
IEOR 151 – LECTURE 7

MATCHING GAMES

1 Resource Allocation with Friction

Matching games are found in many markets in which there is a “resource” to be allocated, but there is some *friction* that prevents efficient allocation of the resources. It may be the case that the resource is held by members in group A and is to be allocated to members in group B , or it may be that the resource is held by members in group A and allocated amongst themselves. Reducing the friction can improve the social welfare of all market participants, and so it is interesting to consider how this can be done so. There are two different (but related viewpoints) to these matching problems. The first is as an operations research approach, and the second is as a game-theoretic approach.

2 Kidney Exchanges

Kidneys play a vital role in clearing the body of waste products and in regulating blood pressure and electrolytes. For some patients there is a progressive reduction in kidney function, which is known as chronic kidney disease (CKD). The most common causes of CKD are diabetes, high blood pressure, or various inflammatory diseases like IgA nephropathy. There may be few symptoms at the earliest stages of CKD; but at the final stage of CKD, which is known as end stage renal disease (ESRD), the kidneys are unable to function at a level that is enough to sustain life. If some form of renal replacement therapy is not given to the patient with ESRD, then their prognosis is very poor.

One of the most common treatments for ESRD is dialysis, which is a treatment that uses artificial equipment to provide the filtration abilities of the kidney. Unfortunately, dialysis is associated with typically poor long-term prognosis and low quality of life because of the invasiveness required by the artificial equipment. Kidney transplantation is generally considered to be a superior treatment because of better prognosis and less reductions in quality of life. An interesting side note is that because of the high costs associated with renal replacement therapy, Medicare was extended in 1972 to provide coverage for nearly all patients (irrespective of age) with ESRD.

A kidney transplantation is a procedure in which a patient with ESRD receives a kidney to replace lost kidney. The transplanted kidney may be provided by a deceased-donor or a living-donor. Living-donor transplantation usually has better prognosis for the patient, and the kidney may be provided by a family member or a non-family member; these two cases are known as living-related and living-unrelated, respectively. Unfortunately, even though kidney transplantation is a superior

treatment for ESRD, there are not enough kidneys available.

Even when a living donor is willing to donate a kidney to a patient with ESRD, it is often the case that the kidney of the potential donor is not *compatible* with the body of the intended recipient. In fact, there are at least three levels of compatibility that are typically checked before a kidney transplant procedure is performed. To overcome limitations imposed by compatibility, kidney exchanges have been employed.¹

2.1 BLOOD TYPE COMPATIBILITY

Human blood contains a number of substances that are on the surface of red blood cells, and a blood type represents a combination of antigens (a substance that binds to an antibody) on the surface of red blood cells. There are several systems to describe these combinations, but the most important for the purposes of kidney transplantation is ABO system. In the ABO system, blood types are categorized into types A, B, AB, and O. The blood type restricts when a potential donor is compatible with the intended recipient, and these relationships are summarized in the table below.

Donor	Recipient
Type O	Type O, A, B, AB
Type A	Type A, AB
Type B	Type B, AB
Type AB	Type AB

Table 1: Compatible Blood Types for the Donor and Recipient

2.2 HLA COMPATIBILITY

Just as red blood cells had specific antigens on their surface, tissues in the human body also have different antigens on the surface of their constitutive cells; these are known as human leukocyte antigens. Six antigens have been identified as being particularly important, with three being inherited from each parent. Because of this genetic component, it is rare for two people to have a six-antigen match. However, a perfect HLA match is not required for transplantation. In some cases, a kidney has been successfully transplanted without matching antigens; in other cases, a kidney with a six-antigen match has been rejected by the body of the recipient. However, generally speaking a greater degree of HLA type matching is associated with better prognosis and also an opportunity for decreasing the usage of immunosuppression drugs.

2.3 CROSS MATCH

There are a lot characteristics not captured by blood typing and HLA typing, but clinical tests can detect additional incompatibilities. These so-called *cross match* tests are often a final step be-

¹D. Abraham, A. Blum, and T. Sandholm, “Clearing algorithms for barter exchange markets: Enabling nationwide kidney exchanges,” Proceedings of the 8th ACM Conference on Electronic Commerce, pp. 295–304.

fore transplantation is conducted, and they proceed by mixing blood and tissue from the potential donor and intended recipient. If there is a positive cross match, then transplant should *not* be performed because the kidney will be incompatible. But because these results are not easy to characterize independently, the potential *cross match* results are not usually initially considered in kidney exchanges.

2.4 KIDNEY EXCHANGE MODEL

Consider a situation in which a potential donor D_1 has an intended recipient R_1 , but there is an incompatibility in the kidney. In the past, this would mean that a transplant would not occur. But what if there was another potential donor D_2 with incompatible recipient R_2 , such that D_1 were compatible with R_2 and D_2 were compatible with R_1 . These four individuals could then agree to perform the kidney transplants at the same time.

Now instead of two groups of donor-recipients, suppose that there are k such groups. It would be useful if there were a way to match different groups of incompatible pairs, as this would increase the number of people that could receive transplants from willing donors. To do so, we consider the following model. The market is described by a directed graph $G = (V, E)$ with edge weights, where there is a vertex $v_i \in V$ for each donor-recipient and an edge $e_{i,j} = (v_i, v_j) \in E$ from v_i to v_j if the i -th donor-recipient would accept a kidney from the j -th donor-recipient. The weight $w_{i,j}$ is the utility obtained to v_i of obtaining v_j 's kidney. The weights can be different for each edge because there are differences in age (which relates to likelihood to successful transplants) and quality of HLA type matches.

A cycle c in the graph represents a group of donor-recipients that can swap kidneys in a way that each donor-recipient in the cycle gets a kidney from the next donor-recipient. An exchange is a union of disjoint cycles. The weight of a cycle w_c is simply the sum of its edge weights, and the weight of an exchange is the sum of each cycle. The social welfare maximizing exchange is the one with maximum weight. The clearing problem is to find social welfare maximizing exchange under the constraint that all cycles can have length less than or equal to a small constant L . The reason for a constraint on the size of cycle is that surgeries in a transplant cycle must be conducted simultaneously, otherwise it would be possible for a donor-recipient to remove themselves from a cycle after receiving a kidney. $L = 3$ can be performed in practice, but $L \geq 4$ is usually too difficult.

3 National Resident Matching Program

Medical doctors in the United States following a training path that begins with undergraduate education. After graduating with a BS degree, potential doctors enroll in graduate medical school programs. Upon graduating with the MD degree, the next step is to join a medical residency program. Depending on specialization interests, completion of additional fellowship programs are required. Admissions to graduate medical programs follows a similar process to that of admissions to undergraduate programs, and it works fairly well because the pool of applicants is large.

However, the pool of applicants for a residency program is much smaller because applicants must match to specific specialities (e.g., pediatrics, ophthalmology, radiology, etc.). Friction is present in this market because of the relatively small number of applicants, the fact that it is in the interest of applicants to delay making decisions for as long as possible, and the fact that it is in the interest of programs to fill open positions as quickly as possible. This friction historically caused residency programs to begin the process of filling open positions up to two years in advance, and led to much dissatisfaction from both the applicants and programs.

3.1 SIMPLIFIED NRMP MODEL

In lecture, we will consider a simplified model of the NRMP that does not consider cases of couples, second-year programs, and unfilled positions. In this model, there are m residency programs and n applicants. We will denote the i -th applicant as v_i and the j -th program as p_j . The i -th residency program has s_i open positions.

Furthermore, both the programs and applicants have respective preferences. For the i -th residency program, the preferences are denoted by an ordering \succ_{p_i} ; this means that if the k -th program prefers applicant v_i to v_j , then $v_i \succ_{p_k} v_j$. Similarly, the preferences of the k -th applicant are denoted by an ordering \succ_{v_k} ; so if the k -th applicant prefers program p_i to p_j , then $p_i \succ_{v_k} p_j$. An important feature of this model is that preferences are unique in the sense that no program or applicant can be preferred at the same level.

The problem is to match applicants to positions, so that no applicant matches to more than one position and so that preferences are maximized in an appropriate sense. It turns out that one group's preferences must be prioritized over the preferences of the other group. The current system prioritizes the preferences of the applicants over those of the programs.