Lab 10: Solving Simple TSP in Excel

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Traveling Salesman Problem (TSP)

Objective

- To find a minimum-length route that begins at some node—it could be any node in the network—and ends at the same node, visiting each node exactly once.
Heuristic Algorithms

- Minimum spanning tree based heuristic
- Nearest insertion heuristic
- Christofides’ heuristic
A Minimum Spanning Tree Based Heuristic

• **Step 1**: Construct a minimum spanning tree
• **Step 2**: Let the *root* be an arbitrary vertex
• **Step 3**: Traverse all the vertices by depth-first search, record the sequence of vertices (both visited and unvisited)
• **Step 4**: Use shortcut strategy to generate a feasible tour
Worst-case Analysis

Let $L^{\text{MST}}$ denotes the length of the tour generated by above strategy, then we have

$$L^{\text{MST}} \leq 2W^* \leq 2L^*$$

Where $W^*$ denotes the length of the minimum spanning tree.
And this bound is tight.
Nearest Insertion Heuristic

- **Step 1**: Choose an arbitrary node $v$ and let the cycle $C$ consist of only $v$.
- **Step 2**: Find a node outside $C$ closest to a node in $C$; call it $k$.
- **Step 3**: Find an edge $\{i, j\}$ in $C$ such that $d_{ik} + d_{kj} - d_{ij}$ is minimal.
- **Step 4**: Construct a new cycle $C$ by replacing $\{i, j\}$ with $\{i, k\}$ and $\{k, j\}$.
- **Step 5**: If the current cycle $C$ contains all the vertices, stop. Otherwise, go to **Step 2**.
Worst Case Analysis

The length of current tour $C$ is no more than the twice of the dark line length.

The dark line is nothing, but a minimum spanning tree (exactly same as Prim’s Algorithm), thus

$$L^{MST} \leq 2W^* \leq 2L^*$$
Christofides’ Heuristic

1. Find the minimum spanning tree
2. Find the odd-degree nodes of the MST
3. Find the optimal matching of the odd-degree nodes and add links of the matching to the MST
4. Draw an Euler tour on the resulting graph
Step 3

Figure 10.22. MST with optimal matching of odd degree (shaded) nodes
Step 4

Figure 10.23. Christofides’ heuristic result after removing multiple visits to nodes E, H, and I.
Optimal Matching -- Example

Figure 10.3. Example undirected network
Example

Figure 10.4. Simple network of Figure 10.3 with odd-degree nodes highlighted
Example

Figure 10.5. Optimal matching of the odd-degree nodes of Figure 10.4
Finding Optimal Matching

**TABLE 10.1.** Pairs of odd-degree nodes than can be paired from the network of Figure 10.4

<table>
<thead>
<tr>
<th>Pair Nodes</th>
<th>Repeat Links</th>
<th>Total Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>B, C</td>
<td>BD, DC</td>
<td>25</td>
</tr>
<tr>
<td><strong>B, E</strong></td>
<td>BE</td>
<td>12</td>
</tr>
<tr>
<td>B, G</td>
<td>BD, DG</td>
<td>23</td>
</tr>
<tr>
<td>C, E</td>
<td>CD, DE</td>
<td>28</td>
</tr>
<tr>
<td><strong>C, G</strong></td>
<td>CD, DG</td>
<td>22</td>
</tr>
<tr>
<td>E, G</td>
<td>EG</td>
<td>13</td>
</tr>
</tbody>
</table>

The bolded pairs indicate the optimal matching.
TSP Formulation

- Inputs and sets
  - $N$: the set of nodes in the network
  - $d_{jk}$: the distance between nodes $j$ and $k$ (to avoid self-loops, set $d_{jj} = \infty$ for all $j$ in $N$)

- Decision variables
  - $X_{jk}$: 1 if the tour goes directly from $j$ to $k$; 0 otherwise
TSP Formulation

\[
\text{Min} \quad \sum_{j \in N} \sum_{k \in N} d_{jk} X_{jk}
\]

s.t. \[
\sum_{k \in N} X_{jk} = 1 \quad \forall j \in N
\]

\[
\sum_{j \in N} X_{jk} = 1 \quad \forall k \in N
\]

\[
\sum_{j \in S} \sum_{k \in S} X_{jk} \leq |S| - 1 \quad \forall S \subset N, 2 \leq |S| \leq |N| - 1
\]

\[
X_{jk} = \{0, 1\} \quad \forall j \in N; k \in N
\]
Example
Example

- Solving without “subtour elimination constraints”

Total distance = 1005
Example

- Luckily, we don’t need $2^{11} - 11 - 2 = 2035$ “subtour elimination constraints” for this example.
Example

Solution

Total distance = 1020
Exercise

- Solve the example in Excel Solver.
- Show me the result and take attendance

Hint: add subtour elimination constraints below

\[ A - F - A \quad X_{AF} + X_{FA} \leq 1 \]
\[ B - D - E - B \quad X_{BD} + X_{DE} + X_{EB} \leq 2 \]
\[ B - E - D - B \quad X_{BE} + X_{ED} + X_{DB} \leq 2 \]
\[ C - G - C \quad X_{CG} + X_{GC} \leq 1 \]
\[ H - K - H \quad X_{HK} + X_{KH} \leq 1 \]
\[ I - J - I \quad X_{IJ} + X_{JI} \leq 1 \]