# IEOR 151 – Homework 5
# Due Friday, October 11, 2013 in class

1. Consider the following optimization problem

$$\begin{aligned} &\min x \\ &\text{s.t. } x \in \mathbb{R} \\ &\quad -x^3 \leq 0. \end{aligned}$$

   (a) If $x^* \in \{x \in \mathbb{R} : -x^3 \leq 0\}$ is the global minimizer of this optimization problem, then what are Fritz John conditions? (2 points)
   (b) Determine all possible combinations of $(\lambda_0, \lambda_1, x^*)$ for which the Fritz John conditions hold. (3 points)
   (c) Show that the LICQ does not hold at all feasible points? (1 point)
   (d) If the global minimizer $x^*$ satisfies the KKT conditions, then write down the KKT conditions. If the global minimizer $x^*$ does not satisfy the KKT conditions, explain why. (2 points)
   (e) Rewrite the optimization problem so that the LICQ holds for all feasible points. (2 points).
   (f) What are the KKT conditions for the rewritten optimization problem where the LICQ holds? (2 points)
   (g) Determine all possible combinations of $(\lambda_1, x^*)$ for which the KKT conditions hold. Based on these combinations, compute the global minimizer of the optimization problem? (3 points)

2. Consider the following parametric optimization problem

$$\begin{aligned} V(\theta) = &\min_x \theta x \\ &\text{s.t. } x \in [-1, 1] \end{aligned}$$

   (a) Is the objective jointly continuous in $(x, \theta)$? Is the constraint set $[1, 1]$ continuous in $\theta$? Hint: You do not have to do any calculations. (2 points)
   (b) Compute the minimizer $x^*(\theta)$ for $\theta \in [-1, 1]$? (3 points)
   (c) Plot $x^*(\theta)$. Is it upper hemi-continuous? (2 point)
   (d) Compute the value function $V(\theta)$ for $\theta \in [-1, 1]$? (1 point)
   (e) Plot $V(\theta)$. Is it continuous? (2 point)
   (f) Do these results agree with the Berge Maximum Theorem? Is this expected? (1 point)