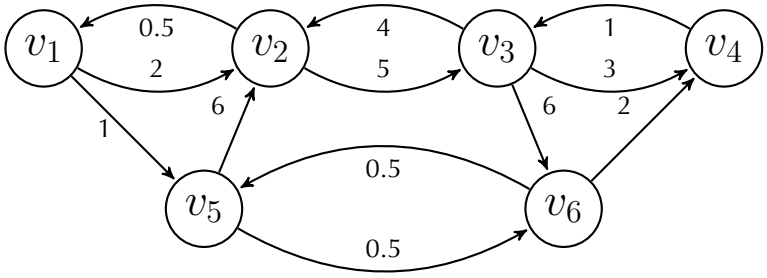
IEOR 151 – Homework 4
Due Wednesday, October 2, 2013 in class

1. Consider the following graph representation of a kidney exchange. Find the social welfare maximizing exchange under the constraint that all cycles can have length less than or equal to $L = 3$. (5 points)



First, we list all cycles of length $L \leq 3$ and compute the weight of these cycles. Next, we determine all sets of disjoint cycles and compute their weight. Lastly, the solution is the set of disjoint cycles with maximal weight. The steps are shown below, and the social welfare maximizing exchange is the set of disjoint cycles B, E.

Cycle Label	Cycles of $L \leq 3$	Cycle Weight	Disjoint Cycles	Weight
A	$v_1 \to v_2 \to v_1$	2.5	A, D, F	7.5
B	$v_1 \to v_5 \to v_2 \to v_1$	7.5	A, E	11.5
C	$v_2 \to v_3 \to v_2$	9	A, F	3.5
D	$v_3 \to v_4 \to v_3$	4	B, D	11.5
E	$v_3 \to v_6 \to v_4 \to v_3$	9	B, E	16.5
F	$v_5 \to v_6 \to v_5$	1	C, F	10
			D, F	5
			E	9
			F	1

2. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)

   For this problem, suppose the applicants' preferences are given by:

George	**Jerry**	**Elaine**
1. City	1. City	1. City
2. General	2. General	2. General

   Suppose that each residency program has only 1 open position, and that the preferences of the programs are given by

City	**General**
1. Elaine	1. Elaine
2. Jerry	2. Jerry

   The results are given by the following table.

City	**General**
~~Jerry~~	Jerry
Elaine	

3. Extra Credit: Based on the presentation given by Prof. Sandholm, briefly explain in three to five sentences how the kidney exchange model can be modified to capture the dynamic nature of the problem. (5 points)