Industrial Engineering & Operations Research, UC Berkeley

IEOR269 Integer Programming and Combinatorial Optimization

Semester: Spring 2004 *Instructor: Alper Atamtürk*

Midterm Exam – Due: March 17, 2004, 5PM (4175 Etcheverry)

1. Let A be a rational valued $m \times n$ matrix and b be a rational valued m–dimensional column vector. Let $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ be a nonempty polyhedron and $x^* \in P$.

   (a) Prove that for x^*, the violation of any Chvátal-Gomory (CG) inequality $\zeta = y^T Ax^* - \lfloor y^T b \rfloor$, where $y \in \mathbb{R}^m_+$, $y^T A$ integral, is upper bounded by a constant.

   (b) Let $s^* = b - Ax^*$ and call a CG inequality *complementary* (with respect to x^*) if $y_i s^*_i = 0$ for all $i = 1, \ldots, m$. Prove that there is a polynomial algorithm that determines whether there is a complementary CG inequality violated by x^* or not (and in the former case outputs such an inequality).

2. Consider the affine transformation $T(x) = QA^{-1/2}(x - a)$ defined in the ellipsoid method of Khachiyan. Prove that

$$T(\hat{E}(A, a)) = \hat{E}(I, 0).$$

3. Prove that if H and G are two faces of a polyhedron P of dimension r and $r + s$ ($r \geq 0, s > 0$ and both integer), respectively, and H is a face of G, then there exists a sequence of faces $\{F_i\}_{i=0}^s$ such that

   (a) $F_0 = H$, $F_s = G$; and

   (b) F_i is a facet of F_{i+1} for $i \in \{0, \ldots, s-1\}$.