1. The scheduled output of a fab in a particular week was as follows:

Product 1 1,000 units
Product 2 2,000 units
Product 3 20,000 units
Product 4 10,000 units
Product 5 5,000 units

The actual output in the week was as follows:

Product 1 900 units
Product 2 1,900 units
Product 3 21,000 units
Product 4 1,000 units
Product 5 5,500 units

The LIPAS (line-item performance against schedule) is an on-time delivery metric defined as the fraction of items with scheduled output in a time period whose output in the period meets or exceeds the scheduled output. What is the LIPAS score for that week?

On time delivery score is the fraction of products delivered on time. For this case, products 3 and 5 are on time, while products 1, 2 and 4 are not. So the LIPAS score is $2/5 = 0.4$.

2. The LIPAS metric for on-time delivery presented in class (and also widely used in industry) is the fraction of all product types that were delivered on time. (A product type is considered on-time if actual output equals or exceeds scheduled output.) This metric is insensitive to the amount of shortage for each product.

Develop a new metric for on-time delivery that reflects the amount of shortage for each product. The metric should have the properties that (1) a score of 1.0 means all product types were on time, (2) no credit is given for excess production of any product type, but there is no penalty for excess production, either, and (3) if the shortage of any product increases while shortages of all other products are held constant, then the metric score decreases.

First, consider the case of on-time delivery in a single time period. Let $d_i$ denote the scheduled output quantity of product $i$. Let $p_i$ denote the actual output quantity of product $i$. The fraction of product $i$ delivered on time is

$$f_i = \frac{\text{Min} \{d_i, p_i\}}{d_i}.$$
We could do a weighted average of the on time delivery scores for all products to arrive at an on-time delivery score for the factory. That is,

\[
OTD = \frac{\sum d_i f_i}{\sum d_i},
\]

or, equivalently,

\[
OTD = \frac{\sum \min \{ d_i, p_i \}}{\sum d_i}
\]

Note the numerator sums over the portion of actual production that was demanded, and the denominator sums over all demand.

Next, we consider the case of a stream of time periods in which surplus in period \( t-1 \) can be used to make up for shortfall in period \( t \). Let \( d(t) \) denote the demand for product \( i \) in period \( t \) and let \( p(t) \) denote the actual output of product \( i \) in period \( t \). Let \( I_i(0) \) denote the inventory (shortage if negative) at time 0, i.e., the start of period 1. The fraction of product \( i \) delivered on time in period \( t \) is

\[
f_i(t) = \frac{\min \{ d_i(t), p_i(t) + \max \left[ 0, I_i(0) + \sum_{\tau=1}^{t-1} p_i(\tau) - \sum_{\tau=1}^{t-1} d_i(\tau) \right] \}}{d_i(t)},
\]

and the on-time delivery metric becomes

\[
OTD(t) = \frac{\sum d(t) f(t)}{\sum d_i(t)}.
\]

3. A control chart is being set up to track the resistivity after the ion implantation process step. Five sample measurements are made from one wafer per lot. The average of the measurements and the range of the measurements are computed. One hundred observations from 20 lots have been collected. The average mean is 97.5, and the average range of the wafer measurements is 5.21.

(a) Assuming the process was stable during processing of the 20 lots, determine three-sigma control limits for R and X-bar charts.

\[\mu = 97.5, \ \bar{R} = 5.21, \ n = 5\]
\[\sigma = 5.21 / d_2 = 5.21 / 2.326 = 2.24\]

X-bar chart:
\[\text{UCL} = 97.5 + 3(2.24)/5^{0.5} = 100.5\]
\[\text{LCL} = 97.5 - 3(2.24)/5^{0.5} = 94.5\]

R-chart:
LCL = $d_3 \bar{R} = 0$
UCL = $d_4 \bar{R} = 2.1(5.21) = 10.99$

(b) Suppose the mean of the resistivity shifts to 100.0. What is the probability that the X-bar chart will make a Type 2 error in the next lot? (A Type-2 error means the process is out of control but the control chart does not detect this.)

$$\text{Prob}\{94.5 \leq \bar{X} \leq 100.5 \mid \mu = 100\}$$

$$= \text{Prob}\{ (94.5-100.0)/\{2.24/5^{0.5}\} \leq Z \leq (100.5-100.0)/\{2.24/5^{0.5}\} \}$$

$$= \text{Prob}\{ -5.490 \leq Z \leq 0.4992 \}$$

$$= \Phi(0.4992) - \Phi(-5.490) = \Phi(0.4992) = 0.6914$$

(c) What is the probability the X-bar chart will make Type 2 errors in each of the next five lots?

$$(0.6914)^5 = 0.158$$

4. A process is monitored using an X-bar chart with UCL = 13.8 and LCL = 8.2. The process standard deviation is estimated to be 6.6. If the X-bar chart is based on three-sigma limits,

(a) What is the estimate of the process mean?

UCL = $\mu + 3\sigma / n^{0.5}$, LCL = $\mu - 3\sigma / n^{0.5}$

$\mu = (\text{UCL} + \text{LCL}) / 2 = 11$

(b) What is the size of each of the sampling subgroups?

UCL - LCL = $6\sigma / n^{0.5}$

$n^{0.5} = (6*6.6) / (13.8 - 8.2) = 7.071$

$n = 50$

5. An R-chart is used to monitor the variation in the weights of packages of chocolate chip cookies produced by a large national producer of baked goods. An analyst has collected a baseline of 200 observations to construct the chart. Suppose the computed value of $\bar{R}$ (the average value of the range) is 3.825.

(a) If subgroups of size six are used, compute the value of three-sigma limits for this chart.

$\bar{R} = 3.825, n = 6$
\[ \sigma = 3.825 / d_2 = 3.825 / 2.534 = 1.51 \]

LCL = \( d_3 \times \bar{R} = 0 \)

UCL = \( d_4 \times \bar{R} = 2 \times 3.825 = 7.65 \)

(b) If an X-bar chart based on three-sigma limits is used, what is the difference between the UCL and the LCL?

UCL = \( \mu + 3\sigma / n^{0.5} \), LCL = \( \mu - 3\sigma / n^{0.5} \)

UCL - LCL = 6\*\( \sigma / 6^{0.5} = 3.70 \)

6. At a particular manufacturing step, the important parameter for process control is the deposition thickness. This is measured at five points on a single wafer from each manufacturing lot passing through the step. The mean of this parameter is 380 and the standard deviation is 54.

(a) Assuming the estimates of the process mean and standard deviation are valid (i.e., the process was in statistical control during the time data was collected to compute them), specify upper and lower control limits for X-bar and R charts.

In X-bar chart:

\[
\text{UCL} = \mu + \frac{3\sigma}{\sqrt{n}} = 380 + \frac{3(54)}{2.236} = 452.45, \quad \text{LCL} = \mu - \frac{3\sigma}{\sqrt{n}} = 380 - \frac{3(54)}{2.236} = 307.55
\]

In R-chart:

\[ \bar{R} = d_2(\sigma) = 2.326(54) = 125.6 \]

LCL = \( d_3 \times \bar{R} = 0 \), UCL = \( d_4 \times \bar{R} = 2.11(125.6) = 265.02 \)

(b) Suppose the process mean suddenly shifts by 27. What is the probability that there will be Type II errors occurring for both of the next two manufacturing lots?

Prob of Type II error in first lot given the mean shifts by \( \delta \sigma \) is

\[
\beta = P \left\{ \left| \bar{X} - \mu \right| \leq \frac{k\sigma}{\sqrt{n}} \bigg| E(\bar{X}) = \mu + \delta \sigma \right\}
\]

\[
= P \left\{ -\frac{k\sigma}{\sqrt{n}} \leq \bar{X} - \mu \leq \frac{k\sigma}{\sqrt{n}} \bigg| E(\bar{X}) = \mu + \delta \sigma \right\}
\]
In this case, $k = 3$, $\delta = 0.5$ and $n = 5$. Using the table in the back of the notes, we find

$\beta = \Phi(1.882) − \Phi(-4.118) = 0.97$

The probability of a Type II error in both of the next two lots is $\beta^2 = 0.941$.

7. The thickness of a film deposited on wafers at a particular process step is subject to statistical process control. The thickness is measured at five points on one wafer per lot. The upper control limit is 132 angstroms and the lower control limit is 96 angstroms.

(a) What kind of control chart(s) should be used to track this parameter? Assume in the following questions that this kind of chart is in use.

X-bar and R charts should be used.

(b) What are the mean and standard deviation of the film thickness?

$\mu = (132 + 96)/2 = 114$ angstroms.

$132 − 96 = 36 = 6\sigma / \sqrt{5}$ or $\sigma = 13.42$.

(c) What is the average range of the five measurements?

$R = d_2(\sigma) = 2.326(13.42) = 31.21$

(d) Suppose the process mean suddenly shifts upward by 10 angstroms. What is the probability the mean shift will NOT be detected in the next five lots? (Assume the only control rule is ordinary UCL and LCL for single-lot measurements.)

Prob of Type II error in first lot given the mean shifts by $\delta\sigma$ is

$$
\beta = P\left\{ \left| \bar{X} - \mu \right| \leq \frac{k\sigma}{\sqrt{n}} \mid E(\bar{X}) = \mu + \delta\sigma \right\}
= P\left\{ -\frac{k\sigma}{\sqrt{n}} \leq \bar{X} - \mu \leq \frac{k\sigma}{\sqrt{n}} \mid E(\bar{X}) = \mu + \delta\sigma \right\},
$$
\[
\begin{align*}
&= P\left\{-k - \delta \sqrt{n} \leq \frac{\bar{X} - \mu - \delta \sigma}{\sigma / \sqrt{n}} \leq k - \delta \sqrt{n} \mid E(\bar{X}) = \mu + \delta \sigma\right\} \\
&= P\{-k - \delta \sqrt{n} \leq Z \leq k - \delta \sqrt{n}\} \\
&= \Phi(k) - \Phi(-k) - \Phi(-k) - \Phi(k) \\
&= \Phi(k - \delta \sqrt{n}) - \Phi(-k - \delta \sqrt{n}).
\end{align*}
\]

In this case, \( k = 3 \), \( \delta = 10/13.42 = 0.745 \) and \( n = 5 \). Using the table in the back of the notes, we find
\[
\beta = \Phi(1.334) - \Phi(-4.666) = 0.9089
\]
The probability of a Type II error in all of the next five lots is \( \beta^5 = 0.620 \).

(e) Suppose the process mean does not shift. What is the probability of a false alarm in the next lot?

Prob of Type I error in first lot is
\[
\alpha = P\left\{ \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} > \frac{k \sigma}{\sqrt{n}} \mid E(\bar{X}) = \mu \right\} \\
= 2P\left\{ \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} < -\frac{k \sigma}{\sqrt{n}} \mid E(\bar{X}) = \mu \right\} \\
= 2P\left\{ \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \leq -k \mid E(\bar{X}) = \mu \right\} \\
= 2\Phi(-k) = 2\Phi(-3) = 2(0.0014) = 0.0028.
\]

8. Recent data on rework in the photolithography process at a particular fab are as follows.

<table>
<thead>
<tr>
<th>Shift #</th>
<th># of wafers processed</th>
<th># of wafers reworked</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>650</td>
<td>39</td>
</tr>
<tr>
<td>3</td>
<td>550</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
<td>18</td>
</tr>
</tbody>
</table>

During which shifts was photo rework in statistical control?
This is a p-chart problem with varying subgroup sizes, so we can apply the standardized variate \( Z \) on each shift (\( \bar{p} \) is 0.049 for these data):

\[
Z = \frac{p - \bar{p}}{\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}}
\]

The calculations of \( p \) and \( Z \) for the four shifts are as follows:

<table>
<thead>
<tr>
<th>Shift #</th>
<th>n</th>
<th>( p )</th>
<th>( \bar{p} )</th>
<th>( Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>20/500 = 0.04</td>
<td></td>
<td>-0.932</td>
</tr>
<tr>
<td>2</td>
<td>650</td>
<td>39/650 = 0.06</td>
<td></td>
<td>1.299</td>
</tr>
<tr>
<td>3</td>
<td>550</td>
<td>36/550 = 0.0655</td>
<td></td>
<td>1.793</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
<td>18/600 = 0.03</td>
<td></td>
<td>-2.156</td>
</tr>
</tbody>
</table>

All shifts have a \( |Z| \) value lower than 3. Assuming 0.49 is a good estimate of the long-run mean rework rate, rework was in statistical control during all four shifts.

9. A control chart is being set up to track the number of particles deposited on wafers after etching. Particles are counted on one blank wafer using a wafer surface scanning machine before processing each lot. One hundred observations have been collected. The sample mean number of particles per wafer is 25. The sample variance is 26.2.

(a) Determine three-sigma control limits for an appropriate control chart. What kind of chart is this?

Use a c-chart, mean = 25.

\[
\begin{align*}
\text{UCL} &= 25 + 3*(25^{0.5}) = 40 \\
\text{LCL} &= 25 - 3*(25^{0.5}) = 10 \quad \text{(but LCL may be irrelevant in this case.)}
\end{align*}
\]

(b) Suppose the mean shifts to 30.0. What is the probability the control chart will make Type 2 errors in both of the next two lots?

\[
\text{Prob. of one Type 2 error} = \text{Prob.}\{10 \leq c \leq 40 \mid \mu = 30\} \\
= \text{Prob.}\{ (10 - 30) / 30^{0.5} \leq Z \leq (40 - 30) / 30^{0.5} \} \\
= \text{Prob.}\{ -3.65 \leq Z \leq 1.826 \} = \text{Prob.}\{ Z \leq 1.826 \} = 0.966
\]

The prob. of two Type 2 errors in a row is \((0.966)^2 = 0.933\).
(c) Suppose one of the observations from the sample was 45 particles. Was this point in statistical control? How should the control limits be modified?

Mean was 25 in 100 observations. But the observation of 45 is above UCL and so it was not in statistical control. We should re-calculate the mean after discarding this point. The new mean is \( \frac{2500 - 45}{99} = 24.8 \). The new control limits are

\[
\begin{align*}
UCL &= 24.8 + 3 \times (24.8^{0.5}) = 39.74, \text{ or } 40 \\
LCL &= 24.8 - 3 \times (24.8^{0.5}) = 9.86, \text{ or } 10 \text{ (but LCL may be irrelevant in this case)}
\end{align*}
\]

So we don’t need to change the control limits.