IEOR 151 – Homework 3 Due Wednesday, September 25, 2013 in class

- 1. Suppose 5 different hypothesis tests have been conducted, with *p*-values of: Test 1 (p = 0.07), Test 2 (p = 0.001), Test 3 (p = 0.015), Test 4 (p = 0.005), Test 5 (p = 0.05).
 - (a) Using the Bonferroni correction, which tests should be accepted or rejected when the family-wise error rate is $\alpha = 0.05$. (2 points)

Since there are five tests, the Bonferroni correction states that a hypothesis should be rejected if $p < \alpha/5 = 0.01$. Thus, Tests 2 and 4 should be rejected and Tests 1, 3, and 5 should be accepted.

(b) Using the Holm–Bonferroni method, which tests should be accepted or rejected when the family-wise error rate is $\alpha = 0.05$. (3 points)

We begin by arranging the *p*-values in increasing order: 0.001, 0.005, 0.015, 0.05, 0.07. We need to determine the smallest k such that the k-th *p*-value in the arranged list is greater than $Q_k = \alpha/(5 + 1 - k)$. For k = 1, ..., 5, the rounded values of Q_k are 0.01, 0.0125, 0.017, 0.025, and 0.05. In this case, k = 4 is that smallest k. As a result, we reject hypothesis corresponding to the first three *p*-values in the ordered list and accept the remaining. Thus, Tests 2, 3, and 4 should be rejected and Tests 1 and 5 should be accepted.

- 2. Suppose that three groups whose measurements are expected to be Gaussian are compared, and an *F*-test gives p = 0.01. The *p*-values for the pairwise comparisons are given by $p_{12} = 0.010$, $p_{13} = 0.007$, and $p_{23} = 0.030$.
 - (a) Using the Bonferroni correction, which tests should be accepted or rejected when the family-wise error rate is $\alpha = 0.05$. (3 points)

Suppose that we perform the *F*-test at the significance level of $\alpha/2$, then we reject the null hypothesis corresponding to the *F*-test. Now using the Bonferroni correction, we should reject any pairwise tests whose *p*-values are below $(\alpha/2)/3 = 0.008$. Thus, we should reject p_{13} and accept p_{12} and p_{23} .

(b) Using the Holm–Bonferroni method, which tests should be accepted or rejected when the family-wise error rate is $\alpha = 0.05$. (3 points)

Suppose that we perform the F-test at the significance level of $\alpha/2$, then we reject the null hypothesis corresponding to the F-test. Next, we arrange the pairwise p-values in increasing order: 0.007, 0.010, and 0.030. We need to determine the smallest k such that the k-th p-value in the arranged list is greater than $Q_k = (\alpha/2)/(3 + 1 - k)$. For $k = 1, \ldots, 3$, the rounded values of Q_k are 0.008, 0.0125, and 0.025. In this case, k = 3 is that smallest k. As a result, we reject hypothesis corresponding to the first two p-values in the ordered list and accept the remaining. Thus, p_{13} and p_{12} should be rejected and p_{23} should be accepted.