Problem 1
Parameters:

\[
\begin{align*}
\lambda &= 6000 \\
K &= $25 \\
I &= 0.5
\end{align*}
\]

All units discount:

\[
Q_1 = \sqrt{\frac{2(25)(6000)}{0.5(4.5)}} = 346.4 \Rightarrow \text{ok}
\]

\[
Q_2 = \sqrt{\frac{2(25)(6000)}{0.5(4.75)}} = 355.4 < 3000 \Rightarrow \text{not feasible}
\]

\[
Q_3 = \sqrt{\frac{2(25)(6000)}{0.5(5)}} = 365.15 < 6000 \Rightarrow \text{not feasible}
\]

Total costs: at solution and break-points:

\[
G(346) = 25(6,000/346) + 0.5(5)(346)/2 + 6,000*5 = 30,866
\]

\[
G(3,000) = 25(6,000/3,000) + 0.5(4.75)(3000)/2 + 6,000*4.75 = 32,112.5
\]

\[
G(6,000) = 25(6,000/6,000) + 0.5(4.5)(6000)/2 + 6,000*4.5 = 33,775
\]

Incremental units discount:

\[
Q_1 = \sqrt{\frac{2(25)(6000)}{0.5(5)}} = 346.4 \Rightarrow \text{ok}
\]

\[
Q_2 = \sqrt{\frac{2(6000)[25 + (5.00 - 4.75)(1500)]}{0.5(4.75)}} = 1421.6 \Rightarrow \text{infeasible}
\]

\[
Q_3 = \sqrt{\frac{2(6000)[25 + (5.00 - 4.50)(1500) + (4.75 - 4.50)(1500)]}{0.5(4.5)}} = 2476.6 \Rightarrow \text{infeasible}
\]

Total costs: at above solution points:

\[
G(346) = 30,866 (\text{same as in all-units discount})
\]

The manager should order 346 units every time and make about 17 orders a year. Also, the manager could order from either company since the costs are the same.

Problem 2
Parameters:

\[
\begin{align*}
c &= $8 \\
\text{selling price} &= $10 \\
\lambda &= 300/\text{day} \times 350 \text{days} = 105,000/\text{year} \\
\text{lead time} &= 2 \text{ days} \\
I &= 0.8
\end{align*}
\]
For weekly demand:
\( \mu = 300 \)
\( \sigma = 60 \)
so demand during the lead time has parameters
\( \mu_{LT} = 300 \times 2 = 600 \)
\( \sigma_{LT} = \sqrt{2} \times 60 = 84.85 \)

\( K = 5000 \) if sending one truck for each order
\( Q = \) multiple of 100

a) \( \alpha = 0.02 \) (continuous review model using Type 1 service)
If we use one truck per order:
\[ Q^* = \sqrt{\frac{2 \times (750) \times (105,000)}{0.80 \times 8}} = 4960 \) (EOQ)
Since \( Q \) must be a multiple of 100, we round it up to 5000
\[ F(R) = 0.02 \Rightarrow z = -2.06 \Rightarrow R = \mu + z \sigma_{LT} = 600 - 2.06(84.85) = 425.2 \approx 426 \]
Hence \((Q,R) = (5000,426)\)

b) \( \)fill-rate=99.9\% (continuous review model with Type 2 service)
\( Q_0 = 4960 \) (same as \( EOQ \))
\( n(R) = (1 - \beta)Q = 4.960 \)
\( \sigma L(z) = 4.960 \)
\[ L(z) = \frac{4.960}{84.85} = 0.0585 \]
\[ z = 1.18 \]
\[ F(R) = 0.8810 \]

Do the iterations:
\[ Q_1 = \frac{4.960}{1 - 0.8810} + \sqrt{\frac{2(750)(105,000)}{0.8(8)} + \left[ \frac{4.960}{1 - 0.8810} \right]^2} = 5002.64 \]
\[ n(R) = 5.00 \]
\[ L(z) = \frac{5.00}{84.85} = 0.589 \]
\[ z = 1.18 \]

\( z \) has converged to 1.18, so we stop
We need to adjust Q downward to 5000 because of the truck capacity, and technically, we should find the best R for Q = 5000, but \( n(R) = 5.00 \) used in the last iteration is correct for Q = 5000, so we are done.

\[
R = 600 + 1.18(84.85) = 700.12 \rightarrow 701
\]

Hence \((Q,R) = (5000, 701)\)

c) We need to make an assumption about whether substituting the name brand product causes a stock-out of that product (name brand) or whether a stock-out leads to a lost sale. If we assume that a substitution does not cause a stock-out of the name brand product, then 
\[ p = 9 - 8 = 1 \] (giving out an item that costs $9 instead of one that costs $8)

Following the Q-R method,
\[
F(R) = 1 - \frac{0.8 \times 8 \times 4960}{1 \times 105,000} = 0.6977
\]

\[ z = 0.52 \]

\[ L(z) = .1917 \]

\[ n(R) = \sigma L(z) = 84.85(.1917) = 16.27 \]

\[
Q_1 = \sqrt{\frac{2(105,000)(750 + 1(16.27))}{0.8(8)}} = 5014.3 \rightarrow 5000
\]

\[
F(R) = 1 - \frac{6.4(5000)}{1(105,000)} = 0.695
\]

\[ z = 0.51 \]

z is very close to the result in previous iteration, so we stop.

\[ R = 600 + 0.51(84.85) = 643.3 \]

So we have \((Q,R) = (5000, 644)\)

**Problem 3**

Parameters:

K=50, c=6, i=0.8, Lead time=1 day

a) 
\[
T = \sqrt{\frac{2(50)}{0.8(6)(105000)}} = 0.0141yr = 4.93 \text{ days}
\]

Send a truck every 5 days

b) 
shortage cost=8-6=$2
overage cost = inventory holding cost for 5 days = 0.8(6)*(5/350)=0.0686

\[ F(S) = \frac{2}{2+0.0686} = 0.9668 \]

Your answer will differ if you assumed a different value of T, but the key point is that any excess inventory is held for the duration of the order cycle.
c). Demand during order cycle + lead time (5 days + 1 day) ~ N(1800, $\sqrt{6 \times 60}$)
   The warehouse has asked for a probability of no stock out during the lead time of 98% > 0.9668 (from part b), hence we use $F(S) = 0.98$ to find $z = 2.06$, $S = 1800 + 2.06 \times (\sqrt{6 \times 60}) = 2103$
   So supplier should “order” up to 2103 units every 5 days.

4. Weekly demand:
   \[ P\{D=0\} = 0.3 \]
   \[ P\{D=1\} = 0.5 \]
   \[ P\{D=2\} = 0.2 \]
   \[ c = $3 \]
   Selling price = $8
   Penalty for shortage = $2
   \[ i = 1 \text{ per year} = 0.02 \text{ per week} \text{ so } h = 0.02($3) = $0.06 \text{ per week} \]
   \[ K = 5 \text{ (if not in multiple of 10 reams)} \]

   a) \[
   \begin{pmatrix}
   0.3 & 0 & 0.7 \\
   0.5 & 0.3 & 0.2 \\
   0.2 & 0.5 & 0.3 \\
   0 & 0.2 & 0.3 \\
   \end{pmatrix}
   \]

   b) \[
   \begin{array}{cccccc}
   \text{demand} & \text{ordering} & \text{inventory} & \text{shortage} & \text{total} & \text{probability} \\
   0 & 0 & 0.06(1) & 0 & 0.06 & 0.3 \\
   1 & 5+3(4) & 0 & 0 & 17 & 0.5 \\
   2 & 5+3(5) & 0 & 2(1) & 22 & 0.2 \\
   \end{array}
   \]
   Expected cost = (.3)(.06)+(.5)(17)+(.2)(22) = $12.92

   c) Average demand = 0.9 reams per week = 46.8 reams per year
   If $K = 5$ (if we don’t order in multiples of 10 reams), then $EOQ = \sqrt{\frac{(2)(5)(46.8)/(1*3)}{1*3}} = 12.49$. So, if we had to pay the ordering (setup) cost, we would want to order about 12 reams at a time. But we can avoid paying the setup cost by ordering 10 reams at a time. The only question remaining is how low we should let the inventory level go before ordering. The newsvendor fractile is $2/[2+.02(3)] = 0.971$, so considering the tradeoff between shortage costs and the cost of holding for one week (i.e., the order cycle), we want to make sure that we satisfy demand with very high probability.

   So $Q = 10$, $R = 2$ is a reasonable option if the lead time is one week and we are willing to review the inventory continuously.

   Another option that is easier to implement is $Q = 10$, $s = 2$. That is, use periodic review (rather than continuous review) and order 10 whenever inventory drops below $s$. (We have not discussed the (Q,s) policy in class, but it may be a sensible policy when certain batch sizes are much more convenient than others and we want to use the same batch size each time, but we don’t want to review inventory continuously.)