IEOR 151 – Homework 6
Due Monday, October 21, 2013 in class

1. Suppose a fast food restaurant would like to purchase veggie burger patties from a food distributor. The restaurant's utility for the patties is given by $S(q) = 1000ln(1+q)$. The fixed costs for the food distributor are $8,000$, and if the distributor is inefficient (efficient) then its marginal costs are 0.10 (0.08). Assume that the restaurant believes that there is a 80% chance that the food distributor is efficient.

   (a) What are the first-best production levels? (2 points)

   Note that equating marginal utility to marginal costs for the inefficient distributor gives

   $$\begin{aligned} q_1^I : S'(q_1^I) = \theta^I &\Rightarrow q_1^I : \frac{1000}{1+q_1^I} = 0.10 \\ &\Rightarrow q_1^I = 9999. \end{aligned}$$

   Similarly, equating marginal utility to marginal costs for the efficient distributor gives

   $$\begin{aligned} q_1^E : S'(q_1^E) = \theta^E &\Rightarrow q_1^E : \frac{1000}{1+q_1^E} = 0.08 \\ &\Rightarrow q_1^E = 12499. \end{aligned}$$

   (b) What are the contracts to implement the first-best production levels? (2 points)

   These contracts allow for zero information rent, meaning that the inefficient distributor should be offered the contract

   $$(q_1^I = 9999, t_1^I = \theta^I q_1^I + F = 0.10 \cdot 9999 + 8000 = 8999.90),$$

   and the efficient distributor should be offered the contract

   $$(q_1^E = 12499, t_1^E = \theta^E q_1^E + F = 0.08 \cdot 12499 + 8000 = 8999.92).$$

   (c) How much profit would the efficient distributor make if the restaurant offers a menu of contracts $\{(q_1^I, t_1^I), (q_1^E, t_1^E)\}$? (1 point)

   The profit would be

   $$t_1^I - \theta^E q_1^I - F = 8999.90 - 0.08 \cdot 9999 - 8000 = 199.98.$$

(d) What are the second-best production levels? (2 points)

The production level for the efficient agent remains unchanged $q_2^E = q_1^E = 12499$, and the production level for the inefficient agent decreases to

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1-\nu}(\theta^I - \theta^E) \Rightarrow q_2^I : \frac{1000}{1+x} = 0.10 + \frac{0.8}{1-0.8}(0.10 - 0.08)$$
$$\Rightarrow q_2^I = 5554.$$

(e) What is the menu of contracts for the second-best production levels? (2 points)

The transfer for the efficient agent is

$$t_2^E = \theta^E q_2^E + (\theta^I - \theta^E) q_2^I + F = 0.08 \cdot 12499 + (0.10 - 0.08) \cdot 5554 + 8000 = 9111.00,$$

and the transfer for the inefficient agent is

$$t_2^I = \theta^I q_2^I + F = 0.10 \cdot 5554 + 8000 = 8555.40.$$

Summarizing, the menu of contracts are $\{(q_2^E = 12499, t_2^E = 9111.00), (q_2^I = 5554, t_2^I = 8555.40)\}$.

(f) What is the information rent of the efficient distributor for the menu of contracts for the second-best production levels? Is this higher or lower than the profit gained for the menu of contracts for the first-best production levels? (2 points)

The information rent for the efficient distributor is

$$U^E = t^E - \theta^E q^E - F = 9111.00 - 0.08 \cdot 12499 - 8000 = 111.08.$$

This is lower than the profit gained for the menu of contracts for the first-best production levels 199.98.