
IEOR 265 – Homework 1

Due Thursday, April 2, 2015 in class

In the paper: P. Cortez, A. Cerdeira, F. Almeida, T. Matos, and J. Reis, Modeling wine preferences by data mining from physicochemical properties, *Decision Support Systems*, vol. 47, no. 4:547-553, the authors considered the problem of modeling wine preferences. Wine can be evaluated by experts who give a subjective score, and the question the authors of this paper considered was how to build a model that relates objective features of the wine (e.g., pH values) to its rated quality. For this homework, we will use the data set available at: <http://ieor.berkeley.edu/~ieor265/homeworks/winequality-red.csv>

Use the following methods to identify the coefficients of a linear model relating wine quality to different features of the wine: (1) ordinary least squares (OLS), (2) ridge regression (RR), (3) lasso regression, (4) exterior derivative estimation (EDE) estimator. Make sure to include a constant (intercept) term in your model, and choose the tuning parameters using cross-validation. You may use any programming language you would like to. For your solutions, please include (i) plots of tuning parameters versus cross-validation error, (ii) coefficients (labeled by the feature) computed by each method, (iii) the minimum cross-validation error for each method, and (iv) the source code used to generate the plots and coefficients. Some hints are below:

- a constant (intercept) term can be included in OLS by solving

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta} \end{bmatrix} = \arg \min_{\beta_0, \beta} \left\| Y - \begin{bmatrix} \mathbb{1}_n & X \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta \end{bmatrix} \right\|_2^2$$

- RR, lasso have one tuning parameter, while EDE has two tuning parameters
- RR (with an intercept term) can be formulated as

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta} \end{bmatrix} = \arg \min_{\beta_0, \beta} \left\| \begin{bmatrix} Y \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbb{1}_n & X \\ 0 & \mu \cdot \mathbb{I} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta \end{bmatrix} \right\|_2^2,$$

where μ is a tuning parameter.

- EDE (with an intercept term) can be formulated as

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta} \end{bmatrix} = \arg \min_{\beta_0, \beta} \left\| \begin{bmatrix} Y \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbb{1}_n & X \\ 0 & \mu \cdot \Pi \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta \end{bmatrix} \right\|_2^2.$$

Here, μ is a tuning parameter and $\Pi = V_2 V_2'$, where the singular value decomposition (SVD) of X is

$$X = US [V_1 \ V_2]'$$

with $V_1 \in \mathbb{R}^{p \times d}$, $V_2 \in \mathbb{R}^{p \times (p-d)}$, and the singular values in S are listed in decreasing order.