IEOR 151 – Lecture 7
Matching Markets

1 Resource Allocation without Money

In a typical market, resources (including goods and services) are allocated on the basis of payments. However, there are many instances when payments cannot be used to allocate goods. In such cases, it is interesting to study how to allocate resources (which are scarce by definition) to maximize social welfare. There are two different (but related viewpoints) to these matching problems. Below, we describe two examples of matching markets, and these examples highlight two different perspectives to the design of such markets. The first takes an operations research approach, and the second uses a game-theoretic approach.

2 Kidney Exchanges

Kidneys play a vital role in clearing the body of waste products and in regulating blood pressure and electrolytes. For some patients there is a progressive reduction in kidney function, which is known as chronic kidney disease (CKD). The most common causes of CKD are diabetes, high blood pressure, or various inflammatory diseases like IgA nephropathy. There may be few symptoms at the earliest stages of CKD; but at the final stage of CKD, which is known as end stage renal disease (ESRD), the kidneys are unable to function at a level that is enough to sustain life. If some form of renal replacement therapy is not given to the patient with ESRD, then their prognosis is very poor.

One of the most common treatments for ESRD is dialysis, which is a treatment that uses artificial equipment to provide the filtration abilities of the kidney. Unfortunately, dialysis is associated with typically poor long-term prognosis and low quality of life because of the invasiveness required by the artificial equipment. Kidney transplantation is generally considered to be a superior treatment because of better prognosis and less reductions in quality of life. An interesting side note is that because of the high costs associated with renal replacement therapy, Medicare was extended in 1972 to provide coverage for nearly all patients (irrespective of age) with ESRD.

A kidney transplantation is a procedure in which a patient with ESRD receives a kidney to replace lost kidney. The transplanted kidney may be provided by a deceased-donor or a living-donor. Living-donor transplantation usually has better prognosis for the patient, and the kidney may be provided by a family member or a non-family member; these two cases are

known as living-related and living-unrelated, respectively. Unfortunately, even though kidney transplantation is a superior treatment for ESRD, there are not enough kidneys available.

Even when a living donor is willing to donate a kidney to a patient with ESRD, it is often the case that the kidney of the potential donor is not *compatible* with the body of the intended recipient. In fact, there are at least three levels of compatibility that are typically checked before a kidney transplant procedure is performed. To overcome limitations imposed by compatibility, kidney exchanges have been employed.[1]

2.1 BLOOD TYPE COMPATIBILITY

Human blood contains a number of substances that are on the surface of red blood cells, and a blood type represents a combination of antigens (a substance that binds to an antibody) on the surface of red blood cells. There are several systems to describe these combinations, but the most important for the purposes of kidney transplantation is ABO system. In the ABO system, blood types are categorized into types A, B, AB, and O. The blood type restricts when a potential donor is compatible with the intended recipient, and these relationships are summarized in the table below.

Donor	Recipient
Type O	Type O, A, B, AB
Type A	Type A, AB
Type B	Type B, AB
Type AB	Type AB

Table 1: Compatible Blood Types for the Donor and Recipient

2.2 HLA COMPATIBILITY

Just as red blood cells had specific antigens on their surface, tissues in the human body also have different antigens on the surface of their constitutive cells; these are known as human leukocyte antigens. Six antigens have been identified as being particularly important, with three being inherited from each parent. Because of this genetic component, it is rare for two people to have a six-antigen match. However, a perfect HLA match is not required for transplantation. In some cases, a kidney has been successfully transplanted without matching antigens; in other cases, a kidney with a six-antigen match has been rejected by the body of the recipient. However, generally speaking a greater degree of HLA type matching is associated with better prognosis and also an opportunity for decreasing the usage of immunosuppression drugs.

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[1] D. Abraham, A. Blum, and T. Sandholm, "Clearing algorithms for barter exchange markets: Enabling nationwide kidney exchanges," Proceedings of the 8th ACM Conference on Electronic Commerce, pp. 295–304.

2.3 Cross Match

There are a lot characteristics not captured by blood typing and HLA typing, but clinical tests can detect additional incompatibilities. These so-called *cross match* tests are often a final step before transplantation is conducted, and they proceed by mixing blood and tissue from the potential donor and intended recipient. If there is a positive cross match, then transplant should *not* be performed because the kidney will be incompatible. But because these results are not easy to characterize independently, the potential *cross match* results are not usually initially considered in kidney exchanges.

2.4 Kidney Exchange Market

Consider a situation in which a potential donor D_1 has an intended recipient R_1, but there is an incompatibility in the kidney. In the past, this would mean that a transplant would not occur. But what if there was another potential donor D_2 with incompatible recipient R_2, such that D_1 were compatible with R_2 and D_2 were compatible with R_1. These four individuals could then agree to perform the kidney transplants at the same time. The more general question is: Given a list of potential donors and intended recipients, along with medical information describing the individuals, how can we match donor-recipient pairs to maximize the number and quality of kidney transplants.

3 National Resident Matching Program

Medical doctors in the United States following a training path that begins with undergraduate education. After graduating with a BS degree, potential doctors enroll in graduate medical school programs. Upon graduating with the MD degree, the next step is to join a medical residency program. Depending on specialization interests, completion of additional fellowship programs are required. Admissions to graduate medical programs follows a similar process to that of admissions to undergraduate programs, and it works fairly well because the pool of applicants is large.

However, the pool of applicants for a residency program is much smaller because applicants must match to specific specialties (e.g., pediatrics, ophthalmology, radiology, etc.). Inefficiencies can occur in this market because of the relatively small number of applicants, the fact that it is in the interest of applicants to delay making decisions for as long as possible, and the fact that it is in the interest of programs to fills open positions as quickly as possible. These conditions historically caused residency programs to begin the process of filling open positions up to two years in advance, and led to much dissatisfaction from both the applicants and programs. This led to the creation of a centralized clearing-house that processes preferences of both hospitals and individuals and generates a single matching of individuals to residency programs.

3.1 Game-Theoretic Considerations

There are some game-theoretic considerations that should be taken into account when designing the market. The first is known as *stability*; this is the notion that the matching must be such that no individual and hospital could decline the match generated by the centralized process, and select another match that has higher preference for both than the broken match. The second is known as being *strategy-proof*; this is the notion that the market should be designed in such a way that participants are encouraged to report their true preferences. In other words, we would prefer that the market be designed in a manner that the best game-theoretic strategy is to be truthful.