as well as the shape of the demand functions at each node; firms determine generation quantities to compete in a Nash-Cournot manner in stage two; and the System Operator (SO) determines how to dispatch electricity within the network to maximize total social welfare. Note that the generation firms take into consideration the SO's actions, it is rational to assume that generation firms and the SO move simultaneous in the spot market.

A. Notation:

We consider the set of nodes, transmission lines, zones, generation firms and their generation facilities, and the states of the spot market.

- N: The set of all nodes or buses.
- Z: The set of all zones. Moreover, $z(i)$ represents the zone where node i resides.
- L: The set of transmission lines whose probabilities of congestion in the spot market are strictly positive. These lines are called flowgates.
- C: The finite set of all states of the spot market.
- G: The set of all generation firms. N_g denotes the set of nodes which generation firm g has facilities attached to.

In the two-settlement markets, generators determine how much to commit in the forward contracts and how much to generate in the spot market. The system operator determines how to adjust the consumption level at each node in the spot market. The variables related to the forward markets are:

- $x_{g,z}$: The forward quantity committed from firm g to zone z.

The variables related to the spot markets are:

- q_i^c: The spot quantity generated at node i in state c.
- r_i^c: Adjustment quantity at node i by the system operator in state c.

The following exogenous parameters are considered in our formulation:

- $\underline{q}_i^c, \overline{q}_i^c$: lower and upper capacity bounds of generation facility at node i in state c.
- $\bar{u}$: the spot price cap.
- $\bar{h}$: the forward price cap.
- $p_i^c(\cdot)$: Inverse demand function at node i in state c. We denote $\bar{p}^c$ as the common price intercept across all nodes in each state c, and b_i being the slopes:

$$p_i^c(q) = \bar{p}^c - b_i q \quad i \in N, c \in C$$

  We assume that the inverse demand at each node shifts inwards or outwards in different states, but the slope does not change.

- $C_i(\cdot)$: Generation cost function at node i. We assume that the cost function is convex quadratic where

$$C_i(q) = d_i q + \frac{1}{2} s_i q^2$$

  with given positive d_i and non-negative s_i.

- K_l^c : capacity limit of line l in state c.
- $D_{l,i}^c$: Power transfer distribution factor in state c on line l with respect to node i.

- $Pr(c)$: Probability of state c in the spot market.
- δ_i: ($\delta_i \geq 0, \sum_{i:z(i)=z} \delta_i = 1$) the weights used to settle the zonal prices.

B. The Spot Market

For the sake of generality we allow different levels of granularity in the financial settlements (with equal granularity being a special case). This is motivated by the fact that in real markets we observe different granularity levels in spot and long term forward markets (for example in PJM, the western hub representing the weighted average price over nearly 100 nodes is the most liquid forward market). Specifically, the network underlying the nodal spot market is divided into a set of zones, each of which is a cluster of connected nodes. This suggests three pricing schemes: spot nodal prices, spot zonal prices (used to settle zonal forward contracts) and forward (zonal) prices.

Spot nodal prices are the prices at which generation and loads are settled at their respective nodes. In state c of the spot market, the total consumption at node i is $r_i^c + q_i^c$, which is sum of the quantity generated by the generator and the (export or import) adjustment made by the SO. Because loads can never be negative, we restrict

$$q_i^c + r_i^c \geq 0 \quad i \in N$$

Consumers evaluate their consumptions of $q_i^c + r_i^c$ at price $p_i^c(q_i^c + r_i^c)$ according to the inverse demand functions. The actual spot nodal prices are $\min\{\bar{u}, p_i^c(q_i^c + r_i^c)\}$ due to the spot price caps.

The spot zonal price u_z^c at a zone z in state c is defined as the weighted average of nodal prices in the zone with predetermined weights δ_i. Typically we expect theses weights to reflect historical load ratios and be updated periodically, however they are treated as constants in our model so that contracting, production and consumption decisions do not affect these weights. In mathematical terms the zonal spot price is given as:

$$u_z^c = \sum_{i:z(i)=z} \delta_i \min\{\bar{u}, p_i^c(q_i^c + r_i^c)\} \quad z \in Z$$

The forward zonal prices h_z are the prices at which forward commitments are agreed upon in the respective zones. We assume that in equilibrium no profitable arbitrage is possible between forward and spot zonal prices. This implies that the forward zonal price is equal to the expected spot zonal prices. That is:

$$\begin{aligned} h_z &= E^c[u_z^c] \\ &= \sum_{c \in C} Pr(c) u_z^c \quad z \in Z \end{aligned} \tag{2}$$

With a forward price cap, a threshold is put on the forward prices:

$$h_z \leq \bar{h} \quad z \in Z. \tag{3}$$

In each state c of the spot market, generation firms decide variables q_i^c: the output from each of its plants. These outputs can not be below the minimal output, neither can they exceed

the respective capacities of the plants in that state. Hence the generators face the constraints:

$$\underline{q}_i^c \leq q_i^c \leq \overline{q}_i^c \quad i \in N_g$$

Each generator g's revenue in state c of the spot market is her generation quantities paid at spot nodal prices and the financial settlement of her forward commitments settled at the difference between the forward zonal prices and spot zonal prices. Her profit π_g^c is given by:

$$\pi_g^c = \sum_{i \in N_g} q_i^c \min\{\overline{u}, p_i^c(r_i^c + q_i^c)\} + \sum_{z \in Z}(h_z - u_z^c)x_{g,z}$$
$$- \sum_{i \in N_g} C_i(q_i^c)$$

To avoid discontinuity in generators' profit function (see for example [16]), We have assumed here that the generators do not consider the impact of their decisions on the settlement of transmission rights. Each generator g solves the following program in state c of the spot market:

$$\max_{q_i^c} \quad \pi_g^c$$

subject to:

$$h_z \leq \overline{h} \quad z \in Z \tag{3}$$
$$\underline{q}_i^c \leq q_i^c \leq \overline{q}_i^c, \quad i \in N_g \tag{4}$$
$$q_i^c + r_i^c \geq 0, \quad i \in N_g \tag{5}$$

The SO decides in each state c of the spot market how to dispatch the energy within the network (i.e., import and export quantities at each node) given the production decisions by generators. She makes the adjustment r_i^c at each node i. Her dispatch must satisfy the network thermal constraints on power flows. We model electricity flows on transmission lines through Power Transfer Distribution Factors (PTDFs) using a Direct Current (DC) approximation of Kirchoff's law [6]. The PTDF is the proportion of flow on a particular line resulting from an injection of one unit at a particular node and a corresponding one-unit withdrawal at the reference "slack bus". The network feasibility constraints are

$$-K_l^c \leq \sum_{i \in N} D_{l,i}^c r_i^c \leq K_l^c, \quad l \in L$$

The SO also maintains real time balance of loads and outputs, that is

$$\sum_{i \in N}(q_i^c + r_i^c) = \sum_{i \in N} q_i^c$$

or equivalently

$$\sum_{i \in N} r_i^c = 0$$

The SO's objective is to maximize the social welfare defined by the area under the consumers' inverse demand function

minus total generation cost. She solves the following mathematical program in each state c of the spot market:

$$\max_{r_i^c} \quad \sum_{i \in N}\left[\int_0^{r_i^c + q_i^c} p_i^c(\tau_i)d\tau_i - C_i(q_i^c)\right]$$

subject to:

$$r_i^c + q_i^c \geq 0, \quad i \in N \tag{5}$$
$$\sum_{i \in N} r_i^c = 0 \tag{6}$$
$$-K_l^c \leq \sum_{i \in N} D_{l,i}^c r_i^c \leq K_l^c, \quad l \in L \tag{7}$$

Since the generators' decision variables q_i^c are treated as constant parameters in the SO's decision, the term $C_i(q_i^c)$ can be dropped from the objective function without affecting its optimal solution.

The constraint (3) is excluded in spot market decision problems because it has been considered by the generators in the forward market.

C. Spot market smooth formulation

The generators' and the system operator's decision problems in the spot market do not have straightforward optimality conditions due to the non-smooth function characterizing the spot prices. In this sub-section, we reformulate these problems by removing the minimization terms of spot nodal prices. It is accomplished by considering separately two cases.

1) High spot caps: The first case is of high spot caps, i.e. $\overline{u} \geq \overline{p}^c$. Due to the constraint (5), it must hold that

$$\overline{p} \geq p_i^c(q_i^c + r_i^c)$$

Thus the spot nodal prices are

$$\min\{p_i^c(q_i^c + r_i^c), \overline{u}\} = p_i^c(q_i^c + r_i^c),$$

and the spot zonal prices are

$$u_z^c = \sum_{i:z(i)=z} \delta_i \min\{\overline{u}, p_i^c(q_i^c + r_i^c)\}$$
$$= \sum_{i:z(i)=z} \delta_i p_i^c(q_i^c + r_i^c) \quad z \in Z.$$

High spot price caps are hence not binding, and we drop them from the generators' decision problems. So the generators' spot decision problems become

$$\mathcal{G}_g^c: \quad \max_{q_i^c} \quad \pi_g^c$$

subject to:

$$h_z \leq \overline{h} \quad z \in Z \tag{3}$$
$$\underline{q}_i^c \leq q_i^c \leq \overline{q}_i^c, \quad i \in N_g \tag{4}$$
$$q_i^c + r_i^c \geq 0, \quad i \in N_g \tag{5}$$

where

$$\pi_g^c = \sum_{i \in N_g} p_i^c(r_i^c + q_i^c)q_i^c + \sum_{z \in Z}(h_z - u_z^c)x_{g,z} - \sum_{i \in N_g} C_i(q_i^c)$$
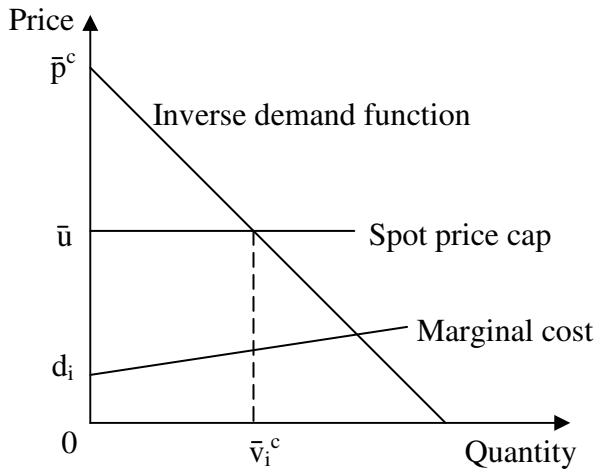
Fig. 1. Low spot price caps

For the system operator, she still faces the decision problem:

$$\mathcal{S}^c : \quad \max_{r_i^c} \quad \sum_{i \in N} \int_0^{r_i^c + q_i^c} p_i^c(\tau_i) d\tau_i$$

subject to:

$$r_i^c + q_i^c \geq 0, \quad i \in N \tag{5}$$

$$\sum_{i \in N} r_i^c = 0 \tag{6}$$

$$-K_l^c \leq \sum_{i \in N} D_{l,i}^c r_i^c \leq K_l^c, \quad l \in L \tag{7}$$

2) Low spot price caps: When the spot price caps are low, i.e. $\bar{u} < \bar{p}^c$, they intersect the inverse demand functions, and are possibly binding (see figure 1). Although spot nodal price $\min\{p_i^c(q_i^c + r_i^c), \bar{u}\}$ is a minimization function and the generators' objective is to be maximized, we can not apply a "min-max" formulation to it. This is because of the uncertainty of the relative magnitude between $x_{g,z}$ and q_i^c in π_g^c, which we should not restrict. To overcome this issue, we introduce artificial variables

- v_i^c: inframarginal quantity from $q_i^c + r_i^c$ at node i which the generators evaluate at the spot price cap.

Following from its definition, v_i^c must satisfies

$$0 \leq v_i^c \leq q_i^c + r_i^c$$

Moreover, quantity v_i^c can not be to the right of the intersection point of the price cap and the inverse demand functions, i.e.

$$v_i^c \leq \bar{v}_i^c = \frac{\bar{p}^c - \bar{u}}{b_i^c}$$

Due to the SO's objective to maximize social welfare, the optimal spot outcomes will set v_i^c as high as possible up to $\min\{\bar{v}_i^c, q_i^c + r_i^c\}$, before setting the remaining quantity $q_i^c + r_i^c - v_i^c$ to the decreasing line segment of the inverse demand functions. In another word, it can not be true that

$$v_i^c < \min\{\bar{v}_i^c, q_i^c + r_i^c\} \text{ and } q_i^c + r_i^c - v_i^c > 0.$$

Otherwise, the social surplus is not maximized. The spot nodal prices are thus functions of q_i^c, r_i^c and v_i^c: $\bar{u} - b_i^c \cdot (q_i^c + r_i^c - v_i^c)$.

The system operator's social welfare maximization problem becomes:

$$\hat{\mathcal{S}}^c : \max_{r_i^c, v_i^c} \quad \sum_{i \in N} \left(\int_0^{v_i^c} p_i^c(\tau_i) d\tau_i + \int_{\bar{v}_i^c}^{\bar{v}_i^c + r_i^c + q_i^c - v_i^c} p_i^c(\tau_i) d\tau_i \right)$$

subject to:

$$\sum_{i \in N} r_i^c = 0 \tag{6}$$

$$-K_l^c \leq \sum_{i \in N} D_{l,i}^c r_i^c \leq K_l^c, \quad l \in L \tag{7}$$

$$0 \leq v_i^c \leq \bar{v}_i^c, \quad i \in N \tag{8}$$

$$r_i^c + q_i^c - v_i^c \geq 0, \quad i \in N \tag{9}$$

The generation firms' spot profit maximization problems are:

$$\hat{\mathcal{G}}_g^c : \quad \max_{q_i^c} \quad \pi_g^c$$

subject to:

$$h_z \leq \bar{h} \quad z \in Z \tag{3}$$

$$\underline{q}_i^c \leq q_i^c \leq \bar{q}_i^c, \quad i \in N_g \tag{4}$$

$$q_i^c + r_i^c - v_i^c \geq 0, \quad i \in N_g \tag{9}$$

where the generators' spot profits are

$$\pi_g^c = \sum_{i \in N_g} (\bar{u} - b_i^c \cdot (q_i^c + r_i^c - v_i^c)) q_i^c$$

$$+ \sum_{z \in Z} (h_z - u_z^c) x_{g,z} - \sum_{i \in N_g} C_i(q_i^c)$$

D. Spot market outcomes

The spot market is described by the pair of $\mathcal{G}_g^c$ and $\mathcal{S}^c$ if spot price caps are no less than the price intercepts of the inverse demand functions, or otherwise the pair of $\hat{\mathcal{G}}_g^c$ and $\hat{\mathcal{S}}^c$. These four problems are all strictly concave-maximization programs, which implies that their first-order necessary conditions (the KKT conditions) are also sufficient. Thus, the spot market outcomes can be replaced by their KKT conditions.

In the spot market equilibrium, the shadow prices of the constraint (3) regarding each zone are equal across all states for each generator. If not, the generators could gain profits by decreasing their outputs in states with lower shadow prices and increase the outputs in states with higher shadow prices. Moreover, the shadow prices in each zone are equal for different generators, because they are also the marginal profit for new entry due to forward price caps. We define these shadow prices, the Lagrangian multipliers to the constraint (3), as η_z.

Let ρ_{i-}^c and ρ_{i+}^c be the Lagrangian multiplier to both directions of the constraint (4), μ_i^c to the constraint (5) and (9), α^c to the constraint (6), λ_{l-}^c and λ_{l+}^c to both directions of constraint (7), and β_{i-}^c and β_{i+}^c to both directions of constraint (8), we have KKT conditions as follows:

1. For the pair of $\mathcal{G}_g^c$ and $\mathcal{S}^c$

$(KKT1):$ $\quad \sum_{j \in N} r_j^c = 0$

$$\bar{p}^c - b_i^c q_i^c - b_i^c r_i^c - \alpha^c + \mu_i^c$$
$$+ \sum_{t \in L} (\lambda_{t-}^c - \lambda_{t+}^c) D_{t,i}^c = 0$$

$$0 \le \lambda_{l-}^c \perp \sum_{j \in N} D_{l,i}^c r_j^c + K_l^c \ge 0$$

$$0 \le \lambda_{l+}^c \perp K_l^c - \sum_{j \in N} d_{l,j}^c r_j^c \ge 0$$

$$0 \le \mu_i^c \perp q_i^c + r_i^c \ge 0$$

$$\bar{p}^c - 2b_i^c q_i^c - b_i^c r_i^c - d_i - s_i q_i^c + \mu_i^c$$
$$+ (1 - Pr(c))\delta_i b_i^c x_{g,z(i)} + \rho_{i-}^c - \rho_{i+}^c$$
$$+ Pr(c)\delta_i b_i^c \eta_z = 0$$

$$0 \le \eta_z \perp \bar{h} - h_z \ge 0$$

$$0 \le \rho_{i-}^c \perp q_i^c - \underline{q}_i^c \ge 0$$

$$0 \le \rho_{i+}^c \perp \bar{q}_i^c - q_i^c \ge 0$$

2. for the pair of $\hat{\mathcal{G}}_g^c$, and $\hat{\mathcal{S}}^c$

$(KKT2):$ $\quad 0 \le \mu_i^c \perp q_i^c + r_i^c - v_i^c \ge 0$

$$\sum_{j \in N} r_j^c = 0$$

$$\bar{p}^c - b_i^c v_i^c - \bar{u} + b_i^c \cdot (r_i^c + q_i^c - v_i^c) - \alpha^c + \mu_i^c$$
$$+ \sum_{t \in L} (\lambda_{t-}^c - \lambda_{t+}^c) D_{t,i}^c = 0$$

$$\bar{u} - b_i^c \cdot (r_i^c + q_i^c - v_i^c) + \beta_{i-}^c - \beta_{i+}^c - \mu_i^c = 0$$

$$0 \le \lambda_{l-}^c \perp \sum_{j \in N} D_{l,i}^c r_j^c + K_l^c \ge 0$$

$$0 \le \lambda_{l+}^c \perp K_l^c - \sum_{j \in N} D_{l,j}^c r_j^c \ge 0$$

$$0 \le \beta_{i-}^c \perp v_i^c \ge 0$$

$$0 \le \beta_{i+}^c \perp \bar{v}_i^c - v_i^c \ge 0$$

$$\bar{u} - 2b_i^c q_i^c - b_i^c \cdot (r_i^c - v_i^c) - d_i - s_i q_i^c + \mu_i^c$$
$$+ (1 - Pr(c))\delta_i b_i^c x_{g,z(i)} + \rho_{i-}^c - \rho_{i+}^c$$
$$+ Pr(c)\delta_i b_i^c \eta_z = 0$$

$$0 \le \eta_z \perp \bar{h} - h_z \ge 0$$

$$0 \le \rho_{i-}^c \perp q_i^c - \underline{q}_i^c \ge 0$$

$$0 \le \rho_{i+}^c \perp \bar{q}_i^c - q_i^c \ge 0$$

$$(10)$$

E. the Forward Market

In the forward market, network feasibility is ignored and the forward contracts are settled. Each firm g takes all her rivals' forward quantities as given, and determines her own best forward quantities to maximize her expected spot utility. Assuming the firms are risk neutral, their forward objectives are to maximize their expected spot profits subject to the KKT conditions which represent the anticipated equilibrium in the spot market. The generators should also consider constraint

(3) in the forward market so that their forward contracting decisions will not result in expected spot zonal prices, or the forward prices, above the forward price cap. Thus each firm g's optimization problem in the forward market is a stochastic MPEC problem:

$$\max_{x_{g,z}} \quad E^c[\pi_g^c] = \sum_{c \in C} Pr(c)\pi_g^c$$

subject to:

$$\times_{z \in Z} x_{g,z} \in X_g \quad z \in Z$$

and constraints (KKT1) and (KKT2)

where X_g defines the set of allowable non-negative forward positions for firm g.

Combining the generators' MPEC problems, the equilibrium problem in the forward market is an EPEC. Furthermore, this EPEC problem is a stochastic EPEC due to the uncertainty of exogenous data.

Note that variables r_i^c and α^c can be eliminated from the KKT conditions, we further define

- x_g: The vector of firm g's forward variables.
  $x_g = [x_{g,z}, z \in Z]$
- y: $y = [y^c, c \in C]$ where y^c is the vector of lagrangian multipliers for all inequality constraints in the generators' and the SO's decision problems in the spot market.
- w: $w = [w^c, c \in C]$ where w^c is the slackness of the constraints corresponding to y^c.

Then spot market KKT conditions becomes the following parametric LCP with respect to w and y with x_g being the parameters

$$w = a + \sum_g A^g x_g + My, \quad 0 \le w \perp y \ge 0 \qquad (11)$$

where a, A^g, and M are suitable vector and matrices derived from constraints (KKT1) and (KKT2).

Now, the generators' forward objectives $E^c[\pi_g^c]$ can be expressed as functions $f_g(x_g, x_{-g}, y, w)$ with respect to x_g, y, and w, where "$-g$" denotes $G\backslash\{g\}$ and x_{-g} denotes all other firms' design variables except for generator g's. The firms' forward problems can be represented as:

$$\mathcal{F}_g: \quad \min_{x_g, y, w} \quad f_g(x_g, x_{-g}, y, w)$$

subject to :

$$x_g \in X_g \qquad (12)$$

$$w = a + A^{-g} x_{-g} + A^g x_g + My \qquad (13)$$

$$0 \le w \perp y \ge 0 \qquad (14)$$

Here, x_g are design variables, y, w are state variables, x_{-g} are parameters. The constraints (13) and (14) are a rewriting of the LCP constraint (11) by separating x_g and x_{-g}. Moreover, the constraints (13) and (14) are shared among all generators.

We observe from the problems $\mathcal{S}^c$, $\mathcal{G}_g^c$, $\hat{\mathcal{S}}^c$ and $\hat{\mathcal{G}}_g^c$ that M is positive semi-definite. Note that both problems $\mathcal{G}_g^c$ and $\mathcal{S}^c$ have optimal solutions for any set of x_g, the KKT conditions (KKT1) and (KKT2) have thus feasible solutions, so do the LCP constraints (11) as a transformation of the KKT conditions. Furthermore, due to theorem 3.1.7 in [7], the LCP problem (11) satisfies the w-uniqueness condition.
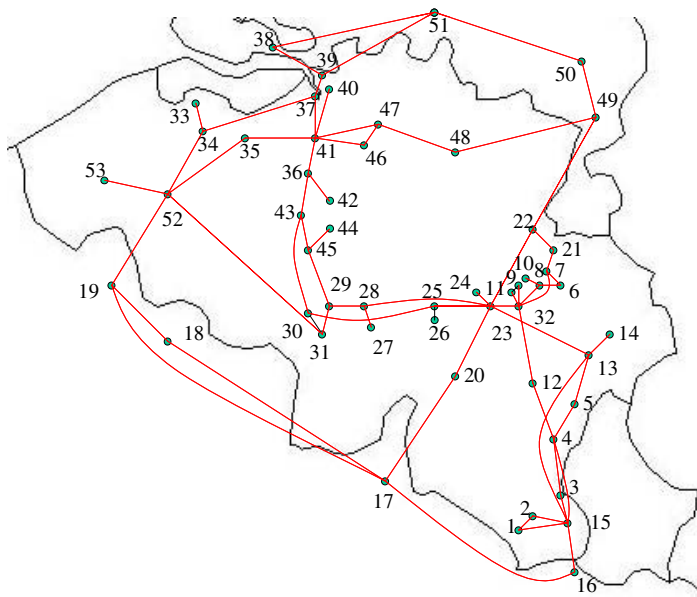
Fig. 2.   Belgian high voltage network

For simplicity, we further assume that the exogenous parameters in our formulation are perturbed in such a way that the spot decision problems have non-degenerate optimal solutions. Following again from theorem 3.1.7 in [7], the LCP constraints (11) have a unique y solutions as well. This property is stated as follows.

A1: The LCP constraints (11) have unique solutions of w and y corresponding to all feasible $\{x_g\}$.

Another observation from problems $\mathcal{F}_g$ is that

A2: the objective function are quadratic.

Properties A1 and A2 guarantee that the solution approaches in [21] are also applicable here.

IV. THE BELGIAN ELECTRICITY MARKET

We use the Belgian electricity network to illustrate the economics results. For the completeness of the network, we incorporate some lines in the Netherlands and France. The network has 92 380kV and 220kv transmission lines, some of which are parallel lines between the same pair of nodes. For computational purpose, parallel lines are combined to single lines with adjusted thermal capacities and resistances, thus the network is reduced to 71 transmission lines linking 53 nodes (see figure 2). Insignificant lower voltage lines and small generation plants have been excluded from this example.

We assume that there are six states in the spot market. The first state is a state in which the demands are at the shoulder, all generation plants operate at their full capacity, all transmission lines rated at full thermal limits. The second state is the same as the first state except that it has on-peak demand. Off-peak state 3 differs from state 1 by the very low demand levels. State 4 denotes the contingency of the transmission line [31,32] being out of service. State 5 and 6 capture the unavailability of two plants at node 10 and 41 respectively. The assumed probabilities of these states are given in table I.

We assume that there are two generation firms competing in the forward market and the spot market, and two zones in

the network with nodes 1 through 32 being in zone 1 and the remaining nodes being in zone 2. Firm 1 owns plants at nodes 7, 9, 11, 31, 32, 33, 35, 37, 41, 47, and 53, and firm 2 owns plants at nodes 10, 14, 22, 24, 40, 42, 44, and 48. There are no generation plants on other nodes. The corresponding information of generation plants is listed in table II. More details are given in table II describing the nodal information on inverse demand functions, first-order marginal generation costs, generation capacities and historical load ratios (in this example, marginal costs are assumed constant). As to thermal limits, we ignore the intra-zonal flows and focus only on the flowgates of lines [22,49], [29,45], [30,43], and [31,52]. We consider five test cases:

- Case 1: price-uncapped single-settlement system, i.e. single settlement without (spot) price caps.
- Case 2: price-capped single-settlement system, i.e. single settlement with (spot) price caps.
- Case 3: price-uncapped two-settlement system, i.e. two settlements without spot and forward price caps.
- Case 4: forward-capped two-settlement system, i.e. two settlements with only forward price caps. We set the forward price cap to 425, about 10% below the prices in the price-uncapped single-settlement system.
- Case 5: spot-capped two-settlement system, i.e. two settlements with only spot price caps. The spot price cap is set to 600.

Case 1 and 2 are essentially equivalent to a two-settlement system with the allowable forward contracts forced to zero quantity.

We restrict the generators' forward positions to their total generation capacity in the respective zones. Table III reports the spot zonal prices of these five scenarios. We find that

- Both spot and forward price caps reduce the magnitude of spot prices as compared to the corresponding single-settlement market.
- Spot price cap reduces generators' incentives to commit forward contracts compared to price-uncapped two-settlement system. The spot price cap causes the generators to commit about 70% compared to the price-uncapped two-settlement markets. This is because the spot price caps themselves already reduce spot prices leaving less incentives for generators to commit forward contracts.
- Forward price caps increase generators' incentives for forward contracting. The generators commitment in the forward-capped two-settlement markets is about 180% compared to the same markets without caps. Under the forward price caps, the generators are in fact net electricity buyer in the spot market. This is because, knowing that entry would occur if the forward prices are too high, the incumbents have to act more competitively to deter it. They will only play an equilibrium in which the forward prices are below the forward price cap.
- Compared to the price-uncapped two-settlement system, the spot cap results in more production in on-peak states, therefore the on-peak prices are lower; but, the reduced forward contracting due to spot caps cause different

TABLE I

STATES OF THE BELGIAN NETWORK

State	Probability	Type and description
1	0.50	Shoulder state: Demands are at shoulder.
2	0.20	On-peak state: All demands are on the peak.
3	0.20	Off-peak state: All demands are off-peak.
4	0.03	Shoulder demands with line breakdown: Line [31,32] goes down.
5	0.03	Shoulder demands with generation outage: Plant at node 10 goes down.
6	0.04	Shoulder demands with generation outage: Plant at node 41 goes down.

TABLE II

NODAL INFORMATION.

Node Id	demand slope	marginal cost	capa-city	Node Id	demand slope	marginal cost	capa-city
1	1	N/A	0	28	1	N/A	0
2	0.82	N/A	0	29	0.93	N/A	0
3	1.13	N/A	0	30	0.85	N/A	0
4	1	N/A	0	31	1	180	712
5	0.93	N/A	0	32	1	580	95
6	0.85	N/A	0	33	0.88	20	496
7	1	450	70	34	0.5	N/A	0
8	1	N/A	0	35	1	250	1053
9	0.88	180	460	36	0.73	N/A	0
10	0.9	180	121*	37	1	100	1399
11	1	200	124	38	0.85	N/A	0
12	0.73	N/A	0	39	1	N/A	0
13	1	N/A	0	40	1.15	100	1378
14	0.85	130	1164	41	1	210	522**
15	1	N/A	0	42	0.79	180	385
16	1.3	N/A	0	43	0.68	N/A	0
17	1	N/A	0	44	1.03	200	538
18	0.79	N/A	0	45	1	N/A	0
19	0.68	N/A	0	46	1	N/A	0
20	1.05	N/A	0	47	1	100	32
21	1	N/A	0	48	0.73	220	258
22	1.1	190	602	49	1.2	N/A	0
23	1	N/A	0	50	1.5	N/A	0
24	0.75	100	2985	51	1	N/A	0
25	1	N/A	0	52	1	N/A	0
26	0.8	N/A	0	53	1	200	879
27	1.13	N/A	0				

*,**: these numbers are zeros in state 5 and 6 respectively.
N/A: the marginal costs are not applicable to zero capacities.

TABLE III

SPOT ZONAL PRICES COMPARISONS

	Case 1		Case 2		Case 3		Case 4		Case 5	
	zone 1	zone 2	zone 1	zone 2	zone 1	zone 2	zone 1	zone 2	zone 1	zone 2
state 1	428.29	429.04	428.29	429.04	423.95	424.79	363.30	363.03	425.27	425.60
state 2	849.75	844.72	600.00	600.00	845.18	841.02	823.53	823.75	600.00	600.00
state 3	232.30	232.30	232.30	232.30	224.24	224.24	189.45	189.45	229.46	229.46
state 4	428.20	429.14	428.20	429.14	419.75	420.97	405.46	406.32	424.38	424.56
state 5	429.96	430.78	429.96	430.78	421.55	422.53	407.19	407.93	426.11	426.12
state 6	431.35	432.31	431.35	432.31	423.34	424.49	409.34	410.47	428.77	428.14
Expected	473.55	473.01	423.56	424.07	468.03	467.73	425.00	425.00	421.19	421.34

results in off-peak states, i.e. lower production and higher nodal prices. On the contrary, the increased forward contracting due to forward price caps results in more production and lower spot prices compared to the price-uncapped two-settlement markets.

Note that the forward prices are decreasing functions in forward commitments and spot generation quantity, the generators will succeed in deterring entry by playing an equilibrium with forward prices lower than the forward cap whenever then have sufficient capacity. On the other hand, if the generators don't have enough capacity, the forward contracts will be signed at prices higher than the forward price cap, and entry

to the markets is inevitable. This result is consistent with Newbery [15].

V. Concluding Remarks

In this paper, we extend our model in [20] to the case in which either the forward prices or the spot prices are capped. The forward caps represent an approximation to competitive entry by new generators. We formulate the Cournot equilibrium in the price-capped two-settlement markets as a stochastic equilibrium problem with equilibrium constraints. We also consider the spot market with uncertainty of demand, generation capacity, and thermal limits of transmission lines.

The main goal in this paper is the development of an effective formulation to analyze capping alternatives under a variety of scenarios in the framework of two-settlement markets. We run test cases based on the Belgian electricity market. The resulting equilibrium reveals less incentives from the generators to commit forward contracts due to the spot price caps, and more incentives due to forward price caps. However, the spot zonal prices under both cap types still decrease as compared to the respective single-settlement cases. Moreover, these two cap types result in different behaviors of spot production and spot energy prices. Additional interesting analysis concerning social welfare, generator's profits as well as other aspects of the market affected by the presence of price caps, will be the subject of future work.

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