IEOR151

Lab 8: Review of Linear Programming and Introduction to Excel Solver

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Linear Programming

- Primal and dual in canonical forms

$$\begin{aligned} Max \quad & \sum_{j \in J} c_j X_j \\ s.t. \quad & \sum_{j \in J} a_{ij} X_j \le b_i \quad \forall i \in I \\ & X_j \ge 0 \quad \forall j \in J \end{aligned}$$

$$\begin{aligned} Min \quad & \sum_{i \in I} b_i W_i \\ s.t. \quad & \sum_{i \in I} a_{ij} W_i \ge c_j \quad \forall j \in J \\ & W_i \ge 0 \quad \forall i \in I \end{aligned}$$

Example in Textbook 2.5.2

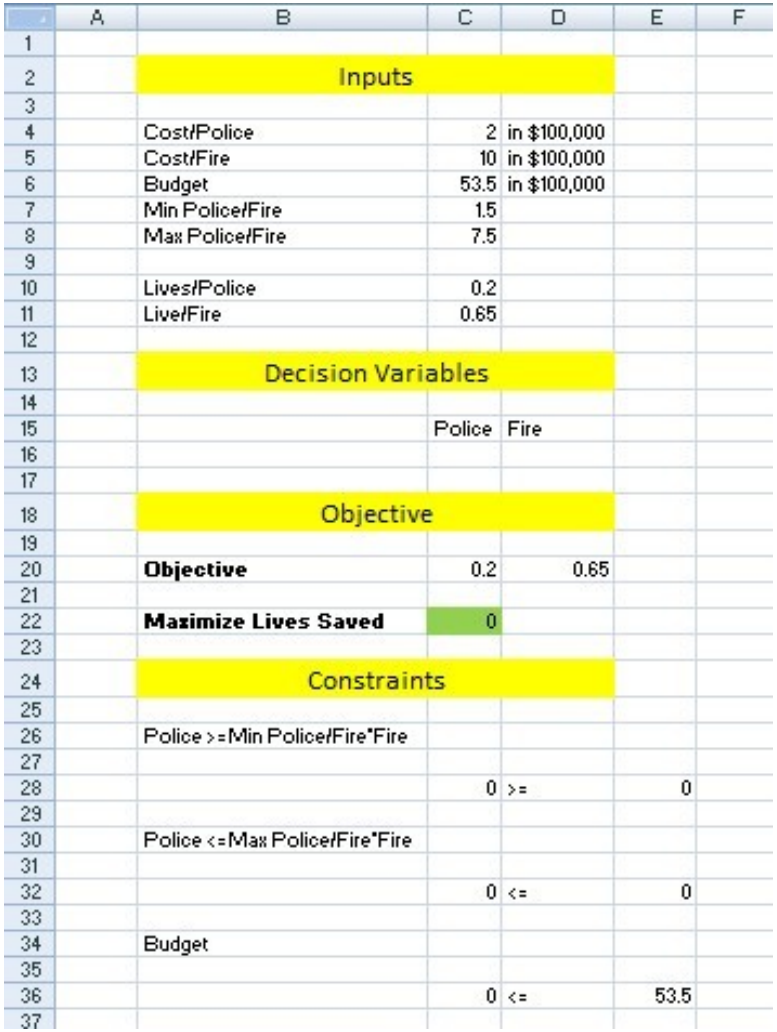
- Consider a problem of deciding how to allocate a budget for municipal services between police and fire protection. In the simple model, each police patrol costs $200,000 per year and each fire truck costs $1,000,000 per year including the cost of the fire station. The city has only $5,350,000 to allocate to the combined police and fire budgets. In addition, contracts with the unions representing the two city services stipulate that there must be at least 1.5 times as many police patrol units as there are fire trucks and that there cannot be more than 7.5 times as many police units as there are fire units.
- The goal is to maximize the number of lives saved over a year. We expect 0.2 lives saved per year per police patrol unit and 0.65 lives saved per fire truck.

2.5.2 Formulation

$$
\begin{aligned}
Max\ & 0.2 \cdot Police + 0.65 \cdot Fire \\
s.t.\ & 200 \cdot Police + 1000 \cdot Fire \leq 5350 \\
& -1.0 \cdot Police + 1.5 \cdot Fire \leq 0 \\
& 1.0 \cdot Police - 7.5 \cdot Fire \leq 0 \\
& Police \geq 0 \\
& Fire \geq 0
\end{aligned}
$$

Solving the Problem in Excel

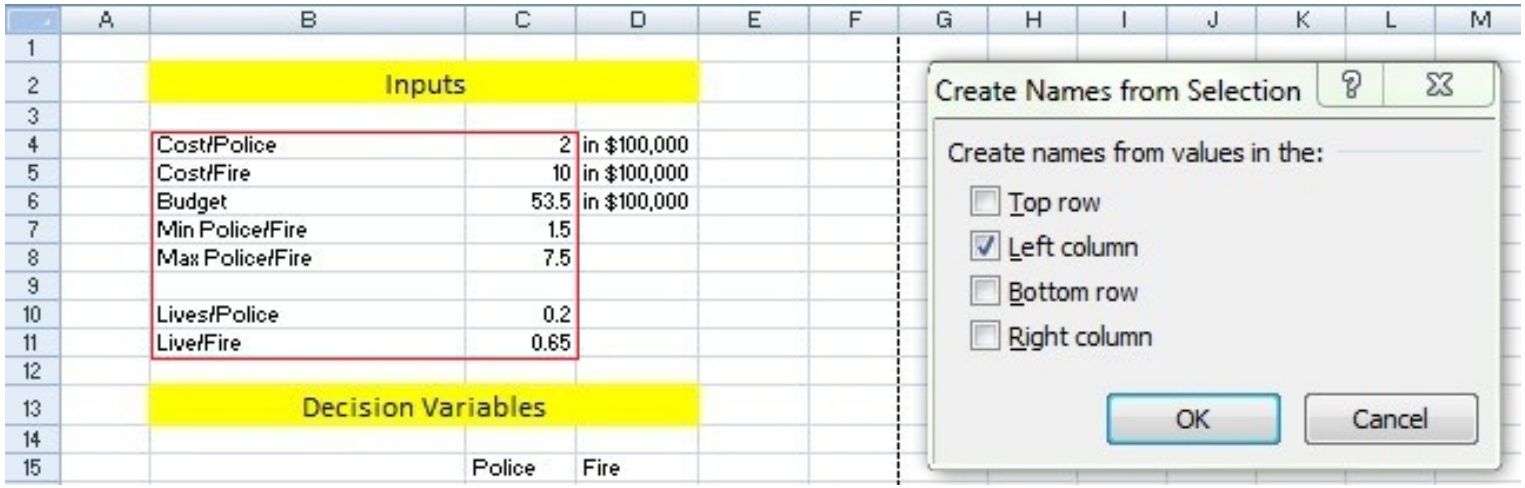
- Build model in Excel



	A	B	C	D	E	F
1						
2		Inputs				
3						
4		Cost/Police	2	in $100,000		
5		Cost/Fire	10	in $100,000		
6		Budget	53.5	in $100,000		
7		Min Police/Fire	1.5			
8		Max Police/Fire	7.5			
9						
10		Lives/Police	0.2			
11		Live/Fire	0.65			
12						
13		Decision Variables				
14						
15			Police	Fire		
16						
17						
18		Objective				
19						
20		**Objective**	0.2	0.65		
21						
22		**Maximize Lives Saved**	0			
23						
24		Constraints				
25						
26		Police >=Min Police/Fire*Fire				
27						
28			0	>=	0	
29						
30		Police <=Max Police/Fire*Fire				
31						
32			0	<=	0	
33						
34		Budget				
35						
36			0	<=	53.5	
37						

Solving the Problem in Excel

- Name the cells
  - Formulas -> Create from Selection

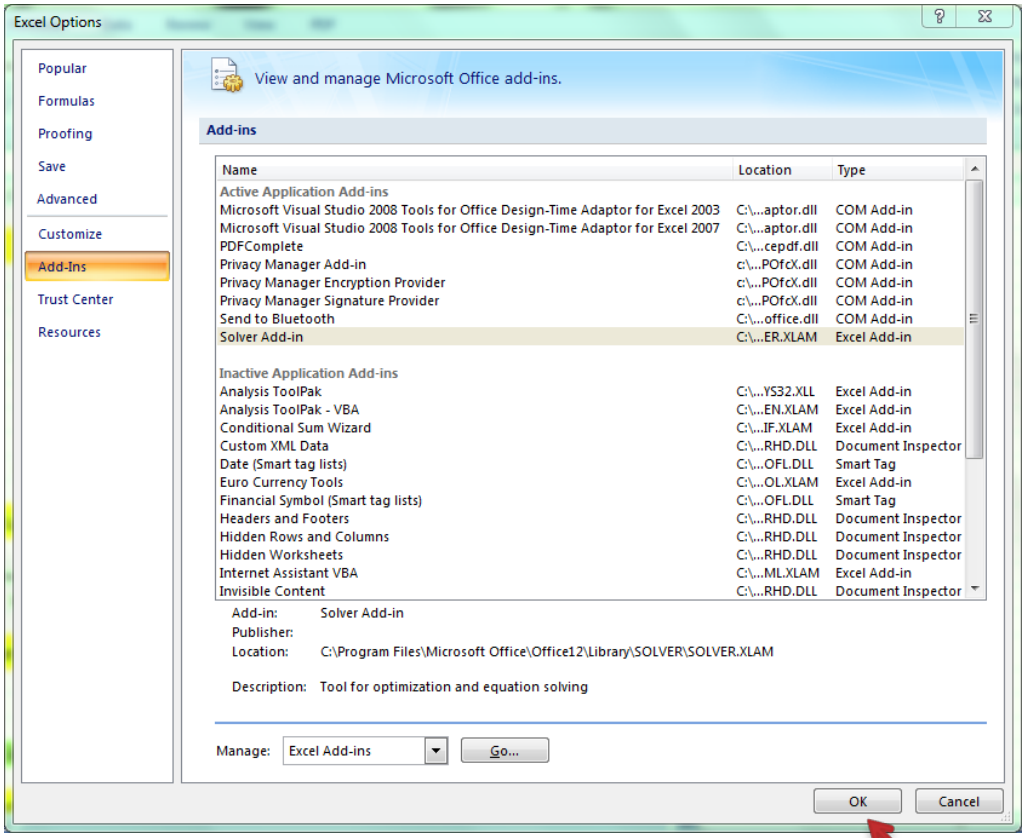


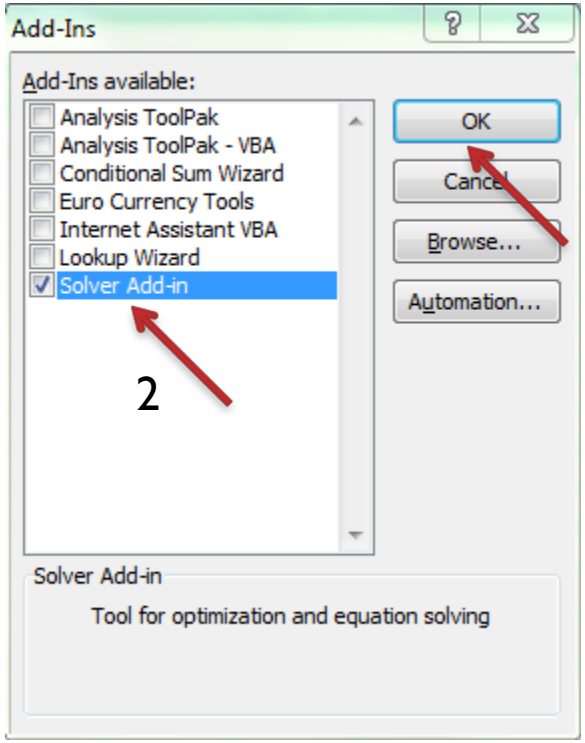
	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		Inputs											
3													
4		Cost/Police	2	in $100,000									
5		Cost/Fire	10	in $100,000									
6		Budget	53.5	in $100,000									
7		Min Police/Fire	1.5										
8		Max Police/Fire	7.5										
9													
10		Lives/Police	0.2										
11		Live/Fire	0.65										
12													
13		Decision Variables											
14													
15			Police	Fire									

- Edit the cells with formulas (using names)

Solving the Problem in Excel

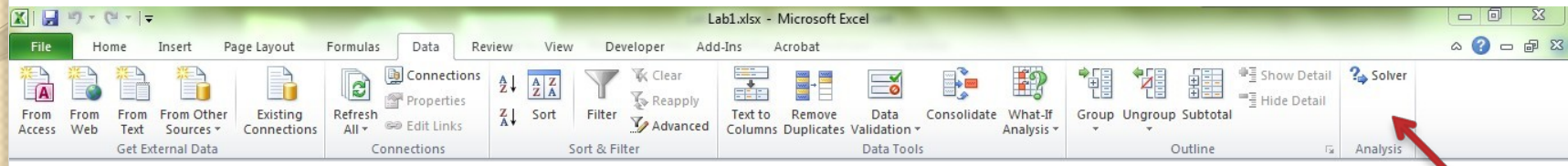
- Setup solver (File->Options->Add-Ins->Solver Add-In)

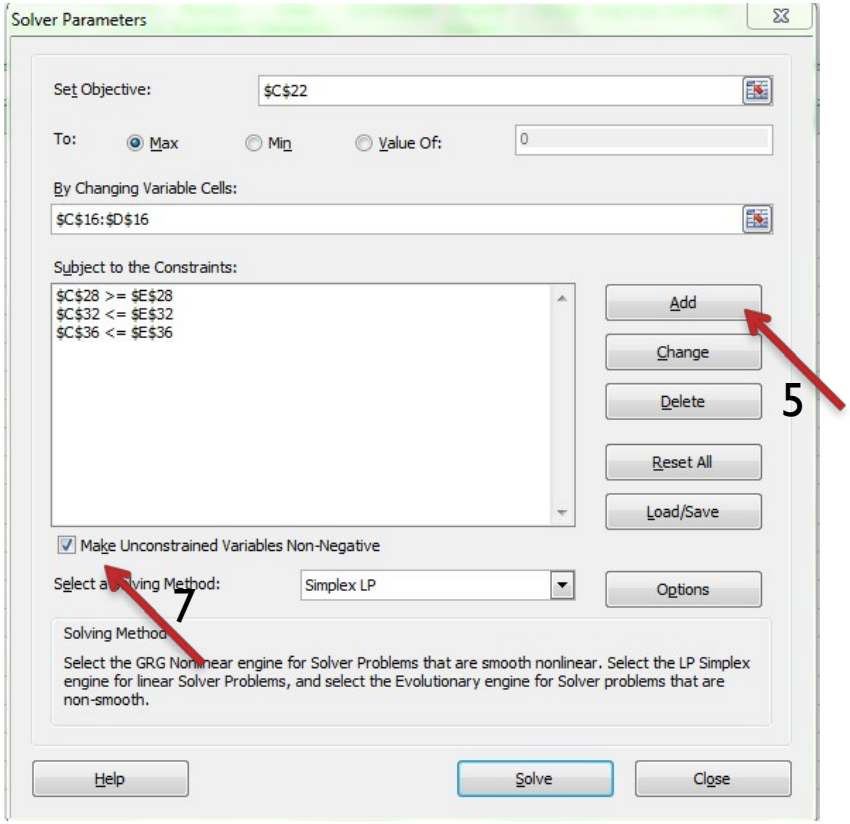


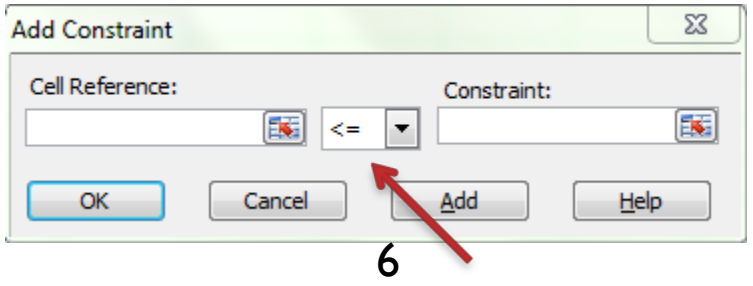


Solving the Problem in Excel

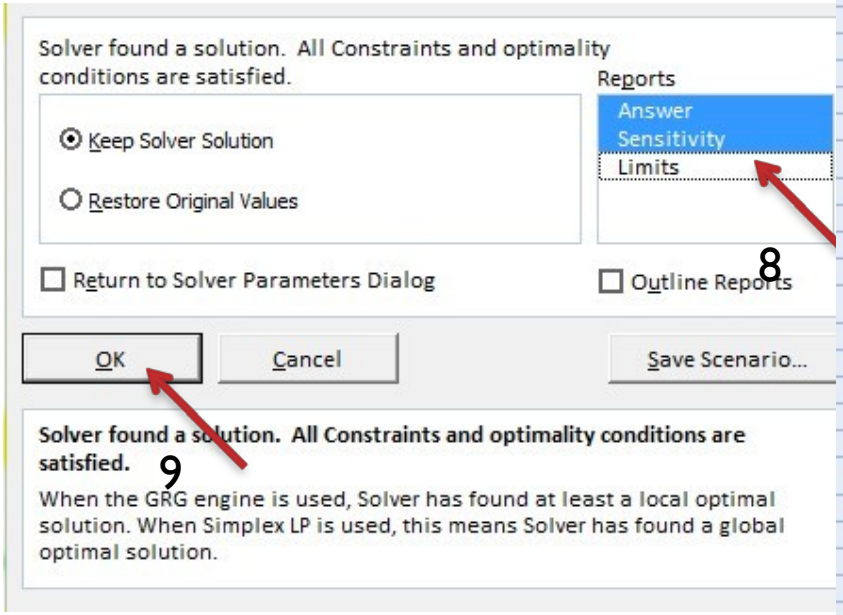
- Setup solver







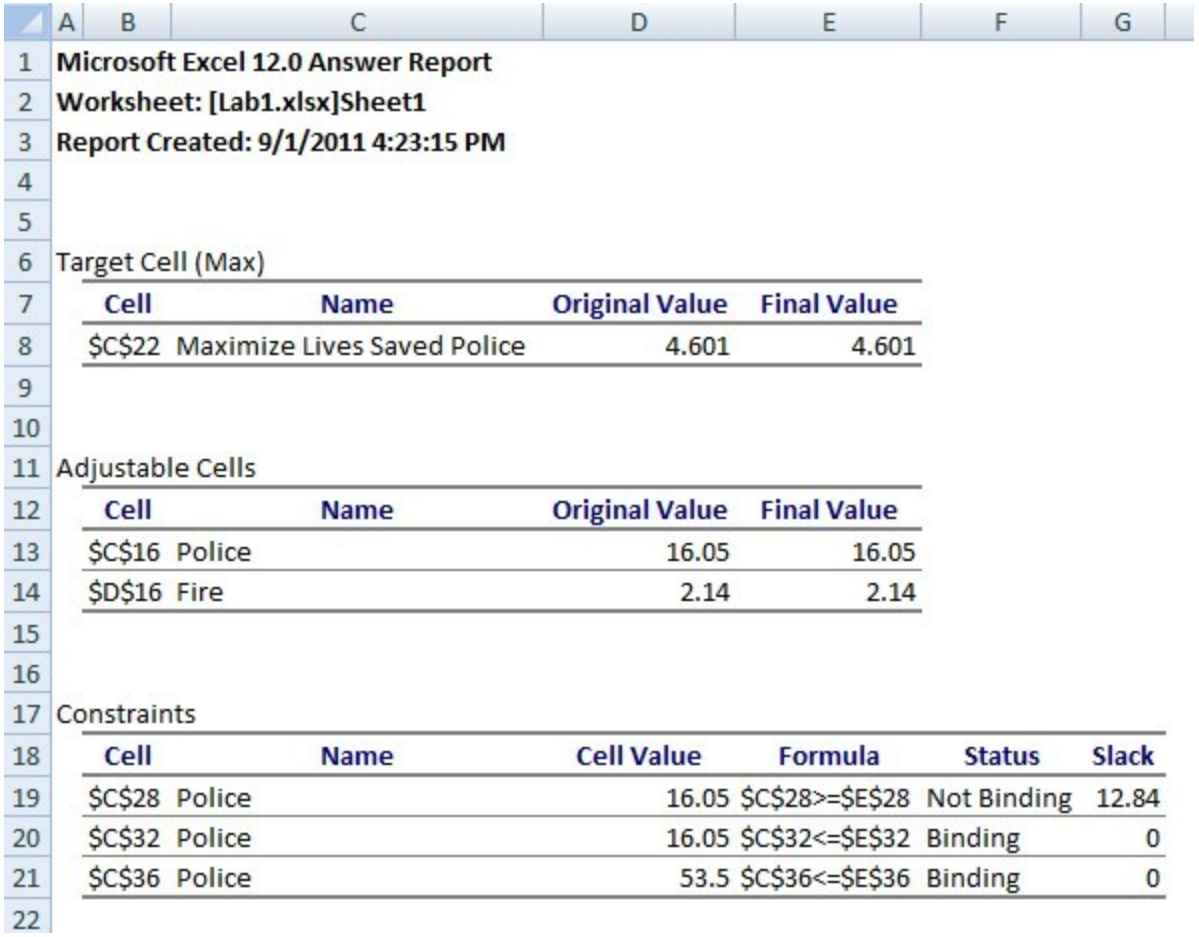
Solving the Problem in Excel



	A	B	C	D	E
1					
2		**Inputs**			
3					
4		Cost/Police	2	in $100,000	
5		Cost/Fire	10	in $100,000	
6		Budget	53.5	in $100,000	
7		Min Police/Fire	1.5		
8		Max Police/Fire	7.5		
9					
10		Lives/Police	0.2		
11		Live/Fire	0.65		
12					
13		**Decision Variables**			
14					
15			Police	Fire	
16			16.05	2.14	
17					
18		**Objective**			
19					
20		**Objective**	0.2	0.65	
21					
22		**Maximize Lives Saved**	4.601		
23					
24		**Constraints**			
25					
26		Police >=Min Police/Fire*Fire			
27					
28			16.05	>=	3.21
29					
30		Police <=Max Police/Fire*Fire			
31					
32			16.05	<=	16.05
33					
34		Budget			
35					
36			53.5	<=	53.5
37					

Solving the Problem in Excel

- Answer report



Microsoft Excel 12.0 Answer Report
Worksheet: [Lab1.xlsx]Sheet1
Report Created: 9/1/2011 4:23:15 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$C$22	Maximize Lives Saved Police	4.601	4.601

Adjustable Cells

Cell	Name	Original Value	Final Value
$C$16	Police	16.05	16.05
$D$16	Fire	2.14	2.14

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$C$28	Police	16.05	$C$28>=$E$28	Not Binding	12.84
$C$32	Police	16.05	$C$32<=$E$32	Binding	0
$C$36	Police	53.5	$C$36<=$E$36	Binding	0

Solving the Problem in Excel

- Sensitivity report

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [Lab1.xlsx]Sheet1
Report Created: 9/1/2011 4:22:59 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$16	Police	16.05	0	0.2	1E+30	0.07
$D$16	Fire	2.14	0	0.65	0.35	2.15

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$28	Police	16.05	0	0	12.84	1E+30
$C$32	Police	16.05	0.028	0	26.75	24.69230769
$C$36	Police	53.5	0.086	53.5	1E+30	53.5

2.5.2 Mixed Integer Program

$$
\begin{aligned}
Max\ & 0.2 \cdot Police + 0.65 \cdot Fire \\
s.t.\ & 200 \cdot Police + 1000 \cdot Fire \le 5350 \\
& -1.0 \cdot Police + 1.5 \cdot Fire \le 0 \\
& 1.0 \cdot Police - 7.5 \cdot Fire \le 0 \\
& Police \text{ integer} \\
& Fire \text{ integer} \\
& Police \ge 0 \\
& Fire \ge 0
\end{aligned}
$$

Add Integer Constraints

Select "int"

Decision variables

Don't ignore integer constraints