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# IEOR 151 – Lecture 23

## Longterm Staffing

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### 1 Static Model with Single Skill

In the first model we consider, there are a set of projects that each require a certain number of person-months of labor. Each project requires the use of multiple skills, but each employee of the company has only one skill. Because employees have already been hired, the marginal cost for their use on a project is zero. Any excess person-months of work that must be required can be met by purchasing the services of a contractor, but different skills have different contractor costs. Given this model, we define the following inputs:

- $I$  – set of projects;
- $J$  – set of skills;
- $s_j$  – supply person-months of skill  $j \in J$  that are available;
- $r_{ij}$  – demand required person-months of skill  $j \in J$  that are needed on project  $i \in I$ ;
- $c_j$  – cost per person-month of contractor with skill  $j \in J$ ;

and decision variables:

- $X_{ij}$  – number of person-months of internal labor allocated to project  $i \in I$  with skill  $j \in J$ ;
- $Y_{ij}$  – number of person-months of contractor labor allocated to project  $i \in I$  with skill  $j \in J$ .

The optimization problem to be solved is a linear program:

$$\begin{aligned} \min \quad & \sum_{j \in J} c_j \sum_{i \in I} Y_{ij} \\ \text{s.t.} \quad & X_{ij} + Y_{ij} \geq r_{ij}, \forall i \in I, j \in J \\ & \sum_{i \in I} X_{ij} \leq s_j, \forall j \in J \\ & X_{ij} \geq 0, \forall i \in I, j \in J \\ & Y_{ij} \geq 0, \forall i \in I, j \in J. \end{aligned}$$

The objective states that we wish to minimize the marginal cost due to hiring contractors with their different pricing based on skill. The first constraint says that we would like to have enough person-months allocated to meet the demanded number, the second constraint says that we cannot more internal labor than is available, and the last two constraints enforce that the amount of labor allocated is positive.

## 2 Static Model with Multiple Skills

The second model we consider is an extension of the previous model. Here, we assume that each employee has multiple skills. In this model, skills can in *packages*. So for instance, one skill package for some construction workers might be able to do plumbing and electrical work, whereas the skill package for other construction works may be masonry and woodwork. Given this model, we define the following additional inputs:

- $K$  – set of skill packages;
- $a_{jk}$  – binary variable that is 1 if skill package  $k \in K$  includes skill  $j \in J$ , and is otherwise 0;
- $s_k$  – supply person-months of workers with skill package  $k \in K$  that are available;

and decision variables:

- $X_{ijk}$  – number of person-months of internal labor with skill package  $k \in K$  allocated to project  $i \in I$  with skill  $j \in J$ .

The optimization problem to be solved is a linear program:

$$\begin{aligned}
 & \min \sum_{j \in J} c_j \sum_{i \in I} Y_{ij} \\
 & \text{s.t.} \quad \sum_{k \in K} a_{jk} X_{ijk} + Y_{ij} \geq r_{ij}, \forall i \in I, j \in J \\
 & \quad \sum_{i \in I} \sum_{j \in J} X_{ijk} \leq s_k, \forall k \in K \\
 & \quad X_{ijk} \geq 0, \forall i \in I, j \in J, k \in K \\
 & \quad Y_{ij} \geq 0, \forall i \in I, j \in J.
 \end{aligned}$$

The objective states that we wish to minimize the marginal cost due to hiring contractors with their different pricing based on skill. The first constraint says that we would like to have enough person-months allocated to meet the demanded number, the second constraint says that we cannot more internal labor than is available, and the last two constraints enforce that the amount of labor allocated is positive.

### 3 Temporal Model with Multiple Skills

The third model we consider is an extension of the previous model. Here, we assume that project supplies, demands, and decision variables depend upon time. We will use a subscript  $t$  to denote this, and the optimization problem to be solved is a linear program: The optimization problem to be solved is a linear program:

$$\begin{aligned}
 & \min \sum_{t \in T} \sum_{j \in J} c_{jt} \sum_{i \in I} Y_{ijt} \\
 & \text{s.t.} \quad \sum_{k \in K} a_{jk} X_{ijkt} + Y_{ijt} \geq r_{ijt}, \forall i \in I, j \in J, t \in T \\
 & \quad \sum_{i \in I} \sum_{j \in J} X_{ijkt} \leq s_{kt}, \forall k \in K, t \in T \\
 & \quad X_{ijkt} \geq 0, \forall i \in I, j \in J, k \in K, t \in T \\
 & \quad Y_{ijt} \geq 0, \forall i \in I, j \in J, t \in T.
 \end{aligned}$$

The objective states that we wish to minimize the marginal cost due to hiring contractors with their different pricing based on skill. The first constraint says that we would like to have enough person-months allocated to meet the demanded number, the second constraint says that we cannot more internal labor than is available, and the last two constraints enforce that the amount of labor allocated is positive.

### 4 Ethical Staffing

Something that often gets lost in these models is that each decision variable represents parts of the life of multiple human beings. But because typical models do not include constraints that reflect this, it is possible for the results of the resulting optimization models to give results that cause inefficiency amongst staff, low morale, and additional life stressors. For instance, some service sectors will purposefully keep staff below 30 hours of work a week so as to not be legally mandated to provide health care and other employee benefits. As another example, some service sectors will schedule staff with very different numbers of hours each week: Some weeks, it may be that a worker is at work for 15 hours, 25 hours, 35 hours, 10 hours, 15 hours, etc. from week-to-week. This adds additional stress to the employee and can cause loss of productivity.

Here, we will consider one model that incorporates these aspects. For simplicity, we will assume that there is only one skill that must be staffed, that the number of available workers varies over time because of vacation, the marginal costs of each employee are given by their number of hours worked, the demands for this skill vary over time, and three decisions to be made are:

1. How many employees should be hired?

2. How many person-hours of employees should be allocated in each time period?
3. How many consultants should be hired for each time period?

Now using the type of model from previous sections, we would solve

$$\begin{aligned}
& \min hu + \sum_{t \in T} (\mu_t X_t + c_t Y_t) \\
& \text{s.t. } X_t + Y_t \geq r_t, \forall t \in T \\
& \quad X_t \leq 40u, \forall t \in T \\
& \quad X_t \geq 0, \forall t \in T \\
& \quad u \in \mathbb{Z}_+ \\
& \quad Y_t \geq 0, \forall t \in T,
\end{aligned}$$

where  $X_t$  is the number of person-hours of employees,  $u$  is the (positive integer) number of employees hired,  $Y_t$  is the number of person-hours of consultants hired,  $h$  is the cost of hiring one person,  $\mu_t$  is the cost of having one person-hour of an employee for time  $t$ , and  $c_t$  is the cost of having one person-hour for time  $t$ . If  $c_t < \mu_t$ , then you would never hire a full-time employee; you would only have contractors. If  $c_t > \mu_t$ , then you would hire some intermediate number of employees. Generally speaking, contractor costs per hour will be higher than that of an employee.

However, the problem with this model is that it may be the case that  $X_t$  significantly varies over time. This can lead to the issues discussed above because of inconsistent weekly schedules for workers. To remedy this, we can modify the cost function to

$$\begin{aligned}
& \min hu + \sum_{t \in T} (\mu_t X_t + c_t Y_t) + \lambda \sum_{t \in T} |X_{t+1} - X_t| \\
& \text{s.t. } X_t + Y_t \geq r_t, \forall t \in T \\
& \quad X_t \leq 40u, \forall t \in T \\
& \quad X_t \geq 0, \forall t \in T \\
& \quad u \in \mathbb{Z}_+ \\
& \quad Y_t \geq 0, \forall t \in T.
\end{aligned}$$

The difference in this model is that we have added the term  $\lambda \sum_{t \in T} |X_{t+1} - X_t|$  to the objective. In some fields, this term is known as total variation, and this additional term is called a total variation regularization. The idea is that this term penalizes for fluctuations over time in the variable  $X_t$ , and the parameter  $\lambda \geq 0$  controls how much such fluctuations are penalized. Having a high value of  $\lambda$  will cause the  $X_t$  to not vary much over time, whereas if  $\lambda = 0$ , then the  $X_t$  is free to vary as much as is optimal for the original model formulation. This is just one formulation of bringing human factors and ethical considerations into the modeling of long-term staffing, and other factors can be incorporated into these models.

## 5 More Information and References

The material in the first three sections of these notes follows that of the course textbook “Service Systems” by Mark Daskin.