IEOR 151 – Homework 1
Due Wednesday, September 11, 2013 in class

1. Two players A and B play a game in which they take turns flipping a coin. A starts the game, and B plays second. The first player to flip a heads wins.

   (a) Compute the probability that A wins if the coin is fair – meaning the probability of heads is $1/2$. (2 points)

   (b) Compute the probability that A wins if the coin is biased, and the probability of heads is p such that $p > 0$. (1 point)

   (c) For what values of p does A have a better chance of winning than B? (1 point)

2. Consider a disease that is found in 0.1% of the population, with a test for this disease that gives a correct diagnosis 90% of the time. Suppose a person is randomly chosen from the population and given a first test. If the first test is positive, then the person is given a second test.

   (a) Compute the probability that the first test is positive given that the person is infected with the disease. (1 point)

   (b) Compute the probability that both tests are positive given that the person is infected with the disease. (1 point)

   (c) Compute the probability that the person is infected given that the first test is positive. (2 points)

   (d) Compute the probability that the person is infected given that both tests are positive. (2 points)

3. Consider an airplane with maximum seating capacity of 100 passengers, and assume that the airline company sells 110 tickets because each passenger has a 10% probability (independently of other passengers) of not showing up for the flight. What is the probability that there are enough seats for all passengers that show up for the flight? Hint: It is not necessary to mathematically simplify the answer. (2 points)

4. Suppose $g(u)$ is twice continuously differentiable at $u = \theta$, $g'(\theta) = 0$, and $g''(\theta) \neq 0$. If $\sqrt{n}[X_n - \theta] \xrightarrow{d} \sigma \mathcal{N}(0, 1)$, then show that $n[g(X_n) - g(\theta)] \xrightarrow{d} \sigma^2 \frac{g''(\theta)}{2} \chi_1^2$, where χ_1^2 is a chi-squared random variable with 1 degree of freedom. Hint 1: If Z is distributed according to $\mathcal{N}(0, 1)$, then Z^2 is distributed according to χ_1^2. Hint 2: Use the Lagrange form of Taylor's Theorem. (5 points)

5. Suppose $\sqrt{n}[\hat{p} - p] \xrightarrow{d} \sqrt{p(1-p)}\mathcal{N}(0,1)$. If $g(u) = 2\arcsin(\sqrt{u})$, then compute the limiting distribution of $\sqrt{n}[g(\hat{p}) - g(p)]$. Note: The function $g(u)$ in this case is what is known as a variance stabilizing transformation. (3 points)