

Optimal transmission switching: economic efficiency and market implications

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Abstract Traditionally, transmission assets for bulk power flow in the electric grid have been modeled as fixed assets in the short run, except during times of forced outages or maintenance. This traditional view does not permit reconfiguration of the transmission grid by the system operators to improve system performance and economic efficiency. The current push to create a smarter grid has brought to the forefront the possibility of co-optimizing generation along with the network topology by incorporating the control of transmission assets within the economic dispatch formulations. Unfortunately, even though such co-optimization improves the social welfare, it may be incompatible with prevailing market design practices since it can create winners and losers among market participants and it has unpredictable distributional consequences in the energy market and in the financial transmission rights (FTR) market. In this paper, we first provide an overview of recent research on optimal transmission switching, which demonstrates the substantial economic benefit that is possible even while satisfying standard N–1 reliability requirements. We then discuss various market implications resulting from co-optimizing the network topology with

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generation and we examine how transmission switching may affect locational Marginal Prices (LMPs), i.e., energy prices, and revenue adequacy in the FTR market when FTR settlements are financed by congestion revenues.

Keywords Optimal transmission switching · Financial transmission rights · Power generation dispatch · Power system economics · Power transmission economics · Revenue adequacy

JEL Classification L94

Notation

Indices and sets

g	Generator
$g(n)$	Set of generators at bus n
k	Transmission element (line or transformer)
$k(n, \cdot), k(\cdot, n)$	Set of transmission elements with bus n as the to bus and the set with bus n as the from bus respectively
m, n	Nodes

Parameters

B_k	Susceptance of transmission element k
c_g	Production cost for generator g
d_n	Real power load at node n
G_k	Conductance of transmission element k
M_k	Big M value for transmission element k
P_g^{\max}, P_g^{\min}	Max and min capacity of generator g
P_k^{\max}, P_k^{\min}	Max and min rating of transmission element k ; typically $P_k^{\max} = -P_k^{\min}$
$\theta^{\max}, \theta^{\min}$	Max and min bus voltage angle difference; typically $\theta^{\max} = -\theta^{\min}$

Variables

P_g	Real power supply from generator g at node n
P_k	Real power flow from node m to node n for transmission element k
Q_k	Reactive power flow from node m to node n for transmission element k
V_n	Bus voltage at node n
z_k	Binary switching variable for transmission element k (0 open/not in service, 1 closed/in service)
ϕ_k	Bus voltage angle difference across transmission element k
θ_n	Bus voltage angle at node n

1 Introduction

The unique physical laws that govern the flow of electric energy in the electrical network imply that switching a line out of service changes the power flows throughout

the network, thereby enabling operational flexibility that can be harnessed to control transmission assets in the short run. This phenomenon is similar to Braess' Paradox, which states that adding capacity to a network may not improve the performance if there are selfish agents that choose their own path. In this case, the flow of electric energy follows Kirchhoff's Laws.

There are various control mechanisms that operators can use to control the flow of electric energy. Flexible AC Transmission System (FACTS) devices can be used to control the flow of electric energy. [Shao and Vittal \(2006a\)](#) proposes the use of FACTS devices as a corrective mechanism to relieve overloading and voltage violations. [Glanzmann and Andersson \(2005\)](#) proposes the use of FACTS devices as a congestion management tool. In this paper, the focus will be on transmission switching, i.e., the opening and closing of circuit breakers in order to temporarily take transmission assets (lines or transformers) in and out of service.

Previous research has demonstrated the benefit of transmission switching as a corrective mechanism for a variety of network conditions. During certain market conditions, it is possible to take transmission elements temporarily out of service to increase transfer capacity or improve voltage profiles ([Ott 2008](#)). For example, it is an accepted practice to improve voltage profiles by opening light-loaded transmission lines at night ([Nauman 2008](#)); at low load levels, the capacitive component of a transmission line is the predominate component and, thus, temporarily removing the line can help reduce high-voltage issues. The Northeast Power Coordinating Council includes "switch out internal transmission lines" in the list of possible actions to avoid abnormal voltage conditions ([Northeast Power Coordinating Council 1997](#); [ISO-NE 2007](#)). Transmission switching has also been proposed as a control method for problems such as over or under voltage, line overloading, loss and/or cost reduction, and system security ([Granelli et al. 2006](#); [Shao and Vittal 2005, 2006b](#); [Bakirtzis and Meliopoulos 1987](#); [Mazi et al. 1986](#); [Schnyder and Glavitsch 1988, 1990](#); [Bacher and Glavitsch 1986, 1988](#); [Fliscounakis et al. 2007](#); [Glavitsch 1985](#)). Numerous Special Protection Systems (SPS) address specific instances of switching for both pre- and post-contingency states. At PJM, SPSs allow the operator to disconnect a line during normal operations but return it to service during a contingency or vice versa ([PJM 2009](#)). [Blumsack et al. \(2007\)](#) showed that there can be a conflict between reliability and congestion by examining the effect of the Wheatstone bridge.

Even though past research has identified the benefits from harnessing the control of transmission assets as a corrective mechanism, traditional security constrained economic dispatch of electricity resources still treats the transmission network as a fixed and static topology while optimizing deployment of generation assets. However, it is well known that the redundancy built into the grid in order to handle the multitude of contingencies over a long planning horizon can, in the short run, create congestion and necessitate costly out of merit dispatch. While it is quite common for operators to occasionally open lines, such practices are employed on an ad hoc basis and are not driven by cost considerations. Furthermore, such options are not included, formally, in the classical formulation of dispatch optimization problems common in system and market operation procedures employed by vertically integrated utilities and Independent System Operators (ISOs).

With the current push to create a smarter grid, recent research suggests that security constrained economic dispatch models should explicitly account for the short term flexibility of transmission assets. The concept of dispatchable transmission was investigated in O'Neill et al. (2005). Optimal transmission switching has been proposed as a way to incorporate the control of transmission assets into the day-ahead dispatch optimization models and this research has demonstrated substantial economic savings for various test cases (Fisher et al. 2008; Hedman et al. 2008, 2009, 2010; Hedman et al., accepted; O'Neill et al., accepted). Although optimal transmission switching focuses on temporarily removing selected transmission lines from service, it does not necessarily imply an economic based degradation in reliable network operations. Indeed Hedman et al. (2009) and Hedman et al. (2010) demonstrated that transmission switching can be beneficial while ensuring the conventional N-1 network reliability criterion. The N-1 criterion requires that the system can survive the failure of any one component (generator, transmission line, transformer, etc.). It has also been demonstrated in Hedman et al. (2010) that co-optimizing the network topology as a part of the day-ahead unit commitment problem can provide substantial savings.

While transmission switching can improve economic efficiency of grid operations, such practice may have unpredictable distributional effects on market participants and undermine some prevalent market design principles that rely on the premise of a fixed network topology. Financial transmission rights (FTRs), which have been widely adopted in the USA as a mechanism for allocating property rights to the electricity grid, are particularly vulnerable to physical changes in the network topology. Modifying the transmission topology for the common good interferes with some of the underlying assumptions that facilitate FTRs markets. Furthermore, such topology co-optimization has unpredictable impacts on Locational Marginal Prices (LMPs), generation rents, load payments, congestion rents, and payments to FTR holders.

Financial transmission rights are instrumental in the electric energy markets since they enable allocation of property rights to the electric power grid and management of congestion risk by energy market participants without inhibiting efficient operation of the grid by the system operator. An FTR entitles its holder to the difference in the LMPs for specific source and sink locations, i.e., the sink price minus the source price, times the quantity (in MW) of the FTR. In electricity markets that are based on the LMP paradigm, congestion charges for transferring 1 MW of power from an injection point to a withdrawal point equal the LMP difference between the two nodes. Hence, a 1 MW FTR between a source and a sink provides a perfect hedge against congestion charges for 1 MW of power transmitted between these two nodes. FTRs may be contracted to be two sided obligations or options, which can be forfeited if their payoff is negative, but they are typically defined as obligations and will be assumed to be so in this paper.

In typical restructured electricity markets in the USA, FTRs are allocated or auctioned so as to meet a simultaneous feasibility test (SFT). The SFT guarantees that if all the outstanding FTRs are exercised simultaneously to support physical transfers between their corresponding sources and sinks then all these transactions can be supported by the physical grid, i.e., no transmission constraint will be violated. When

the topology assumed for the SFT is the same as the topology used in the real time dispatch, it was shown in [Hogan \(1992\)](#) that under certain theoretical conditions, congestion revenues collected by the ISO will be “adequate” in the sense that they will be sufficient to cover the financial settlement of all outstanding FTR obligations. The proof of revenue adequacy is based on the “separating hyperplane principle” of convex optimization and relies on the assumption that the SFT feasible set (nomogram) is convex. This convexity assumption holds for a DC load flow model where the SFT nomogram is represented by a convex polyhedron.

In practice, however, FTR markets are often “revenue inadequate”, i.e., the ISO does not collect sufficient congestion rents to fully compensate the FTR holders. Such shortfalls are often due to changes in the network topology ([Alsac et al. 2004](#)), which can occur when lines fail or are switched offline. Revenue inadequacy can also occur when lines’ thermal capacities are de-rated for various operational reasons and may also be due to the fact that actual power flows are based on an AC model and not the DC approximation. Various ad hoc procedures have been adopted by the different ISOs to address such revenue shortfalls. Some ISOs uniformly discount payments to the FTR holders when revenue shortfalls occur. Such discounting obviously decreases the FTRs’ value; it also undermines the purpose of FTRs as a hedge against price risk. This research is based on the DCOPF model and our discussion on revenue adequacy in this paper refers only to changes in the network topology and does not include other possible causes mentioned above.

In this paper we explore, from an economic perspective, the potential of treating the grid as a flexible topology that can be co-optimized along with generation dispatch, subject to reliability constraints, so as to minimize the cost of serving load. We then examine the implications of such co-optimization by the system operator on various settlements and in particular on the FTR market.

In Sect. 2, we provide an overview of transmission switching and how the control of transmission assets can be incorporated into traditional power flow formulations. In Sect. 2.5, we provide a review of recent research that demonstrates that optimizing the network topology with generation can significantly improve the economic operations, even while maintaining the traditional “N–1 reliability” standards ([Fisher et al. 2008](#); [Hedman et al. 2008, 2009, 2010](#); Hedman et al., accepted). Our analysis also provides an assessment of potential economic gains from smart grid technologies that will enable replacement of the preventive N–1 reliability standard in favor of new reliability concepts such as a corrective “just in time N–1 reliability.” Test results are presented for the IEEE 118 bus test case, the IEEE RTS 96 system, and the ISO-NE 5000 bus electric grid with all showing significant efficiency improvements in economic dispatch through transmission switching. In Sect. 3 we discuss the market consequences that can arise due to transmission switching; Sects. 3.1–3.3 present a review of past research on market implications ([Hedman et al. 2008, 2010](#)). In Sect. 4 we present two illustrative three-bus examples and discuss how transmission switching affects revenue adequacy of FTRs; in Sect. 4 we also analyze the IEEE 118-bus test case to estimate the worst case revenue shortfall that is possible over a year. Section 5 presents a discussion of the policy implications regarding the implementation of transmission switching. Finally, Sect. 6 concludes this paper.

2 Optimal transmission switching

2.1 Optimal power flow

The flow of electric energy follows Kirchhoff's laws. The Alternating Current Optimal Power Flow (ACOPF) problem is the optimization problem that models how electricity flows across the AC electric grid and it is used to optimally dispatch generation in real time. The ACOPF optimization problem is, however, a very difficult problem to solve since it is a non-convex optimization problem with constraints that include trigonometric functions. Equations 1 and 2 represent the equations for the flow of electric power into bus n from transmission line k (line k is connected from bus m to bus n).

$$Q_k = -V_m V_n (G_k \sin(\theta_m - \theta_n) - B_k \cos(\theta_m - \theta_n)) - B_k V_m^2, \quad \forall k \quad (1)$$

$$P_k = V_m^2 G_k - V_m V_n (G_k \cos(\theta_m - \theta_n) + B_k \sin(\theta_m - \theta_n)), \quad \forall k \quad (2)$$

P_k is the real power flow and Q_k is the reactive power flow. V_n , V_m , θ_n , and θ_m are all variables and they represent bus voltages and phase angles respectively for the two buses. Equations 1 and 2 are the power flow equations. Additional constraints that are required for the ACOPF problem include the constraints on the magnitude of the voltage variables, constraints on the angle difference between two connected buses, operational constraints on the generators, capacity constraints on the transmission lines, and node balance constraints. The non-linearity that these equations create significantly complicates the optimization problem.

It is common, both in academic literature and in the industry, to use a linear approximation of the ACOPF problem. The first assumption concerns the voltage variables, V_n and V_m . The voltage levels are generally very close to one since they are based on a per unit calculation; thus, all voltage variables are assumed to have a value of one. This removes one of the non-linearity problems within (1) and (2). The next assumption comes from the fact that the angle difference between two connected buses is typically very small. This then allows one to approximate the sine of a small angle difference by the angle difference itself and the cosine of a small angle difference is approximately one. These two assumptions cause the B_k terms in (1) to cancel and the G_k terms in (2) to cancel. The next simplification is that the remaining reactive power term, Q_k , is ignored. Finally, the resistance is assumed to be zero thereby making the susceptance equal the negative inverse of the reactance. The traditional DCOPF formulation is, therefore, a lossless model; however, there are ways to modify the traditional DCOPF formulation to account for losses. Throughout this paper, the DCOPF problem is assumed to be a lossless model. All of these assumptions then produce the linear approximation of the ACOPF problem, which is known as the Direct Current Optimal Power Flow (DCOPF) problem. Assuming linear or piecewise linear cost function for each generator, the DCOPF problem is a linear program so it is much easier to solve than the non-convex ACOPF problem. With these assumptions, (1) is ignored and (2) collapses into (3) below for the DCOPF.

$$P_k = B_k(\theta_n - \theta_m), \quad \forall k \quad (3)$$

Alternative formulations of the DCOPF problem often eliminate the phase angle variables and express the network constraints (Kirchhoff’s laws) in terms of Power Transfer Distribution Factors (PTDFs). For our purposes, we use (3) to represent the basic DCOPF network flow constraints since PTDFs change whenever the network topology is changed.

2.2 Transmission switching’s impact on the feasible set

To motivate the optimal transmission switching concept, we will use a simple example that illustrates the potential gains from such an option. We first discuss how transmission switching expands the feasible set and thereby enables an economically superior dispatch. In the following section we discuss how such switching affects reliability.

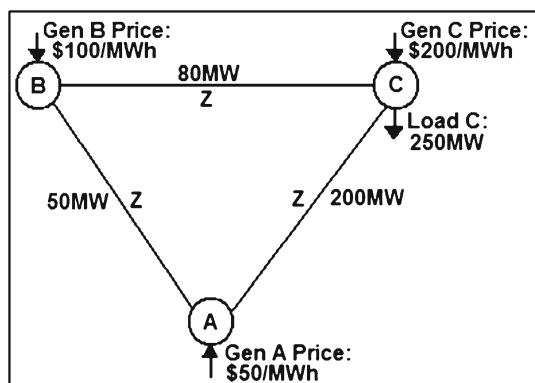
If the impedance of a transmission element is changed, this will change where the power flows in the electrical grid and, thus, this affects which generation dispatches are feasible. Moreover, harnessing the flexibility of transmission assets’ state, in service or out of service, may allow for previously infeasible dispatches to become feasible by expanding the feasible set of solutions. Figure 1 below presents a 3-bus example. All lines in the example have the same impedance but their thermal capacities differ as marked in the figure. The feasible sets in Fig. 2 are defined by transmission constraints. For the original topology, there are three equations that represent the network constraints, (4)–(6). Opening any line will change these constraints and, thus, change the feasible set. With all lines closed, i.e., in service, the feasible set is defined by the set of vertices {0, 1, 2, 3} in Fig. 2. If line A–B is opened, i.e., out of service, the feasible set is {0, 4, 5, 6}.

$$-50 \leq \frac{1}{3} \text{GEN}_A - \frac{1}{3} \text{GEN}_B \leq 50 \tag{4}$$

$$-80 \leq \frac{1}{3} \text{GEN}_A + \frac{2}{3} \text{GEN}_B \leq 80 \tag{5}$$

$$-200 \leq \frac{2}{3} \text{GEN}_A + \frac{1}{3} \text{GEN}_B \leq 200 \tag{6}$$

Fig. 1 3-bus example A



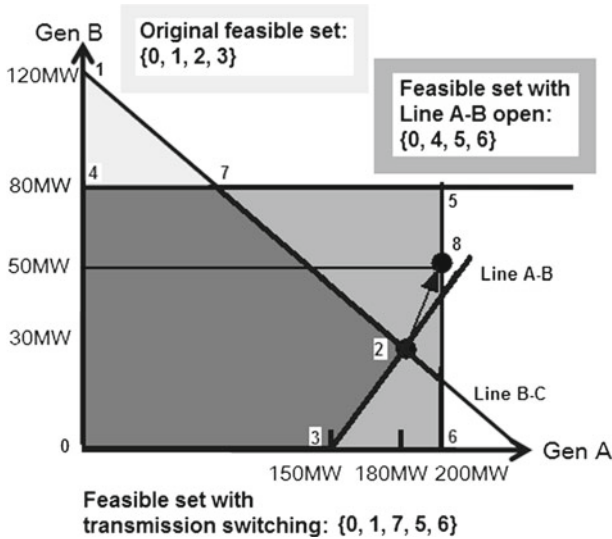


Fig. 2 Feasible sets for Gen A and Gen B, example A

Modifying the topology changes the feasible set. By incorporating the control of transmission elements' state through transmission switching, the feasible set becomes $\{0, 1, 7, 5, 6\}$, which is the non-convex union of the nomograms corresponding to the two states of the line between nodes A and B. Since the original topology can always be chosen, there is the guarantee that any optimized objective will be at least as good as before. In particular, for our example, if the objective were to minimize the total dispatch cost, the original solution would be located at $\{2\}$ where the corresponding dispatch is 30 MW from GEN A, 180 MW from GEN B, and 40 MW from GEN C for a total cost of \$20,000 per hour. By opening line A–B, there is a new feasible solution $\{8\}$, where the corresponding dispatch is 200 MW from GEN A and 50 MW from Gen B, which reduces the total cost to \$15,000 per hour. In general, with transmission switching as an option, any feasible dispatch solution corresponding to a network topology configuration can be chosen and, therefore, harnessing the option of switching lines increases the market surplus for the network. In the following section we discuss how transmission switching can impact reliability; later we provide an overview of work to date on transmission switching and show how the conventional DCOPF problem can be modified to incorporate a switching option for each transmission line.

2.3 Transmission switching and reliability

Transmission switching could not be implemented in practice if it were to violate established reliability requirements. The illustrative example of Sect. 2.2 demonstrated how utilizing the control of transmission elements can be beneficial but that example does not enforce reliability requirements and the proposed line switching solution would

violate conventional reliability criteria. However, in general temporarily removing a transmission line does not necessarily deteriorate reliability. Furthermore, the true concern is whether the specified reliability requirements are met and not whether the reliability of the system is diminished as compared to the case with all lines in service.

There are set reliability requirements and the objective of the operator is to maximize the market surplus subject to the operational and reliability constraints (when the load is inelastic, minimizing the total cost is the same as maximizing the market surplus). With multiple generator dispatch solutions that all meet the reliability requirements, the operator chooses the one that maximizes the market surplus. The conventional operating policy is to enforce reliability requirements as constraints; however, no preference is given to dispatch solutions that exceed these set constraints, i.e., increase the reliability of operations beyond required levels. Hence, implementing a transmission switching protocol, which can increase the market surplus while meeting the reliability constraints, is consistent with that conventional policy even if it degrades overall system reliability. For instance, if the system was initially $N-4$ reliable¹ but with the transmission switching solution it is only $N-3$ reliable, then no reliability requirement would have been violated and, therefore, the superior economic solution, the transmission switching solution, should be chosen. Even if a line is opened during steady-state operations, with a truly smarter, more flexible grid, the opened lines could be put back into service once there is a contingency. Similar practices happen today in ISOs that implement SPSs that involve pre- and post-contingency switching (PJM 2009).

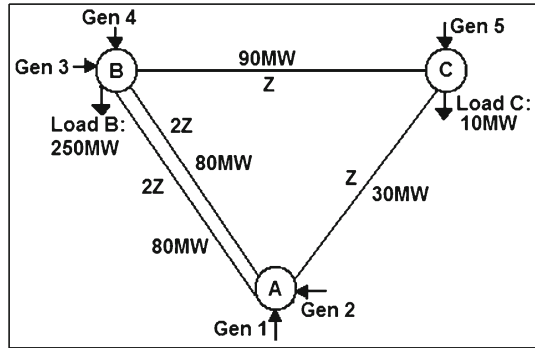
In addition, it is possible that taking a line out of service can actually improve reliability under certain market conditions. The electric grid is a very complex system and it is not possible to judge reliability purely on the topology of the network alone. The reliability of the grid depends on the topology, the load, the generator commitments, the generator ramping abilities, the generators' available capacity, etc. These conditions are always changing and it is not guaranteed that one network topology is the most reliable for all possible network conditions.

Transmission switching allows for previously infeasible generator dispatch solutions to become feasible as demonstrated in the previous section. One of these previously infeasible generator dispatch solutions, which is feasible with transmission switching, may in fact be more reliable than the optimal dispatch solution when all lines are in service. Perhaps this previously infeasible dispatch solution may have different generators committed that have faster ramping capabilities as well as more available capacity. Consequently, examining the grid topology alone is insufficient in regards to judging the reliability of the system; therefore, it is possible that transmission switching may allow one to obtain a more reliable generation dispatch solution.

The following 3-bus example demonstrates that with transmission switching it is possible to have a more reliable system for certain market conditions due to the ability to choose additional generator dispatch solutions. For this example we enforced $N-1$; the system must be able to survive the loss of any line or generator. Note that it is assumed that the two lines between bus A and bus B are in separate right of ways;

¹ $N-k$ reliability means that the system can survive the simultaneous failure of any k elements without violating any constraints on the surviving network.

Fig. 3 3-bus example B



thus, the simultaneous loss of both lines is not included in the list of $N-1$ contingencies. This is a one-period unit commitment problem where we include startup costs and generator ramp rates since we required that there must be adequate 10 min spinning reserve available to survive any contingency. The diagram for the 3-bus example is displayed by Fig. 3 and the generator information is listed in Table 1. Tables 2 and 3 present the optimal dispatch solutions without and with transmission switching respectively.

The first solution with all lines in service is $N-1$ compliant but it would not be able to handle the loss of both generator 3 and generator 4; this is not problematic because there is no regulatory requirement stating that the system must be capable

Table 1 3-bus example B generator information

	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5
Cost \$/MWh	25	20	80	80	100
Startup cost \$	100	100	300	500	400
Gen Min MW	50	50	10	50	10
Gen Max MW	400	100	250	100	150
Ramp rate MW/10 min	200	100	50	50	150

Table 2 Case 1: optimal dispatch solution without transmission switching

	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5	Total dispatch cost
Optimal dispatch MW	Offline	100	40	100	20	\$16,900

Table 3 Case 2: optimal dispatch solution with transmission switching (optimal topology has line A-C open)

	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5	Total dispatch cost
Optimal dispatch MW	160	Offline	10	80	10	\$13,700

of surviving the loss of any two generators. The solution with line A–C open is also N–1 compliant and it has a lower total cost. The transmission switching solution can also survive the loss of both generator 3 and generator 4, unlike the solution with all lines in service. This example demonstrates that merely opening a line does not mean that the system is less reliable. This solution is more reliable with line A–C open than the optimal dispatch solution with line A–C in service; changing the topology of the grid changes what dispatches are possible and reliability is dependent not only on the topology of the network but also on the generation dispatch.

2.4 Overview of optimal transmission switching

Fisher et al. (2008) showed how the traditional DCOPF can incorporate transmission switching into the formulation. Hedman et al. (2009) expanded this problem by enforcing the N–1 reliability requirements to show that transmission switching can be beneficial even while satisfying N–1 reliability requirements. Hedman et al. (2010) presented the problem of co-optimizing unit commitment and the transmission topology with N–1 reliability requirements and analyzed how transmission switching can impact day-ahead unit commitment schedules.

With a smarter grid that has more advanced switching and detection technologies, transmission elements that are open in the optimal dispatch of a network may be available to be switched back into the system as needed; this is referred to as just-in-time transmission by Hedman et al. (accepted). Such advanced technologies give additional flexibility to grid operators in order to improve economic operations as well as improve reliability. Similar switching technologies exist today, such as in PJM’s SPSs. In cases where this may not be possible, transmission switching can be conducted in conjunction with contingency analysis in order to maintain reliability levels while taking advantage of improved network topology solutions, which has been demonstrated by previous research (Hedman et al. 2009, 2010).

Before we change the traditional DCOPF formulation to incorporate transmission switching, we have rewritten (3) as (3a) below to introduce a new variable, ϕ_k , which represents the voltage angle difference for the two buses that transmission element k is connected to. We then use $B_k\phi_k$ to represent the real power flow for transmission element k throughout the optimization problem and, thus, we no longer need P_k to represent the flow on transmission element k . With this change, we must then add (7) in order to ensure ϕ_k is equal to the actual angle difference between buses n and m . We also introduce a new variable, z_k . z_k is a binary variable that represents the status of transmission line k ; a value of one means the line is in service (closed) and a value of zero means the line is temporarily out of service (open).

OPF formulations typically include lower and upper bound constraints on the voltage angle difference, $\theta_n - \theta_m$, for any two buses that are connected by a line, see (8). In the DCOPF formulation, such a constraint can be subsumed by (9), i.e., placing lower and upper bounds on the angle difference for a line places bounds on the power flow for that line. Therefore, the thermal capacity lower and upper bounds, P_k^{\min} and P_k^{\max} , can be replaced by $B_k\theta^{\max}$ and $B_k\theta^{\min}$ if those bounds are tighter than the thermal

capacity constraints for the lines. Therefore, we do not include (8) in the DCOPF formulation; instead, we update P_k^{\min} and P_k^{\max} accordingly.

For the purpose of this paper we will assume that generators’ cost functions are linear, i.e., constant marginal cost², and, hence, the DCOPF problem is a linear program; introducing transmission switching into the DCOPF creates a mixed integer linear programming problem. The first modification is made to the transmission element’s thermal capacity constraints; the original formulation is shown by (9). To incorporate transmission switching, (9) is modified by multiplying the lower and upper bounds by the binary variable z_k to force ϕ_k to be zero if z_k is zero; this modification is shown by (11). The next change occurs with (7), which is rewritten as two inequalities, (12a) and (12b). With this change, when z_k equals one, the line is in service and (12a) and (12b) will then enforce the relationship defined by (7). When z_k equals zero, the line is out of service and, therefore, there should be no constraint that forces ϕ_k to be equal to $\theta_n - \theta_m$; this allows ϕ_k to be equal to zero when z_k equals zero without forcing θ_n to equal θ_m . To properly model this relationship, we use a large multiplier, M_k , which is commonly referred to as a “big M” value. When z_k equals zero, (12a) and (12b) then become lower and upper bounds on the angle difference between buses n and m ; M_k is chosen to be large enough such that these constraints are inactive when z_k equals zero.

The rest of the constraints, as well as the objective, reflect those in the traditional DCOPF. Generator costs are assumed to be linear and the objective, (10), is to minimize the total cost of meeting the demand, which is perfectly inelastic. Equation 13 represents the node balance constraints, which specifies that the power flow into a node (line flow into the node, generator injection) must equal the power flow out of a node (line flow out of the node, demand withdrawal). Equation 14 represents the lower and upper bound constraints on the generators; note that this formulation does not incorporate unit commitment so we assume that the lower bound for all generators is zero. The last equation, (15), specifies the binary constraints on the z_k variables. The full optimal transmission switching DCOPF formulation is listed below by (10)–(15) with the notation listed in Appendix; for further information on optimal transmission switching, see Fisher et al. (2008) and Hedman et al. (2008, 2009, 2010).

$$P_k = B_k(\theta_n - \theta_m) = B_k\phi_k, \quad \forall k \tag{3a}$$

$$\theta_n - \theta_m = \phi_k, \quad \forall k \tag{7}$$

$$\theta^{\min} \leq \theta_n - \theta_m \leq \theta^{\max}, \quad \forall k \tag{8}$$

$$P_k^{\min} \leq B_k\phi_k \leq P_k^{\max}, \quad \forall k \tag{9}$$

Optimal Transmission Switching DCOPF Formulation:

² In reality, generator variable operating cost functions are quadratic in output; however, in practice, such cost functions are approximated by piecewise linear functions represented as block offers at different marginal prices. The DCOPF formulation with piecewise linear cost functions is also a linear programming problem.

$$\text{Minimize : } \sum_g c_g P_g \tag{10}$$

s.t. :

$$P_k^{\min} z_k \leq B_k \phi_k \leq P_k^{\max} z_k, \quad \forall k \tag{11}$$

$$\phi_k - (\theta_n - \theta_m) + (1 - z_k)M_k \geq 0, \quad \forall k \tag{12a}$$

$$\phi_k - (\theta_n - \theta_m) - (1 - z_k)M_k \leq 0, \quad \forall k \tag{12b}$$

$$\sum_{\forall k(n,.)} B_k \phi_k - \sum_{\forall k(.,n)} B_k \phi_k + \sum_{\forall g(n)} P_g = d_n, \quad \forall n \tag{13}$$

$$0 \leq P_g \leq P_g^{\max}, \quad \forall g \tag{14}$$

$$z_k \in \{0, 1\}, \quad \forall k \tag{15}$$

2.5 Review of economic savings due to optimal transmission switching

Previous research has demonstrated substantial economic savings can be obtained by co-optimizing the electric network topology with generation (Fisher et al. 2008; Hedman et al. 2008, 2009, 2010; Hedman et al., accepted). Such research has analyzed transmission switching with a DCOPT, an N-1 DCOPT, and a unit commitment N-1 DCOPT formulation with various test cases: the IEEE 73-bus RTS96 system, the IEEE 118-bus test case, and two large scale, 5000-bus test cases provided to the authors by ISO-NE. Table 4 provides an overview of the best found solutions for each of these test cases; additional sensitivity studies were done with these test cases as well, see Fisher et al. (2008), Hedman et al. (2008, 2009, 2010) and Hedman et al. (accepted),

Table 4 Overview of economic savings from transmission switching for various test cases and formulations

Formulation	IEEE 73-Bus (RTS 96)	IEEE 118-Bus	ISONE 5000-Bus
1 HR DCOPT			
% Savings	–	25% (Fisher et al. 2008; Hedman et al. 2008)	13% (Hedman et al. accepted)
\$ Savings	–	\$512 (Fisher et al. 2008; Hedman et al. 2008)	\$62,000 (Hedman et al. accepted)
1 HR N-1 DCOPT			
% Savings	8% (Hedman et al. 2009)	16% (Hedman et al. 2009)	–
\$ Savings	\$8,480 (Hedman et al. 2009)	\$530 (Hedman et al. 2009)	–
24 HR unit commitment N-1 DCOPT			
% Savings	3.7% (Hedman et al. 2010)	–	–
\$ Savings	\$120,000 (Hedman et al. 2010)	–	–

with all studies showing that optimal transmission switching can provide substantial economic benefits.

We present both the percent savings on the total generation cost as well as the dollar savings; since these test cases vary in sizes and generator costs, the best indicator of the potential of optimal transmission switching is the percent savings. Of the solutions in Table 4, the only solution known to be the optimal solution is the IEEE 118-bus DCOPF result; consequently, the true optimal solutions for the rest of the test cases may provide even more economic savings.

The results in Table 4 enforce the $N-1$ reliability constraints, except for the DCOPF results in the first row. For the results that do not enforce $N-1$ reliability, these solutions still provide insight into possible economic savings from optimal transmission switching. Hedman et al. (accepted) discusses the concept of just-in-time transmission; operators would be able to optimize the network topology for economic gains for steady-state operations and whenever there is a disturbance, the transmission can be switched back into service, if needed, to bring the system back to being $N-1$ reliable. Such an operation would require adequate ancillary services and generator ramping capabilities to be able to reach a feasible dispatch solution once the topology is changed after the contingency. If the electric grid of the future were to include such advanced switching and detection technology, then we could co-optimize the grid for economic gains while knowing that we can switch back to a more reliable grid once there is a contingency.

The solution in Table 4 for the ISONE 5000-bus model had 20 transmission elements temporarily taken out of service by the opening of breakers. Of the 20 elements that were opened, 15 were lines of at least 115 kV; the most common lines opened, seven in total, were 345 kV lines. The lowest voltage lines opened were 69 kV and there were three transformers that were opened. There was a 69 kV parallel line and an 115 kV parallel line that were opened. For the DCOPF IEEE 118-bus results, Fig. 4 shows the 10 best lines to open.

2.6 IEEE 118 optimal transmission switching results

Since the IEEE 118-bus test case does not have a defined load curve, we applied the load curve that is defined for the IEEE 73-bus test case ([Power System Test Case Archive 2010](#)). In addition, the IEEE 118-bus test case dataset in [Power System Test Case Archive \(2010\)](#) does not include thermal capacity ratings for the transmission lines; the information from [Blumsack \(2006\)](#) was used for the thermal capacity limits. Figure 5 presents the load curve in terms of the percentage of the base load levels defined for the IEEE 118-bus test case; the lowest demand level is 34% of the base load and the highest is 100%. Figure 6 shows that the original DCOPF solution is very close to the unconstrained economic dispatch solution.³

The unconstrained economic dispatch solution is a lower bound to both the DCOPF problem as well as the optimal transmission switching DCOPF problem. The maxi-

³ The unconstrained economic dispatch problem is a dispatch problem without transmission network constraints.

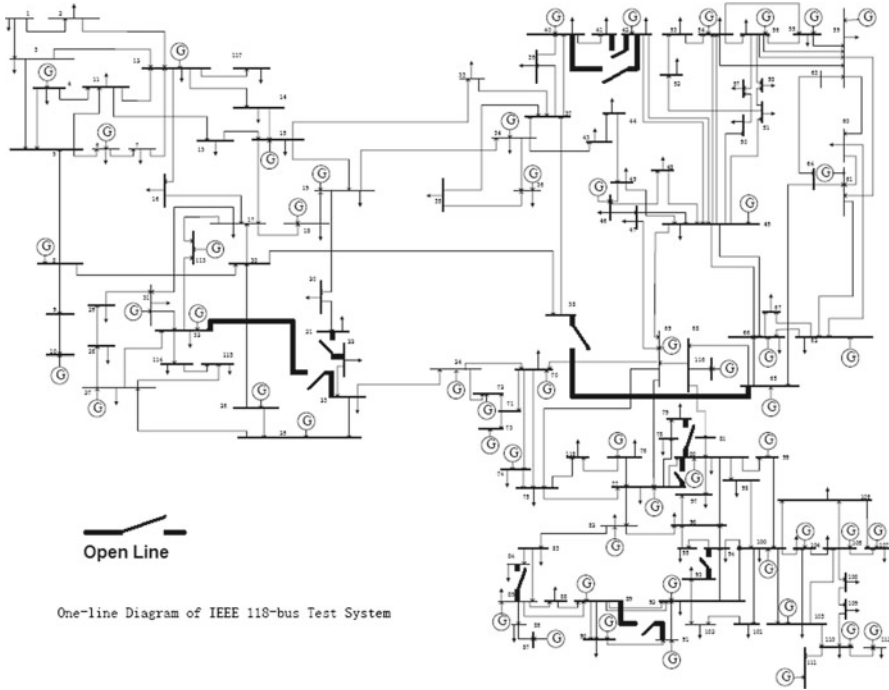


Fig. 4 IEEE 118-bus test case with 10 best lines opened

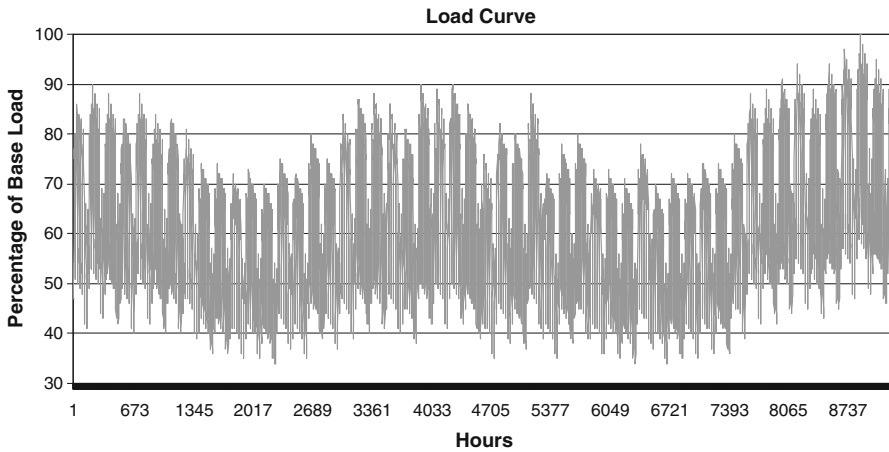


Fig. 5 Yearly load curve

imum potential savings that can ever be obtained from optimal transmission switching is the gap between the DCOPT solution and the unconstrained economic dispatch solution; consequently, the potential savings from optimal transmission switching are limited for this particular test case, the IEEE 118-bus test case, since the DCOPT solution is only a fraction more expensive than the unconstrained economic dispatch

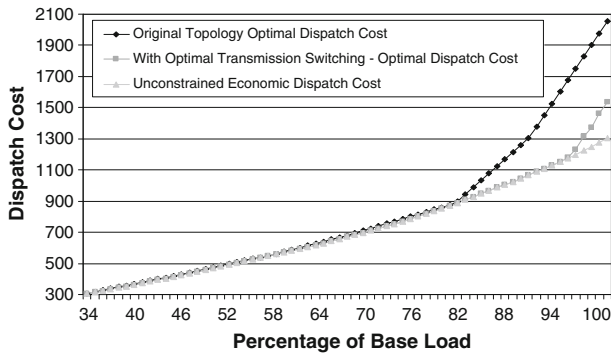


Fig. 6 Optimal dispatch costs for all load levels

solution for most hours. For the entire year, the unconstrained economic dispatch solution is 3.078% less than the DCOPF solution; for most real world networks, this gap would be substantially larger and, thus, the potential savings from optimal transmission switching would be larger as well.

The optimal dispatch costs for the DCOPF, the optimal transmission switching DCOPF, and the unconstrained economic dispatch problems are displayed in Table 5. The best found optimal transmission switching solution has a total dispatch cost that is 3.05% lower than the DCOPF solution; the optimality gap is 0.0083%. The unconstrained economic dispatch solution is 3.07% below the DCOPF solution so optimal transmission switching solution is almost as cheap as the unconstrained economic dispatch solution. For the DCOPF solution, i.e., without transmission switching, there was congestion in the network for each hour; by co-optimizing the network topology with generation, the optimal transmission switching solution produced the unconstrained economic dispatch solution for 8,451 h or 96.4% of the time.⁴ Figure 7 shows the percentage of economic savings for each load level in the year. The lowest percent savings for any hour was 0.05% and the highest was 29.6%. The optimal solution was obtained for 8,460 h. There are 186 lines in the IEEE 118-bus test case; 148 of these lines are opened at least once during the year. One line in particular is opened up for 32 out of the 67% load levels and for 2,996 h.

Note that there are much higher percent savings from optimal transmission switching at the higher load level hours. The percent savings start to increase around the same hour as when the DCOPF solution starts to move farther away from the unconstrained economic dispatch solution, as can be seen by viewing Fig. 6 around percent level 82. The lower bound to the optimal transmission switching DCOPF problem is the unconstrained economic dispatch solution and the upper bound is the DCOPF problem (without switching). Thus, the low percent savings from optimal transmission switching for most of the hours throughout the year for this IEEE test case is

⁴ This is likely a unique result for this IEEE test case since the DCOPF solution is extremely close to the unconstrained economic dispatch solution for most hours; such a result is not expected for practical networks.

Table 5 Dispatch cost for entire year

	Optimal dispatch cost	Percent below DCOPF
DCOPF	\$ 5,607,032.18	–
Optimal transmission switching DCOPF	\$5,436,071.51 (optimality gap: 0.0083%)	3.05
Unconstrained economic dispatch	\$5,434,469.04	3.07

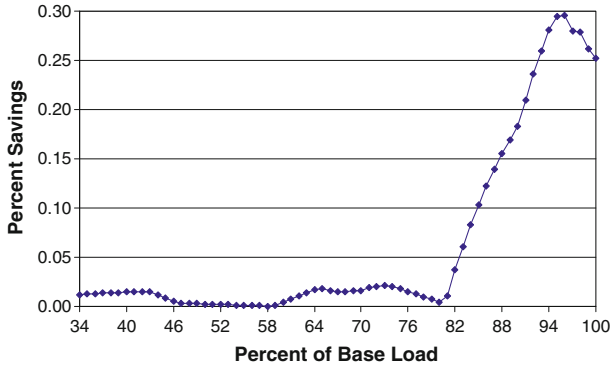


Fig. 7 Percent savings from optimal transmission switching for all load levels

because its upper bound (the DCOPF solution) is extremely close to its lower bound (the unconstrained economic dispatch solution) for most of the load levels. This is a unique result corresponding to this IEEE test case as most large-scale systems will not have such a small gap between the DCOPF solution and the unconstrained economic dispatch solution.

3 Market implications of optimal transmission switching

3.1 Generation cost, generation rent, congestion rent, and load payment

The objective of optimal transmission switching is to minimize the total generation cost; this is the same as maximizing the total market surplus since load is assumed to be perfectly inelastic. The only guarantee that we have is that the total generation cost will not increase. It is not possible to know what will happen to the short term generation rent, the congestion rent, or the load payment. The DCOPF problem without transmission switching is a linear program with the objective to minimize generation cost; it can be shown that its dual has the objective to maximize the load payment minus the generation rent minus the congestion rent. This creates the identity that the sum of the generation cost, the generation rent, and the congestion rent equals the load payment. Therefore, we know that it is not possible that the load payment increases

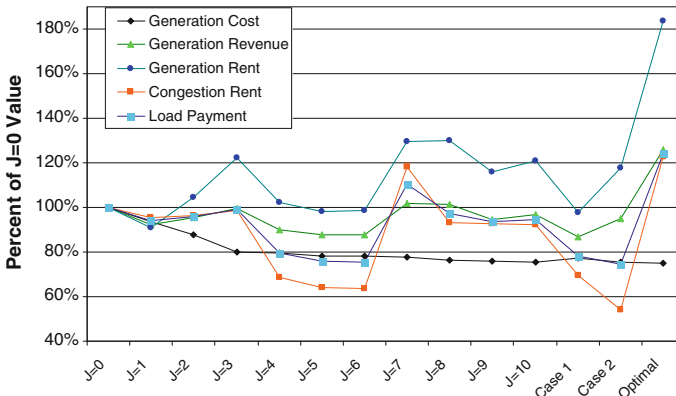


Fig. 8 Generation cost, generation rent, congestion rent, and load payment for various transmission switching solutions—IEEE 118-bus test case (Hedman et al. 2008)

while the generation rent and congestion rent either decrease or stay the same as this would mean that the generation cost would increase.

Figure 8 below illustrates how these terms can vary for various transmission switching solutions for the IEEE 118-bus test case (Hedman et al. 2008). The J index on the x -axis is a reflection of how many lines were opened; $J = 0$ reflects the base case where all lines are in service. The plots reflect how each term varies from one transmission switching solution to the next as compared to that term's value in the base case when all lines are in service; for instance, the load payment for the $J = 0$ solution is \$7,757/h and the load payment is 79% of that value for the $J = 4$ solution. Cases 1 and 2 are solutions found by a heuristic technique and the optimal solution is the last solution listed.

This earlier work from Hedman et al. (2008) emphasizes the potential unpredictable distributional impacts that can occur due to transmission switching. Optimal transmission switching is a challenging combinatorial optimization problem. As is the case in MIP based unit commitment, it is unlikely that the operator will find the optimal solution; rather, the operator will choose the best solution obtained within the available time-frame or for a pre-specified duality gap. Figure 8 shows that neighboring solutions can vary in objective (total generation cost) by trivial amounts but yet the impacts on the market participants can vary drastically and non-monotonically from one near optimal solution to the next. For instance, the optimal solution and the case 2 solution vary in total cost by \$6, or by 0.3%; however, for the optimal solution, the load payment is above 120% of the $J = 0$ solution's load payment while the load payment for case 2's solution is below 80% of the $J = 0$ solution's load payment. In other words the load pays over 50% more with the optimal topology as compared to the topology chosen by case 2. Likewise, the generation rent for the optimal solution is above 180% while it is below 120% for case 2's solution. The congestion rent for the optimal solution is above 120% of the $J = 0$ solution's congestion rent but it is at 60% for case 2's solution. Not only can neighboring solutions have drastically different results for market participants even though their objectives are almost the same, it is extremely difficult to predict which sub-optimal solution will be chosen by the

Table 6 Maximum, average, and minimum change in LMP–IEEE 118-bus test case (Hedman et al. 2008)

	J = 1	J = 2	J = 3	J = 4	J = 5	J = 6
Average % change in LMP	−6	−5	2	3	−1	−1
Max % change in LMP	42	29	63	196	83	79
Min % change in LMP	−105	−216	−51	−100	−106	−106
	J = 7	J = 8	J = 9	J = 10	Optimal	
Average % change in LMP	60	3	−5	−3	102.10	
Max % change in LMP	2159	137	47	74	2271.85	
Min % change in LMP	−61	−42	−201	−69	−66.10	

algorithm used to solve this combinatorial problem. The computational complexities associated with this concept make it incredibly difficult for market participants to be able to predict the chosen topology.

3.2 LMPs

Locational Marginal Prices are the dual variables on the node balance constraints, (Eq. 9), within optimal power flow (OPF) formulations. The optimal transmission switching DCOPF problem is a mixed integer programming problem; MIPs do not have well defined duals like linear programs. Once the optimal transmission switching problem is solved to optimality, all integer variables are fixed to their solution values and the problem can be resolved as a linear program in order to generate the dual and its corresponding dual variables. As previously stated in Sect. 3.1, all that we can guarantee from the optimal transmission switching formulation is that the total generation cost will not increase; LMPs can vary substantially between what is obtained in the traditional DCOPF problem as compared to the LMPs generated from the optimal transmission switching DCOPF problem. Table 6 below shows how the LMPs vary for the optimal transmission switching problem by a percent comparison to the LMPs that are generated for the traditional DCOPF problem. Even though the total generation cost of the system decreases with transmission switching, by changing the topology as well as by changing the generation dispatch solution, LMPs can increase substantially as well as decrease. This result demonstrates that there can be significant winners as well as losers as a result of implementing optimal transmission switching.

Note that the maximum percent change for $J = 7$ and for the optimal solution in Table 6 are extremely high. For both cases, there was an LMP in the base case, the $J = 0$ case, that was very close to zero. Once the grid was modified by optimal transmission switching, these LMPs increased, causing a substantial percentage increase in LMP since the original values were very close to zero. This high percent change is not a general result of optimal transmission switching but rather a result corresponding to potential situations when LMPs are close to zero.

Further discussion on the impact of transmission switching on LMPs can be found in Hedman et al. (2008, 2009). In particular, at the end of section V.B in Hedman et al.

(2009), an example from the RTS 96 test case is given where the second highest LMP, roughly \$85/MWh, for the original topology increases for some of the transmission switching solutions but it decreases dramatically for other transmission switching solutions, down to almost \$20/MWh. These results emphasize how transmission switching can drastically affect LMPs.

3.3 Unit commitment schedule

Hedman et al. (2010) incorporated optimal transmission switching into the day-ahead $N-1$ reliable unit commitment formulation and analyzed this problem on the RTS 96 system. This formulation allows the operator to choose a different network topology for each hour. By doing so, there was a 3.7% cost savings (optimality gap is 3.2%) translating to \$120,000 for a one day period. If similar savings were obtained for every day, this medium sized IEEE test case would receive over \$40 million in economic savings over a year. This research also demonstrated that the network topology does indeed vary hour to hour, which is expected since the market conditions change hour to hour. Without transmission switching, the optimal unit commitment schedule for the following day included two peaker units that were required to be turned on for only 1 h; with transmission switching incorporated into the unit commitment formulation, these two peaker units were not turned on for the entire day. Such a result demonstrates how utilizing the control of transmission assets can help negate the need to perform costly actions, e.g., turn on expensive peaking generators.

3.4 Ensuring a non-confiscatory market in non-convex markets

Uplift payments are used by most ISOs to ensure that generators at least break even on any given day; this is known as ensuring a non-confiscatory market. Previous research has already shown that non-convex markets cannot guarantee a non-confiscatory market with only linear energy payments based on marginal prices (O'Neill et al. 2005). Hence, uplift payments are needed to ensure a non-confiscatory day-ahead market due to the non-convexities in the feasible constraints set and objective function of unit commitment problems (minimum up and down time constraints, positive minimum run levels, startup costs, shutdown costs, and no-load costs).

While transmission switching introduces non-convexities into the OPF problem by making the feasible set non-convex, transmission switching does not create a need for uplift payments to generators as unit commitment causes. Once the optimal topology is determined by optimal transmission switching, the LMPs corresponding to the optimal dispatch under the chosen topology are market clearing and support the resulting dispatch with no restrictions on the generator schedules and no imposition of unrecoverable generator costs due to topology choice. Consequently, the resulting dispatch problem given the chosen topology mimics the same mathematical structure as the original OPF formulation. The addition of optimal transmission switching does not create a need for uplift payments to generators. However, as will be shown in the following section, it can cause revenue inadequacy in the FTRs market, which may require side payments in the FTR market.

4 Transmission switching and revenue adequacy of FTRs

4.1 Revenue inadequacy due to transmission switching

Revenue adequacy for FTRs is maintained for the static DC network (Hogan 1992) but not guaranteed if the network topology changes (Alsac et al. 2004). With optimal transmission switching, the topology of the network may be changed in order to increase the total market surplus. Even if the total market surplus increases as a result of transmission switching, it is still possible to have revenue inadequacy. The following example demonstrates this possibility. Figure 9 shows a simple, stylized 3-bus example where both lines A–B1 and A–B2 are opened to improve the market surplus of the system. Table 7 provides data for the optimal solution with all lines in service and Table 8 provides data for the optimal dispatch solution when the lines between buses A and B are opened. Table 9 provides a possible FTR allocation that meets the SFT and is revenue adequate with the original topology but this allocation is revenue inadequate for the modified topology, which provides a higher market surplus.

Figure 10 shows the feasible set of solutions for point to point transactions A–B and A–C when all lines are closed; the feasible set is represented by sets 1 and 2 in the plot. For simplicity, we restrict the feasible set within Fig. 10 to consider only non-negative point to point transactions from A to B and A to C; the constraints that define the feasible set when all lines are closed are (16)–(18). The constraints that define the feasible set when the two lines between buses A and B are opened are (19)–(20). The grey circle at (60, 45) represents the FTR allocation listed in Table 9; this FTR

Fig. 9 Revenue adequacy 3-bus example C

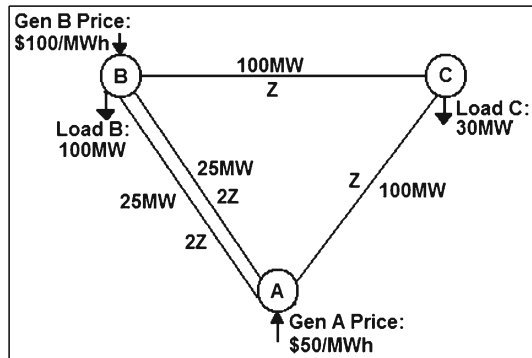


Table 7 3-bus example C optimal dispatch results (no switching)

Bus	Gen Pg (MW)	LMP	Gen cost	Transaction	MW	Cong. rent
A	90	\$50/MWh	\$4,500	A–B	60	\$3,000
B	40	\$100/MWh	\$4,000	A–C	30	\$750
C	0	\$75/MWh	\$0	Total congestion rent		\$3,750
Total generation cost			\$8,500			

Table 8 3-bus example C optimal dispatch results (lines A–B1 and A–B2 open)

Bus	Gen Pg (MW)	LMP	Gen cost	Transaction	MW	Cong. rent
A	100	\$50/MWh	\$5,000	A–B	70	\$3,500
B	30	\$100/MWh	\$3,000	A–C	30	\$1,500
C	0	\$100/MWh	\$0	Total congestion rent		\$5,000
Total generation cost			\$8,000			

Table 9 3-bus example C results—FTR settlements

Source to sink	FTR quantity (MW)	FTR settlements (no switching)	FTR settlements (lines A–B1 and A–B2 open)
A to B	45	\$2,250 (LMP gap: \$50/MWh)	\$2,250 (LMP gap: \$50/MWh)
A to C	60	\$1,500 (LMP gap: \$25/MWh)	\$3,000 (LMP gap: \$50/MWh)
Total FTR settlements		\$3,750	\$5,250

allocation is on the boundary of the feasible set as the upper bound constraint of (16) is binding with this FTR allocation. The optimal solution with all lines in service is represented by the grey square. When we open the two lines between A and B, the new optimal dispatch is located at the white square, which is a part of the new feasible set. The feasible set when these two lines are opened is represented by the light grey region in Fig. 10 defined by the dashed line, i.e., sets 1 and 3. The FTR allocation is no longer a part of this new feasible set; this FTR allocation passes the SFT when all lines are closed but it would not pass the SFT when we open the lines between buses A and B.

$$-50 \leq \frac{2}{3}AB + \frac{1}{3}AC \leq 50 \tag{16}$$

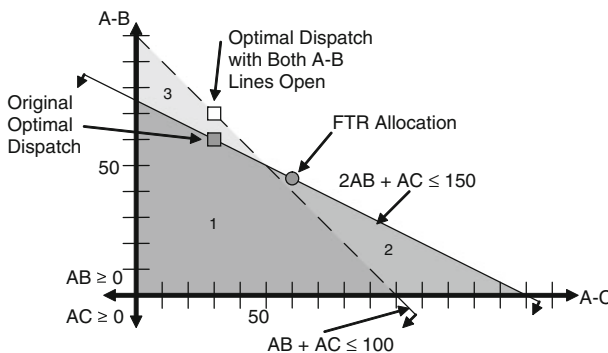


Fig. 10 Feasible set of solutions for point to point transactions A to B and A to C with all lines in service and with lines between buses A and B open

$$-100 \leq \frac{1}{3}AB + \frac{2}{3}AC \leq 100 \tag{17}$$

$$-100 \leq \frac{1}{3}AB - \frac{1}{3}AC \leq 100 \tag{18}$$

$$-100 \leq AB + AC \leq 100 \tag{19}$$

$$-100 \leq AB \leq 100 \tag{20}$$

The example above demonstrates that it is possible to change the topology to improve the market surplus and still have revenue inadequacy for FTRs. There is revenue adequacy with the original topology; with the new topology, the congestion rent is only \$5,000 whereas the FTR holders are owed \$5,250. Even if there is revenue inadequacy, since the total surplus is guaranteed not to decrease with optimal transmission switching, there is the possibility for Pareto improvements for all market participants. This raises the question about the appropriate settlement rules if revenue inadequacy occurs.

4.2 Transmission switching can help restore revenue adequacy

Example C above demonstrates how taking lines out of service can cause revenue inadequacy. This can happen whether lines are deliberately temporarily taken out of service for economic reasons, as is the case above, or the line fails due to a contingency. In this section, we show that in case of a line failure, it may be possible to restore revenue adequacy by adjusting the topology through switching off additional lines. In other words, just as transmission switching can cause revenue inadequacy by changing the network, it can also help restore revenue adequacy, when a contingency occurs, while improving the market surplus.

The results when all lines are closed are provided in Table 7 and the feasible set of solutions for point to point transactions for A–B and A–C is displayed by the sets 1, 2, and 4 in Fig. 11. When one of the lines between buses A and B is opened, the results are then given by Table 10, the feasible set is defined by set 4 in Fig. 11, and the constraints that define the feasible set are (21)–(23). Table 11 provides an FTR allocation that would pass the SFT when all lines are closed and when lines A–B1 and

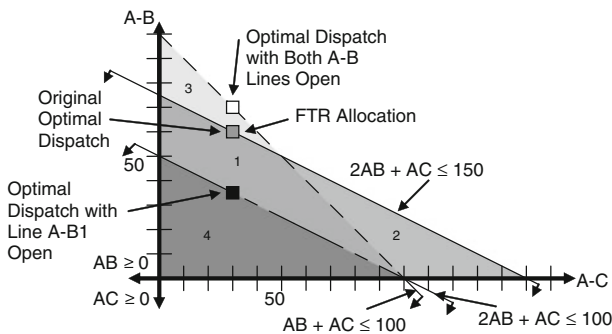


Fig. 11 Feasible set of solutions for point to point transactions A to B and A to C

Table 10 3-bus example C results (line A–B1 open)

Bus	Gen Pg (MW)	LMP	Gen cost	Transaction	MW	Cong. rent
A	65	\$50/MWh	\$3,250	A–B	35	\$1,750
B	65	\$100/MWh	\$6,500	A–C	30	\$750
C	0	\$75/MWh	\$0	Total congestion rent		\$2,500
Total generation cost			\$9,750			

Table 11 3-bus example C results—FTR settlements (line A–B1 open)

Source to sink	FTR quantity (MW)	FTR settlement (no switching)	FTR settlement (line A–B1 open)
A to B	60	\$3,000 (LMP gap: \$50/MWh)	\$3,000 (LMP gap: \$50/MWh)
A to C	30	\$750 (LMP gap: \$25/MWh)	\$750 (LMP gap: \$25/MWh)
Total FTR settlements		\$3,750	\$3,750

A–B2 are open but this allocation causes revenue inadequacy when one of these lines between bus A and bus B is out due to a disturbance. This FTR allocation is located at the grey square in Fig. 11 and the optimal dispatch for the original topology is located at the same grey square. When line A–B1 is opened, the optimal dispatch is located at the black square and when both A–B1 and A–B2 are opened, the optimal dispatch is located at the white square.

$$-25 \leq \frac{1}{2}AB + \frac{1}{4}AC \leq 25 \tag{21}$$

$$-100 \leq \frac{1}{2}AB + \frac{3}{4}AC \leq 100 \tag{22}$$

$$-100 \leq \frac{1}{2}AB - \frac{1}{4}AC \leq 100 \tag{23}$$

With one of the lines between buses A and B opened due to a disturbance, the FTR holders are owed exactly the same amount as when all lines are in service since the LMPs have not changed between these two cases. However, congestion rents are insufficient to cover the FTR settlements when one of the lines between A and B is open. The FTR holders are owed \$3,750 but the congestion rent was only \$2,500.

When line A–B1 is opened, this FTR allocation is no longer a part of the feasible set as is shown by Fig. 11 and it would, therefore, fail the SFT for this topology. However, if both lines between buses A and B are opened, this FTR allocation is once again revenue adequate and part of the feasible set; this is confirmed by the total FTR settlements in Table 12 and the total congestion rent in Table 8. With both lines A–B1 and A–B2 opened, the feasible set for point to point transactions A–B and A–C is represented by sets 1, 3, and 4 in Fig. 11.

In this example, when both lines A–B1 and A–B2 are opened, the ISO has more congestion rent than the required FTR payments. The possibility of such a surplus is consistent with the theoretical result of Hogan (1992). The above example

Table 12 3-bus example C results—FTR settlements (lines A–B1 & A–B2 open)

Source to sink	FTR quantity (MW)	FTR settlements (lines A–B1 & A–B2 open)
A to B	60	\$3,000 (LMP gap: \$50/MWh)
A to C	30	\$1,500 (LMP gap: \$50/MWh)
Total FTR settlements		\$4,500

demonstrates that even if a forced outage causes revenue inadequacy, it may still be possible to not only increase the market surplus but also regain revenue adequacy. If there is an outage that causes revenue inadequacy, further grid modifications may regain revenue adequacy but there is no guarantee that this will be possible as this depends on the FTR allocation and the network conditions.

4.3 Worst case analysis: IEEE 118-Bus test case

The previous two sections present stylized examples and the results build insight regarding how optimal transmission switching may affect revenue adequacy in the FTR markets. In this section, we present results on how optimal transmission switching can affect a standard IEEE test case, the IEEE 118-bus test case ([Power System Test Case Archive 2010](#)). Our objective is to determine the worst case revenue inadequate situation that can potentially occur due to optimal transmission switching.

The traditional SFT has the objective to maximize the bid surplus within the FTR auction. For this study, we instead maximize the total FTR payments, i.e., settlements, due to the FTR holders in order to determine the worst case revenue inadequacy that may occur as a result of optimal transmission switching. We performed this study on the results presented in Sect. 2.6, which include optimal transmission switching solutions for each hour of the year. The LMPs from these solutions are used as inputs for this study as they define the total FTR settlement due to the FTR holders over the entire year if optimal transmission switching is implemented. This optimization problem, therefore, searches for the FTR allocation that passes the SFT for the original network topology with all lines in service while maximizing the total FTR settlement, which is based on the LMPs generated from the optimal transmission switching solutions. Based on this IEEE test case and based on the optimal transmission switching results presented in Sect. 2.6, it is possible to have the FTR settlements be 59% higher than the total congestion rent collected by the ISO; these results correspond to the *best found solution* listed in Table 13 below. Thus, this study demonstrates that revenue inadequacy, even over a long period (a year in this case), can be substantial; however, there is no guarantee that the revenue inadequacy would be this severe as the underlying question is whether this particular FTR allocation is one that would ever be selected in the FTR auction.

For this particular test case, there are multiple transmission switching solutions that are very close in objective to the best found solution we present in Sect. 2.6. In particular, the alternative solution presented in Table 13 is only 0.0096% away from the best found solution; however, this solution generates a different set of LMPs. With

Table 13 Revenue inadequacy study—IEEE 118-bus test case

	Total cost (\$)	Percent savings	FTR settlement (\$)	Congestion rent (\$)	Ratio
Best found solution (Sect. 2.6)	5,436,071.51	3.05	89,537.43	56,405.73	1.587
Alternative solution	5,436,593.46	3.04	42,632.84	45,009.47	0.947

this different set of LMPs, along with the worst case FTR allocation that caused revenue inadequacy for the best found solution, the system would be revenue adequate for the year unlike the best found solution. The best found solution has a revenue shortfall of over \$33,000; the alternative solution is revenue adequate and is only \$520 more expensive than the best found solution. Such a result raises an interesting policy question: should the ISO choose a sub-optimal solution that is a fraction less than the optimal solution but yet does not cause a problem, i.e., revenue inadequacy, in this side financial market, the FTR market. This policy question and other issues are discussed in the following section.

Similar to what is presented in Sect. 3.1, these results confirm that neighboring solutions can have drastically different results for market participants. In this case, there are topology solutions with trivial differences in objective values (total system cost) but yet these topology solutions have substantial differences regarding their impact on the FTR market. Furthermore, since it is unlikely that the operator will be able to solve this difficult combinatorial problem to optimality, this creates a situation where it will be difficult to predict market outcomes. Hence, this example demonstrates the potential unpredictable distributional impacts of transmission switching optimization.

5 Policy implications of revenue adequacy of FTRs and transmission switching

Generally, the role of the ISO is to maximize the market surplus (subject to reliability and operating constraints); with demand modeled as being perfectly inelastic, minimizing the total cost achieves the same objective. Market participants whose revenues depend on LMPs and on FTR settlements may object, however, to co-optimization of the grid topology with generation dispatch due to the unpredictable distributional impacts of such an approach. The ISO itself may be conflicted in that regard since revenue inadequacy in the FTR market as well as the need for uplift payments are often misjudged as a market operation failure. This situation is a manifestation of the complexity associated with network effects in the electric power industry and the difficulties in creating market mechanisms that can deal with such complexities. Here we have an example where efficiency gains can be achieved by a central system operator but we may have to forgo such gains because of the distributional effects and the difficulty of finding a side payment scheme that will translate the system efficiency gains to a Pareto improvement.

One possibility of addressing this dilemma with respect to revenue adequacy in the FTR market is to augment the optimal transmission switching formulation so as to ensure that whatever topology is chosen, that it is also revenue adequate. Adding

such a restriction may exclude, however, the true optimal network topology that maximizes the market surplus from being feasible, which amounts to “leaving money on the table.” Alternatively, revenue inadequacy can be resolved by de-rating FTR settlements, which is the current industry practice when revenue inadequacy is due to contingencies. Such de-rating has been a constant source of disputes, however, and it would be hard to explain to FTR holders that their income is being reduced because the system as a whole is better off with optimal transmission switching.

If the ISO chooses to implement side payments, there is then the question as to how to determine the side payments. Should all market participants contribute to the revenue shortfall? Should a specific class of participants, for instance the consumers, cover the revenue shortfall? As previous research has demonstrated in [Hedman et al. \(2008, 2009\)](#) as well as shown in Sect. 3.1 of this paper, it cannot be guaranteed that any specific group of market participants will benefit from transmission switching. The only guarantee is that the market surplus will not decrease. It is possible that consumers end up, overall, paying more with the optimal network topology that maximizes the market surplus. Is it then fair to make them incur the cost of revenue inadequacy? Should the ISO instead charge the market participants that benefit from transmission switching? Identifying beneficiaries and agreeing upon the resulting benefits are considered to be major obstacles holding back transmission investments and these issues are unlikely to be any easier to handle in the transmission switching arena.

The use of side payments in wholesale electric energy markets already exists today but under a different setting. As Sect. 3.4 discussed, uplift payments are needed to ensure a non-confiscatory day-ahead market. Recent research has proposed that a different pricing system should be adopted to reduce the required uplift payments ([Gribik et al. 2010](#)); this new pricing system is known as the extended LMP or “convex hull based pricing” and it is currently being considered by the Midwest ISO. In principle, a similar approach can be developed for transmission switching by calculating LMPs based on a feasible set, which is the convex hull of the union of all feasible power flows corresponding to the possible transmission switching solutions. However, even if we ignore the computational tractability and implementation issue of such an approach, it is still not clear if it could solve the revenue adequacy problem. Both the optimal transmission switching solution and the outstanding FTR positions represent suboptimal solutions to the OPF subject to the convex hull feasible set upon which the modified LMPs would be based. Hence, it is not clear whether congestion rents that are based on the actual dispatch and these modified LMPs would be higher or lower than the FTR settlements. Further analysis of this question is an interesting theoretical exercise but it is out of the scope of this paper.

There is also the concern over how severe revenue inadequacy may become as a result of transmission switching. Since the market surplus increases there will always be enough additional surplus to compensate the losers but implementing side payments becomes a more heated policy issue when the magnitude of wealth transfers are substantial, which was shown to be possible in Sect. 4.3.

Finally, there is the question as to whether side payments should happen at all and whether the fixed network topology paradigm is a proper benchmark for determining such side payments if at all. Alternatively, should the true, benchmark model be redefined to account for a flexible topology? If side payments are not implemented, can we

expect the FTR markets and the clearing prices in the FTR auction to adjust and reflect this new technology, which increases uncertainty in FTR settlements and degrades the hedging quality of such instruments? Such policy questions do not have simple technical answers. Emerging changes in grid technologies under the umbrella of the “smart grid” and the integration of new resource types, such as renewable energy and storage, may necessitate rethinking of market design principles and market mechanisms that are made obsolete by such innovations.

6 Conclusion

Traditional optimal power flow problems treat transmission as a static asset even though operators do have control over the transmission assets’ state. Harnessing the control of transmission has been shown to be beneficial for a variety of reasons including line overloading, loss reduction, reliability improvements, etc. Furthermore, research has shown that co-optimizing the grid topology with generation has the potential to provide substantial economic savings and these savings can even be achieved without any reliability degradation. Introducing the control of transmission into dispatch problems provides operators with another level of control and, as demonstrated by Sect. 2.3, adding this flexibility expands the feasible set of solutions. However, co-optimizing the network topology with the generation has unpredictable distributional effects and, in particular, may cause revenue inadequacy in the FTR market.

The objective of the paper is to highlight the conflict that can arise between the ISO mandate to maximize the total market surplus and existing market design principles and mechanisms that are not equipped to handle potential gains enabled by technological innovation and rapid changes in the resource landscape. Our main focus in this work has been the FTR market, which is highly vulnerable to manipulation of the grid topology. Though there are many benefits from optimal transmission switching, it is not in line with one of the core assumptions that the FTR markets are based on. As a result, there is the potential for revenue inadequacy due to optimal transmission switching. From a regulatory perspective, the tension between the reliance on decentralized market paradigms that rely purely on incentives and price discovery by market participants vs. centralized markets that try to capture efficiency gains through coordination has been a subject of debate throughout the electricity market reform process. This paper draws attention to one more aspect of electricity markets where the desire to simplify the market design through the use of FTRs may be at odds with the goal of achieving economic efficiency through centralized optimization that enables the exploitation of network effects. We hope that these observations will motivate research on new forms of transmission property rights and congestion hedging approaches as well as on side payment mechanisms that can distribute and convert efficiency gains due to transmission switching so as to achieve Pareto improvements.

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