IEOR 151 – Midterm
October 23, 2013

Name:	
Problem 1:	/8
Problem 2:	/8
Problem 3:	/8
Problem 4:	/8
Problem 5:	/8
Problem 6:	/8
Overall:	/48

Instructions:

1. Show all your intermediate steps.
2. You are allowed a single 8.5x11 inch note sheet.
3. Calculators are **NOT** allowed.

1. For each the following scenarios, would you (i) accept the null hypothesis, (ii) reject the null hypothesis, or (iii) gather additional data and information before making a decision? Explain your reasoning. Note: the numbers in the scenarios below are fictional.

   (a) The null hypothesis is that wireless service providers A and B both provide equal average 4G data download speeds, the difference in average download speeds is 0.5 mbps from an average baseline speed of 1.5 mbps, and $p = 0.029$. (2 points)

   (b) The null hypothesis is that bank tellers who are standing process checks faster than tellers who are sitting, the difference between the average times to process a check is 3.2 seconds, and $p = 0.049$. (2 points)

   (c) The null hypothesis is that the average length of a flight delay in the morning is the same as the average length of a flight delay in the evening, the difference in average delays is 10 minutes more for a pilot in the evening, and $p = 0.051$. (2 points)

   (d) The null hypothesis is that the gravitational constant G for gravity due to the moon and the gravitational constant G for gravity due the sun are identical, and $p = 0.009$. (2 points)

Solutions:

(a) Reject. The difference in average speeds is a large fraction of the baseline, and the p-value is smaller than $\alpha = 0.05$.

(b) Accept. Though the p-value is below $\alpha = 0.05$, the difference in averages is small and would not introduce a significant risk or delay in check processing.

(c) Gather more data. Though the p-value is above $\alpha = 0.05$, a difference of 10 minutes might be a significant negative impact on service quality. More data will help to clarify whether the difference is statistically significant.

(d) Accept. The gravitation constant G is a fundamental physical constant, and we would need significant evidence to conclude that it varies for different celestial bodies.

2. Suppose 5 different hypothesis tests have been conducted, with p-values of: Test 1 ($p = 0.002$), Test 2 ($p = 0.05$), Test 3 ($p = 0.011$), Test 4 ($p = 0.005$), Test 5 ($p = 0.065$). Hint: Some useful values are $0.05/1 = 0.05$, $0.05/2 = 0.025$, $0.05/3 = 0.0167$, $0.05/4 = 0.0125$, and $0.05/5 = 0.01$.

   (a) Using the Bonferroni correction, which tests should be accepted or rejected when the family-wise error rate is $\alpha = 0.05$? (3 points)

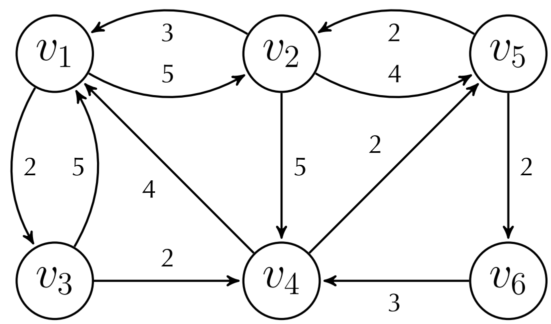
   (b) Using the Holm-Bonferroni method, which tests should be accepted or rejected when the family-wise error rate is $\alpha = 0.05$? (5 points)

Solutions:

(a) Since there are five tests, the Bonferroni correction states that a hypothesis should be rejected if $p < \alpha/5 = 0.01$. Thus, Tests 1 and 4 should be rejected and Tests 2, 3, and 5 should be accepted.

(b) We begin by arranging the p-values in increasing order: 0.002, 0.005, 0.011, 0.05, 0.065. We need to determine the smallest k such that the k-th p-value in the arranged list is greater than $Q_k = \alpha/(5 + 1 - k)$. For $k = 1, \ldots, 5$, the rounded values of Q_k are 0.01, 0.0125, 0.017, 0.025, and 0.05. In this case, $k = 4$ is that smallest k. As a result, we reject hypothesis corresponding to the first three p-values in the ordered list and accept the remaining. Thus, Tests 1, 3, and 4 should be rejected and Tests 2 and 5 should be accepted.

3. Consider the following graph representation of a kidney exchange. Find the social welfare maximizing exchange under the constraint that all cycles can have length less than or equal to $L = 3$. (8 points)



Solutions:

First, we list all cycles of length $L \leq 3$ and compute the weight of these cycles. Next, we determine all sets of disjoint cycles and compute their weight. Lastly, the solution is the set of disjoint cycles with maximal weight. The steps are shown below, and the social welfare maximizing exchange is the set of disjoint cycles C, F.

Cycle Label	Cycles of $L \leq 3$	Cycle Weight	Disjoint Cycles	Weight
A	$v_1 \to v_2 \to v_1$	8	A, G	15
B	$v_1 \to v_2 \to v_4 \to v_1$	14	B	14
C	$v_1 \to v_3 \to v_1$	7	C, E	13
D	$v_1 \to v_3 \to v_4 \to v_1$	8	C, F	16
E	$v_2 \to v_5 \to v_2$	6	C, G	14
F	$v_2 \to v_4 \to v_5 \to v_2$	9	D, E	14
G	$v_4 \to v_5 \to v_6 \to v_4$	7	E	6
			F	9
			G	7

4. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (8 points)

   For this problem, suppose the applicants' preferences are given by:

Maverick	**Iceman**	**Hollywood**	**Goose**
1. City	1. City	1. General	1. State
2. General	2. State	2. City	2. General
3. State	3. General	3. State	3. City

   Suppose that each residency program has only 1 open position, and that the preferences of the programs are given by:

City	**General**	**State**
1. Maverick	1. Goose	1. Goose
2. Iceman	2. Iceman	2. Hollywood
3. Goose	3. Maverick	3. Maverick
4. Hollywood	4. Hollywood	4. Iceman

Solutions:

The results are given by the following table. Hollywood does not match.

City	**General**	**State**
Maverick	~~Hollywood~~	~~Iceman~~
	Iceman	Goose

5. Suppose an orange juice manufacturer would like to purchase oranges from a orange supplier. The restaurant's utility for the oranges is given by $S(q) = 200\sqrt{q}$. The fixed costs for the supplier are 500. If the supplier is inefficient then its marginal costs are 5, and if the supplier is efficient then its marginal costs are 4. Assume that the restaurant believes that there is a 83.33% (5/6) chance that the food distributor is efficient.

   (a) What are the first-best production levels? (1 point)

   (b) What are the contracts to implement the first-best production levels? (1 point)

   (c) How much profit would the efficient distributor make if the restaurant offers a menu of contracts corresponding to the first-best production levels of the inefficient and efficient types? (1 point)

   (d) What are the second-best production levels? (2 points)

   (e) What is the menu of contracts for the second-best production levels? (2 points)

   (f) What is the information rent of the efficient distributor for the menu of contracts for the second-best production levels? (1 point)

Solutions:

(a) Note that equating marginal utility to marginal costs for the inefficient distributor gives

$$q_1^I : S'(q_1^I) = \theta^I \Rightarrow q_1^I : 100/\sqrt{q_1^I} = 5 \Rightarrow q_1^I = 400.$$

Similarly, equating marginal utility to marginal costs for the efficient distributor gives

$$q_1^E : S'(q_1^E) = \theta^E \Rightarrow q_1^E : 100/\sqrt{q_1^E} = 4 \Rightarrow q_1^I = 625.$$

(b) These contracts allow for zero information rent, meaning that the inefficient distributor should be offered the contract

$$(q_1^I = 400, t_1^I = \theta^I q_1^I + F = 5 \cdot 400 + 500 = 2500),$$

and the efficient distributor should be offered the contract

$$(q_1^E = 625, t_1^E = \theta^E q_1^E + F = 4 \cdot 625 + 500 = 3000).$$

(c) The profit would be: $t_1^I - \theta^E q_1^I - F = 2500 - 4 \cdot 400 - 500 = 400$.

(d) The production level for the efficient agent remains unchanged $q_2^E = q_1^E = 625$, and the production level for the inefficient agent decreases to

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1-\nu}(\theta^I - \theta^E) \Rightarrow q_2^I : 100/\sqrt{q} = 5 + \frac{5/6}{1-5/6}(5-4) \Rightarrow q_2^I = 100.$$

(e) The transfer for the efficient agent is

$$t_2^E = \theta^E q_2^E + (\theta^I - \theta^E) q_2^I + F = 4 \cdot 625 + (5 - 1)100 + 500 = 3100,$$

and the transfer for the inefficient agent is

$$t_2^I = \theta^I q_2^I + F = 5 \cdot 100 + 500 = 1000.$$

Summarizing, the menu of contracts are $\{(q_2^E = 625, t_2^E = 3100), (q_2^I = 100, t_2^I = 1000)\}$.

(f) The information rent for the efficient distributor is

$$U^E = t^E - \theta^E q^E - F = 3100 - 4 \cdot 625 - 500 = 100.$$

6. Suppose a hotel has decided to outsource cleaning services to an external company. The hotel's utility for the number of rooms cleaned in one day is given by $S(q) = 500 \ln\ (1 + q)$. (A room can be cleaned more than once to provide greater service to guests.) Assume that the hotel and cleaning company both believe that there is a 208/249 chance that the cleaning company is efficient. Suppose the fixed costs for the external cleaning company are 100. If the cleaning company is inefficient then its marginal costs are 2, and if the cleaning company is efficient then its marginal costs are 1.

   (a) Show that the first-best production levels are $q_1^E = 499$ and $q_1^I = 249$. (2 points)

   (b) Now imagine that the cleaning company does not know if it is efficient or inefficient, because it has recently hired a large number of new employees. In this situation, the hotel designs the contract by solving the following optimization problem

$$\begin{array}{ll} \min\limits_{\{(q^E,t^E),(q^I,t^I)\}} & -\frac{208}{249}(500\ln(1+q^E) - t^E) - \frac{41}{249}(500\ln(1+q^I) - t^I) \\ \text{subject to:} & t^E - q^E - 100 \geq t^I - q^I - 100 \\ & t^I - 2q^I - 100 \geq t^E - 2q^E - 100 \\ & \frac{208}{249}(t^E - q^E - 100) + \frac{41}{249}(t^I - 2q^I - 100) \geq 0. \end{array}$$

   What are the KKT conditions for this optimization problem? (3 points)

   (c) The menu of contracts

$$\{(q^E = 499, t^E = 640), (q^I = 249, t^I = 390)\}$$

   is a minimizer for the optimization problem given above. Show that it satisfies the KKT conditions. (3 points)

Solutions:

(a) Note that equating marginal utility to marginal costs for the inefficient distributor gives

$$q_1^I : S'(q_1^I) = \theta^I \Rightarrow q_1^I : 500/(1 + q_1^I) = 2 \Rightarrow q_1^I = 249.$$

Similarly, equating marginal utility to marginal costs for the efficient distributor gives

$$q_1^E : S'(q_1^E) = \theta^E \Rightarrow q_1^E : 500/(1 + q_E^1) = 1 \Rightarrow q_1^I = 499.$$

(b) The KKT conditions are given by

$$\begin{aligned}
-\tfrac{208}{249}500/(1+q^E) + \lambda_1 - 2\lambda_2 + \tfrac{208}{249}\lambda_3 &= 0\\
\tfrac{208}{249} - \lambda_1 + \lambda_2 - \tfrac{208}{249}\lambda_3 &= 0\\
-\tfrac{41}{249}500/(1+q^I) - \lambda_1 + 2\lambda_2 + 2\tfrac{41}{249}\lambda_3 &= 0\\
\tfrac{41}{249} + \lambda_1 - \lambda_2 - \tfrac{41}{249}\lambda_3 &= 0\\
\lambda_i &\geq 0\\
\lambda_1(t^I - q^I - 100 - t^E + q^E + 100) &= 0\\
\lambda_2(t^E - 2q^E - 100 - t^I + 2q^I + 100) &= 0\\
\lambda_3(-\tfrac{208}{249}(t^E - q^E - 100) - \tfrac{41}{249}(t^I - 2q^I - 100)) &= 0.
\end{aligned}$$

(c) Substituting the menu of contracts into the KKT conditions gives

$$\begin{aligned}
-\tfrac{208}{249} + \lambda_1 - 2\lambda_2 + \tfrac{208}{249}\lambda_3 &= 0\\
\tfrac{208}{249} - \lambda_1 + \lambda_2 - \tfrac{208}{249}\lambda_3 &= 0\\
-\tfrac{41}{249}2 - \lambda_1 + 2\lambda_2 + 2\tfrac{41}{249}\lambda_3 &= 0\\
\tfrac{41}{249} + \lambda_1 - \lambda_2 - \tfrac{41}{249}\lambda_3 &= 0\\
\lambda_i &\geq 0\\
\lambda_1(0) &= 0\\
\lambda_2(-250) &= 0\\
\lambda_3(0) &= 0.
\end{aligned}$$

Thus, we must have that $\lambda_2 = 0$ because of the complimentary slackness conditions. Next, the first equation of the stationarity condition means that we must have

$$\lambda_1 = \tfrac{208}{249}(1 - \lambda_3).$$

Substituting this into the fourth equation of the stationarity condition gives

$$\tfrac{41}{249} + \tfrac{208}{249}(1 - \lambda_3) - \tfrac{41}{249}\lambda_3 = 0 \Rightarrow 1 - \lambda_3 = 0 \Rightarrow \lambda_3 = 1.$$

Thus, we have $\lambda_1 = 0$. Summarizing, the KKT conditions are satisfied by $\{(q^E = 499, t^E = 640), (q^I = 249, t^I = 390)\}$ when $(\lambda_1, \lambda_2, \lambda_3) = (0, 0, 1)$.