1. Suppose a fast food restaurant would like to purchase veggie burger patties from a food distributor. The restaurant’s utility for the patties is given by $S(q) = 1000ln(1 + q)$. The fixed costs for the food distributor are $8,000$, and if the distributor is inefficient (efficient) then its marginal costs are $0.10$ ($0.08$). Assume that the restaurant believes that there is a $80\%$ chance that the food distributor is efficient.

(a) What are the first-best production levels? (2 points)

Note that equating marginal utility to marginal costs for the inefficient distributor gives

$q_I^l : S'(q_I^l) = \theta^I \Rightarrow q_I^l : \frac{1000}{1 + q_I^l} = 0.10$

$\Rightarrow q_I^l = 9999.$

Similarly, equating marginal utility to marginal costs for the efficient distributor gives

$q_E^l : S'(q_E^l) = \theta^E \Rightarrow q_E^l : \frac{1000}{1 + q_E^l} = 0.08$

$\Rightarrow q_E^l = 12499.$

(b) What are the contracts to implement the first-best production levels? (2 points)

These contracts allow for zero information rent, meaning that the inefficient distributor should be offered the contract

$(q_I^l = 9999, t_I^l = \theta^I q_I^l + F = 0.10 \cdot 9999 + 8000 = 8999.90)$,

and the efficient distributor should be offered the contract

$(q_E^l = 12499, t_E^l = \theta^E q_E^l + F = 0.08 \cdot 12499 + 8000 = 8999.92)$.

(c) How much profit would the efficient distributor make if the restaurant offers a menu of contracts $\{(q_I^l, t_I^l), (q_E^l, t_E^l)\}$? (1 point)

The profit would be

$t_I^l - \theta^E q_I^l - F = 8999.90 - 0.08 \cdot 9999 - 8000 = 199.98.$
(d) What are the second-best production levels? (2 points)

The production level for the efficient agent remains unchanged $q_2^E = q_1^E = 12499$, and the production level for the inefficient agent decreases to

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1 - \nu} (\theta^I - \theta^E) \Rightarrow q_2^I : \frac{1000}{1 + x} = 0.10 + \frac{0.8}{1 - 0.8} (0.10 - 0.08)$$
$$\Rightarrow q_2^I = 5554.$$

(e) What is the menu of contracts for the second-best production levels? (2 points)

The transfer for the efficient agent is

$$t_2^E = \theta^E q_2^E + (\theta^I - \theta^E)q_2^I + F = 0.08 \cdot 12499 + (0.10 - 0.08) \cdot 5554 + 8000 = 9111.00,$$

and the transfer for the inefficient agent is

$$t_2^I = \theta^I q_2^I + F = 0.10 \cdot 5554 + 8000 = 8555.40.$$

Summarizing, the menu of contracts are \{(q_2^E = 12499, t_2^E = 9111.00), (q_2^I = 5554, t_2^I = 8555.40)\}.

(f) What is the information rent of the efficient distributor for the menu of contracts for the second-best production levels? Is this higher or lower than the profit gained for the menu of contracts for the first-best production levels? (2 points)

The information rent for the efficient distributor is

$$U^E = t^E - \theta^E q^E - F = 9111.00 - 0.08 \cdot 12499 - 8000 = 111.08.$$

This is lower than the profit gained for the menu of contracts for the first-best production levels 199.98.