## IEOR 151 - Service Operations Design and Analysis



September 30, 2016

1. Consider the following graph representation of a kidney exchange. Find the social welfare maximizaing exchange under the constraint that all cycles can have length less than or equal to $\mathrm{L}=3$. ( 5 points)

2. Consider the case there are 4 coaches $\{1,2,3,4\}$ and 4 athletes $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$ on a track and field team. Match the coaches to the athletes based on the listed preferences. Show intermediate steps of the algorithm. The athletes' preferences are given by:

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ |
| :---: | :---: | :---: | :---: |
| 1 | 4 | 2 | 3 |
| 3 | 3 | 3 | 4 |
| 2 | 2 | 1 | 2 |
| 4 | 1 | 4 | 1 |

Suppose that each coach can only mentor 1 athlete, and the coaches's preferences are:

| $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :---: | :---: | :---: | :---: |
| A | A | A | A |
| B | D | C | B |
| C | C | B | C |
| D | B | D | D |

First, consider the case where the coaches can choose first. Then, consider the case where the athletes can choose first. Does the matching change?
3. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)
For this problem, suppose the applicant's preferences are given by:

| Anil | Alper | Candi | Rhonda |
| :---: | :---: | :---: | :---: |
| Hopkins | General | General | Temple |
| General | Hopkins | Temple | Hopkins |
| Temple | Temple | Hopkins | General |

Suppose that each residency program has only 1 open position, and that the program's preferences are given by:

| General | Hopkins | Temple |
| :---: | :---: | :---: |
| Candi | Rhonda | Rhonda |
| Rhonda | Alper | Candi |
| Anil | Candi | Anil |
| Alper | Anil | Alper |

4. Suppose Apple, Inc would like to purchase processors from Samsung Electronics Co. Apple's utility for the processors is given by $S(q)=550 \sqrt{(1+q)}$. The fixed costs for Samsung Electronics Co. are $\$ 5,500$, and if Samsung is inefficeint (efficient) then its marginal costs are 2.75 (2.00). Assume that Apple, Inc believes that there is a $25 \%$ chance that Samsung Electronics Co. is efficient.
(a) What are the first-best production levels? (2 points)
(b) What are the contracts to implement the first-best production levels? (2 points)
(c) How much profit would Samsung Electronics Co. make if Apple, Inc offers a menu of contracts $\left\{\left(q_{1}^{I}, t_{1}^{I}\right),\left(q_{1}^{E}, t_{1}^{E}\right)\right\}$ (1 point)
(d) What are the second-best production levels? (2 points)
(e) What is the menu of contracts for the second-best production levels? (2 points)
(f) What is the information rent of an efficient Samsung Electronics Co. for the menu of contracts for the second-best production levels? Is this higher or lower than the profit gained for the menu of contracts for the first-best production levels? (2 points)
