Energy Economics 32 (2010) 779–785

Contents lists available at ScienceDirect

Energy Economics

journal homepage: www.elsevier.com/locate/eneco

The inherent inefficiency of simultaneously feasible financial transmission rights auctions

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ARTICLE INFO

Article history:
Received 25 August 2008
Received in revised form 15 January 2010
Accepted 25 January 2010
Available online 6 February 2010

JEL Classification:
C6
G1
L9
Q4

Keywords:
Financial transmission right
Electricity auction
Simultaneous feasibility
Transmission pricing

ABSTRACT

Empirical evidence from the New York ISO shows that the clearing prices for point-to-point congestion revenue rights, also known as financial transmission rights (FTRs), resulting from centralized auctions conducted by Independent System Operators differ significantly and systematically from the realized congestion revenues that determine the accrued payoffs of these rights. The question addressed by this paper is whether such deviations are due to price discovery errors which will eventually vanish or due to inherent inefficiencies in the auction structure.

We show that even with perfect foresight of average congestion rents the clearing prices for the FTRs depend on the bid quantity and therefore may not be priced correctly in the financial transmission right (FTR) auction. In particular, we prove that quantity limits on the FTR bids may cause the auction clearing prices to differ from the bid prices. This phenomenon which is inherent in the theoretical properties of the optimization algorithm used to clear the auction, is further illustrated through numerical simulations with test systems. We conclude that price discovery alone would not remedy the discrepancy between the auction prices and the realized values of the FTRs. Secondary markets or frequent reconfiguration auctions are necessary in order to achieve such convergence.

1. Introduction

Point-to-point financial transmission rights (FTRs) (see Bushnell and Stoft (1997) and Hogan (1992)) and flow-gate rights (FGRs) (see Chao and Peck (1996), Chao and Peck (1997), and Chao et al. (2000)) are two forms of Congestion Revenue Rights (CRRs) outlined in the Standard Market Design put forth by the Federal Energy Regulatory Commission (FERC) of the U.S. The purposes of the CRRs are twofold: a) Create a system of property rights to the transmission system that will offer economic signals for charging/compensating transmission usage/investment and facilitate the implementation of an economically efficient transmission congestion management protocol; b) Offer risk management capability to market participants entering into forward energy transactions so that they can hedge the uncertain congestion rents associated with such transactions. The allocation of FTRs can be done either on the basis of historical entitlements and use of the transmission system or through an auction whose proceeds are distributed to transmission owners or consumers who funded the construction of the system; or, through a combination of the two where unallocated FTRs and FTRs currently held by private parties are auctioned off through a centralized auction conducted periodically by an Independent System Operator (ISO). The latter approach is currently used by the three major ISOs in the northeastern US (New England, New York ISO and Pennsylvania–New Jersey–Maryland).

In this paper we primarily focus on the risk management aspect of FTRs and the extent to which FTRs are efficient instruments for trading and mitigation of congestion risk. In evaluating a financial hedging instruments and its market performance, two questions must be addressed: How good is the hedge? Namely, to what extent does the payoff (or payout) of the instrument offset the fluctuations in the risky cash flow that the instrument is supposed to hedge. How efficient is the market for the instrument? That is, does the forward market price of the instrument reflect the expected risky cash flow hedged by the instrument with the proper risk premium adjustment.

Much of the discussion surrounding FTRs focuses on the first question and indeed FTRs provide a perfect hedge against real-time congestion charges based on nodal prices. A one Megawatt (MW) bilateral transaction between two points in a transmission network is charged (or credited) the nodal price difference between the point of withdrawal and the point of injection. At the same time (assuming that transmission rights are fully funded), a one MW financial transmission right (FTR) between two points is an entitlement (or

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doi:10.1016/j.eneco.2010.01.010

obligation) for the difference between the nodal prices at the withdrawal node and the injection node. Thus regardless of how the system is dispatched, a one MW FTR between two nodes is a perfect hedge against the uncertain congestion charge between the same two nodes[1]. The hedging properties of FTRs make them ideal instruments for converting historical entitlements to firm transmission capacity into tradable rights that hold the owners of such entitlements harmless while enabling them to cash out when someone else can make more efficient use of the transmission capacity covered by these entitlements. In other words, FTRs make it relatively easy to preserve the status quo while opening up the transmission system to new and more efficient use.

From the perspective of new transmission users who view the FTRs as a mechanism to hedge their exposure to congestion risk (as well as old users who are actively evaluating their commercial options with respect to FTR entitlements) the second question is as relevant as the first. A purchaser of FTRs must assess whether the forward price of the instrument indeed reflects the value that it provides in making the decision whether to purchase/hold the instrument or to face the exposure to the real-time congestion charges.

In typical financial and commodity markets, competition and trading liquidity push the forward prices to the expected spot prices with a proper (market based) risk premium adjustment. Such convergence is achieved through a process of arbitrage. Such arbitrage, however, may be more difficult when dealing with FTRs for several reasons. Most importantly, due to the large number of FTR types, the trading liquidity[2] of these instruments is relatively low; and there is virtually no secondary market that enables reconfiguration and re-trading. In order to maintain financial solvency of the system operator who is the counter-party to FTRs, the configuration of FTR types must satisfy "simultaneous feasibility conditions" that are dictated by the system constraints. Consequently, pricing and trading of FTRs is done through a central periodic auction and the liquidity of the FTR depends on the frequency of that centralized reconfiguration auction. It is important to recognize that FTR liquidity cannot be measured in terms of the number of bids in the FTR auctions which merely reflect bid fragmentation. Indeed, the Pennsylvania–New Jersey–Maryland (PJM) ISO have experienced a large volume of FTR bids in their auction which may be misinterpreted as an indication of good liquidity. However, volume does not necessarily imply liquidity. True liquidity in a financial sense is reflected by the frequency of trading opportunities, bid-ask spreads and the ability to sell or buy FTRs for short time segment (e.g. one day or specific hours) which represent only small fractions of the time intervals between reconfiguration auctions.

Furthermore, because of the interaction among the different FTR types through the simultaneous feasibility conditions, prices of the FTRs resulting from the FTR auction as well as the congestion charges hedged by these FTRs are highly interrelated. An efficient market (that correctly prices FTRs) must anticipate not only the uncertainty in congestion prices due to contingencies and load fluctuation but also the shift in the "operating point" within the feasible region which is determined by the economic dispatch procedure.

Empirical evidences reported in Adamson and Englander (2005), Adamson et al. (2008) and Siddiqui et al. (2003) show that the clearing prices for FTRs resulting from centralized auctions conducted by the New York Independent System Operator (NYISO) have differed significantly and systematically from the realized congestion revenues that determined the accrued payoffs of these transmission rights.[3] The question addressed by this paper is whether such deviations are only due to risk premiums and price discovery errors which will eventually vanish, or there are inherent inefficiencies in the auction structure itself that can explain the observed discrepancies.

We address this question by presenting a theoretical analysis that can potentially explain the empirical findings cited above and then we demonstrate the implications of our theoretical results through numerical simulations and sensitivity analysis conducted on a DC-flow approximation model of a six-node system and the IEEE-24 bus Reliability Test System (see Siddiqui et al. (2003) for a general AC-flow formulation) with known outage probabilities of each element and known statistical demand variability. In the example, we simulate the expected value of all point-to-point transmission rights taking into consideration all possible *n-1* transmission contingencies and demand realizations. We then construct a hypothetical FTR auction in which all FTR bids equal the correct expected value of the corresponding congestion rents whereas the bid quantities are bounded by some multiple α of the corresponding average point-to-point transaction volume. In making the latter assumption we do not attempt to model bidding behavior in the FTR auction but rather to illustrate the effect of quantity limits on the bids and the order of magnitude by which bid quantities need to exceed average transaction volumes in order to eliminate price distortions in the FTR auction. The homogeneous scaling of bid quantities is simply convenient. This assumption is intended to approximately model the fact that FTR auction participants generally acquire a targeted number of FTRs according to their physical transaction needs and they do not want to be caught up with excessive FTRs. This fact naturally puts upper bounds on the FTRs quantities that the auction participants bid for.

Similar results can be obtained by assuming alternative patterns of bid quantity limits. In reality such quantity limits arise due to the desire of auction participants to match their FTR holdings to their hedging needs based on their expected use of the transmission system (which corresponds to the case of $\alpha = 1$) and due to credit limits faced by the bidders. The specific quantity limits on FTR bids and their relation to the expected transaction volume will, obviously, vary among FTRs and we do not attempt to predict those limits. In general, however, we anticipate that the ratios of FTR bid quantities to expected transaction volumes will be relatively low since excessive bid quantities relative to use, especially by regulated load serving entities, may be perceived as speculative behavior and frowned upon by regulators who are unlikely to pass through the downside risk of such activities to consumers. The results of our theoretical and computational analysis shed light on the observed discrepancies between realized FTR values and their auction prices.

The organization of our paper is as follows. In Section 2, we formulate an FTR auction model which incorporates the simultaneous feasibility conditions under postulated contingencies on transmission line availability and load variation. We then provide theoretical results on the potential systematic biases in market clearing nodal prices with respect to rational expectations. Numerical examples are presented in Section 3 that confirm our theoretical findings. Finally, we conclude and point out future research directions in Section 4.

2. The point-to-point FTR auction

We consider an FTR auction conducted by a system operator in an electric power grid with n buses and m transmission lines. The auction is cleared under the standard FTR auction rules that treat all FTR bids as simultaneous bilateral transactions that must satisfy all the line

[1] Some ISOs derate FTR settlements in order to cover congestion revenue shortfalls due to transmission contingencies not accounted for in the FTR auction. In such cases, depending on the derating approach, FTRs may not provide acceptable hedges.

[2] Trading liquidity of an FTR can be measured by the ratio of the number of the FTR trades to the actual number of this right, or by the magnitude of the spread between the bid and ask prices of this FTR.

[3] We are not aware of additional empirical works done with market data from the other ISOs. Nevertheless, the design of the FTR markets is similar across all ISOs in the U.S.

operating limits under all *n-1* contingencies and load realizations. The auction is cleared so as to maximize FTR revenues and the prices are set to the marginal clearing bids for each FTR.

We first show that the FTR simultaneous feasibility auction can be represented by an equivalent virtual energy auction. We limit the proof to our case of interest where we assume that all bidders have perfect foresight of the expected value of the locational marginal price (LMP) for energy at all buses of a network. In this special case (assuming perfect competition and rational risk neutral bidders), all FTR auction participants bid only one price f_{ij} for FTR contracts with the same origin i and destination j. Furthermore, for each FTR from bus i to bus j, we can aggregate all bid quantities for this FTR into one single bid quantity $\overline{q}_{ij}$.[4] Let $C\equiv(c_1,c_2,\cdots,c_n)^T$ denote the vector of expected LMPs at the n buses then $f_{ij}\equiv c_j-c_i$ (since the expected value of the difference between two random variables equals the difference of the respective expected values of these variables). Let $\{q_{ij},\forall i,j\}$ denote the awarded FTR quantity from bus i to bus j and $Q\equiv(q_1,q_2,\cdots,q_n)^T$ denote the energy injection/withdrawal vector imputed from all awarded FTR quantities. Then $q_i\equiv\sum_{j\neq i}q_{ij}-\sum_{k\neq i}q_{ki},\forall i\in N$ where N is the set of all buses. We adhere to the convention that a positive q_i represents injection while a negative q_i represents withdrawal. In an FTR auction market, participants are either hedgers who purchase FTRs to hedge the congestion charges of their energy transactions or speculators who trade FTRs for speculative profits subject to trading quantity limits set by risk control measures. None of these participants would bid for an unlimited amount of FTRs. Thus it is natural to assume that the aggregate bid quantity of the FTR from node i to node j is bounded by $\overline{q}_{ij}$ with all bids submitted at the expected settlement price for the corresponding FTR. The clearing mechanism for the FTR auction is formulated as follows. The system operator maximizes the as-bid value of awarded FTRs over all feasible FTR allocation quantities $\{q_{ij},\forall i,j\}$ subject to the corresponding energy dispatch vector Q satisfying power flow constraints under all designated system reliability contingency scenarios. Let R denote the set of all plausible reliability contingencies. Each scenario r R represents the outage of at most one transmission line. The FTR auction is cleared through solving the following optimization problem.

$$\max_{\{q_{ij},\forall i,j\}}\sum_{i\in N}\sum_{j\neq i}f_{ij}\cdot q_{ij} \tag{1}$$

$$\text{s.t. } q_i=\sum_{j\neq i}q_{ij}-\sum_{k\neq i}q_{ki} \qquad \forall i\in N$$

$$-L\le G_r\cdot Q\le L \qquad \forall r\in R$$

$$0\le q_{ij}\le\overline{q}_{ij} \qquad \forall i,j,\text{ and } j\neq i$$

where L is the vector of transmission line capacity limits and G_r is the power transfer distribution factor (PTDF) matrix with bus-n chosen as the swing bus in each contingency scenario r.

By re-arranging terms in the objective function of the FTR auction problem (1), we get the following:

$$\sum_{i\in N}\sum_{j\neq i}f_{ij}\cdot q_{ij}$$

$$\equiv\sum_{i\in N}\sum_{j\neq i}(-c_i+c_j)\cdot q_{ij}$$

$$=-\sum_{i\in N}c_i\cdot\left(\sum_{j\neq i}q_{ij}\right)+\sum_{j\in N}c_j\cdot\left(\sum_{i\neq j}q_{ij}\right) \tag{2}$$

$$=-\sum_{i\in N}c_i\cdot\left(\sum_{j\neq i}q_{ij}-\sum_{j\neq i}q_{ji}\right) \tag{3}$$

$$=-\sum_{i\in N}c_i\cdot q_i \tag{4}$$

The term in Eq. (4) represents the merchandizing surplus in the network, (i.e. total purchase price minus sales price) for all transacted energy Q when all the awarded FTRs are exercised simultaneously. When the willingness-to-pay of all demands at a node and the generation cost at a node are constants (as assumed in our case) the merchandizing surplus equals the social surplus (i.e., the difference between demand willingness-to-pay and supply marginal cost).

Moreover, the constraints for the components of Q (i.e. q_i's) in Eq. (1) imply that Q is a balanced energy dispatch. Namely,

$$e^TQ\equiv\sum_{i\in N}q_i$$

$$=\sum_{j\in N}\left(\sum_{m\neq j}q_{jm}-\sum_{n\neq j}q_{nj}\right)=0 \tag{5}$$

where e is a row vector consisting of n "1"s and a positive/negative q_i indicates an injection/ejection at node i.

Substituting Eqs. (4) and (5) into the FTR auction problem (1), we have shown that Eq. (1) is equivalent to the following virtual energy auction conducted by the system operator to maximize the social surplus of all transacted energy. In particular, the constraints on the FTR bid quantities in Eq. (1) are implemented by converting the quantity bounds of FTR bids to quantity bounds of nodal energy in the virtual energy auction Eq. (6). Specifically the nodal demand/generation at node i is bounded from below by $\underline{q}_i=-\sum_{k\neq i}\overline{q}_{ki}$ and from above by $\overline{q}_i=\sum_{j\neq i}\overline{q}_{ij}$.

$$\max_{Q}-\sum_{i\in N}c_i\cdot q_i \tag{6}$$

$$s.t.\ e^TQ=0$$

$$-L\le G_r\cdot Q\le L \qquad \forall r\in R$$

$$\underline{Q}\le Q\le\overline{Q}$$

where L is defined in Eq. (1), G_r's are the same PTDF matrices as those in Eq. (1), and $\underline{Q}$ and Q denote the n-vectors of upper and lower quantity bounds whose elements are $\underline{q}_i$ and $\overline{q}_i$ ($\forall i$ N), respectively. The FTR award quantities for each pair of nodes (which must be subsequently allocated to all the bidders tied for each award) can be extracted from the optimal dispatch solution Q^* in the virtual optimal power flows by solving the equations:

$$\sum_{j\neq i}q^*_{ij}-\sum_{k\neq i}q^*_{ki}=q^*_i,\ \forall i\in N \tag{7}$$

$$0\le q^*_{ij}\le q_{ij}.$$

The corresponding FTR auction prices are determined as the differences of the corresponding source and sink nodal prices in the virtual energy auction.

Remark. Eq. (7) always have a solution $\{q^*_{ij},\forall i,j\}$ due to the number of variables being larger than the number of equations. Furthermore, $\{q^*_{ij},\forall i,j\}$ is an optimal solution to the FTR auction problem (1).

Thus, an energy auction where energy bids and offers at all nodes equal the corresponding expected locational prices under all transmission contingencies and load scenarios is equivalent to a FTR

[4] The result can be generalized to the more general case where there are multiple bids with different prices for each FTR but the mathematical representation of that general case is more complicated and will be omitted here for clarity.

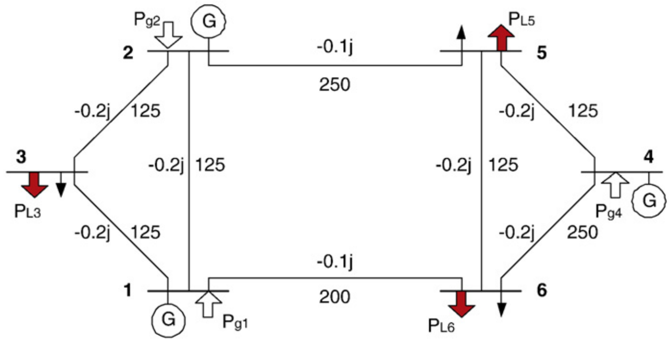

Fig. 1. A 6-bus test system.

Table 1
Bid functions of generation and load.

Bus-ID	Supply bids	Bus-id	Load bids
Bus-1	$10+0.05q$	Bus-3	$37-0.05q$
Bus-2	$15+0.05q$	Bus-5	$75-0.1q$
Bus-4	$42+0.025q$	Bus-6	$80-0.1q$

auction where all FTR bids between two points are equal to their expected payoffs. Such an FTR auction where all market clearing bids for FTRs between any two nodes are identical to the respective expected payoffs of the FTRs over all transmission contingencies and load scenarios would represent a perfect price discovery in an auction market with risk-neutral bidders.

We summarize the above results as a theorem which we just proved.

Theorem 1. Assume perfect knowledge of the expected locational marginal prices for energy in an electricity grid and all bidders being rational and risk-neutral price takers. Then the FTR auction problem (1) is equivalent to the virtual energy auction problem (6).

Theorem 1 states that the FTR simultaneous feasibility auction is represented by an equivalent virtual energy auction as described above. Hence in our subsequent analysis and numerical experiments, without loss of generality, we represent the FTR auction as a virtual energy auction from which we can derive both the expected congestion rents and the FTR clearing prices. Under this scheme, the expected congestion rent between any two network locations is the expected difference of locational energy prices between the two points. Likewise, the FTR clearing price between any two points is the difference between the locational clearing prices for energy in the virtual energy auction. It follows that correct prediction of expected congestion rents between any two points is equivalent to correct prediction of the expected locational energy prices. Thus, an energy auction where energy bids and offers at all nodes equal the corresponding expected locational prices over all transmission contingencies and load scenarios is equivalent to an FTR auction where all FTR bids between two points are equal to their expected payoffs. The outcome of such an FTR auction where all market clearing bids for FTRs between any two nodes are identical to the respective

Table 2
Ex post nodal prices and expected nodal prices.

Scenario	Bus-1	Bus-2	Bus-3	Bus-4	Bus-5	Bus-6
Normal	26.5	26.5	26.5	48.5	48.5	48.5
(L-13)	24.13	24.13	31.25	48.5	48.5	48.5
(L-16)	20.63	25	29.38	50	50	50
(L-25)	24.17	22.27	26.042	47.98	59.41	53.69
(L-45)	26.11	26.48	26.92	48.4	48.56	48.49
$E[P]$	25.40	25.69	27.26	48.60	49.75	49.17

Table 3
FTR auction market clearing nodal prices.

	$\alpha=1$	$\alpha=0.7$	$\alpha=0.5$	FTR Bids
bus-1	25.40	25.69	27.26	25.40
bus-2	25.69	25.69	27.26	25.69
bus-3	27.26	27.26	27.26	27.26
bus-4	48.60	49.17	48.60	48.60
bus-5	49.75	49.17	48.60	49.75
bus-6	49.17	49.17	48.60	49.17

Table 4
FTR price comparison under transmission contingencies only.

FTR \ α	$\alpha=1$	$\alpha=0.7$	$\alpha=0.5$	FTR Bids (exante)
FTR-12	0.28	0	0	0.28
FTR-13	1.86	1.57	0	1.86
FTR-14	23.19	23.48	21.34	23.19
FTR-15	24.34	23.48	21.34	24.34
FTR-16	23.77	23.48	21.34	23.77
FTR-23	1.57	1.57	0	1.57
FTR-24	22.91	23.48	21.34	22.91
FTR-25	24.06	23.48	21.34	24.06
FTR-26	23.48	23.48	21.34	23.48
FTR-34	21.34	21.91	21.34	21.34
FTR-35	22.49	21.91	21.34	22.49
FTR-36	21.91	21.91	21.34	21.91
FTR-45	1.15	0	0	1.15
FTR-46	0.57	0	0	0.57
FTR-56	−0.58	0	0	−0.58

expected payoffs of the FTRs over all transmission contingencies and load scenarios would represent a perfect price discovery when all bids exhibit rational risk-neutral price taking behaviors.

To identify the relationship between the bids/offers and the expected market clearing energy prices in the virtual energy auction, we show that the clearing prices in Eq. (6) depend on the upper and lower quantity bounds of energy bids. Let λ, (μ_r^+,μ_r^-), and (η^+,η^-) be the dual variables associated with the constraints in Eq. (6) where λ is a scalar associated with the energy balance constraint, $(\mu_r^+,\ \mu_r^-)$ $(\forall r\in R)$ are m-vectors associated with the transmission line capacity constraints and $(\eta^+,\ \eta^-)$ are n-vectors associated with the bid quantity bound constraints. The dual problem of the linear programming (LP) problem (6) is as follows (see Luenberger (1984)).

$$\begin{aligned} \min_{\lambda,\mu_r^+,\mu_r^-,\eta^+,\eta^-} \quad & \sum_{r\in R}(\mu_r^+ + \mu_r^-)^T L + (\eta^+)^T\ \overline{Q} + (\eta^-)^T \underline{Q} \\ s.t. \quad & \lambda\cdot e^T + \sum_{r\in R}(\mu_r^+ - \mu_r^-)^T G_r + \eta^+ - \eta^- \geq C^T \\ & \mu_r^+,\mu_r^- \geq 0, \forall r\in R, \text{ and } \eta^+,\eta^- \geq 0. \end{aligned} \tag{8}$$

Proposition 1. If none of the quantity bound constraints in Eq. (6) are binding, then the market clearing nodal prices resulting from the virtual energy auction are equal to the bid vector C.

If a bid quantity bound constraint at a bus i is binding, then the resulting market clearing nodal price P_i differs from the bid price c_i. Specifically, P_i is greater/less than c_i if bus i is a generation/load bus.

Proof. The market clearing nodal price vector P of the FTR auction Eq. (6) is given by:

$$P \equiv \lambda\cdot e^T + \sum_{r\in R}(\mu_r^+ - \mu_r^-)^T G_r. \tag{9}$$

By the duality theory from LP (Luenberger (1984)), the conclusions are drawn through inspecting the dual problem (8) and

Table 5
Load contingencies.

	Node 3	Node 5	Node 6
No-load change	$37.5-0.05q$	$75-0.1q$	$80-0.1q$
Load +25%	$46.875-0.05q$	$93.75-0.1q$	$100-0.1q$
Load −25%	$28.125-0.05q$	$56.25-0.1q$	$60-0.1q$

Table 6
Joint probability distribution of transmission and load contingencies.

	Normal	(L-13)	(L-16)	(L-25)	(L-45)
Base load	0.2	0.04	0.04	0.04	0.04
Load +25%	0.2	0.03	0.03	0.03	0.03
Load −25%	0.2	0.03	0.03	0.03	0.03

Table 7
FTR auction bids and market clearing prices and quantities under load and transmission contingencies.

	Bus-1	Bus-2	Bus-3	Bus-4	Bus-5	Bus-6
P ($)	24.3	25.5	28.5	47.1	53.9	50.8
Q (MW) (FTR)	286.4	210.6	180.0	185.4	210.8	291.6
P ($)	24.3	25.5	28.5	47.1	53.9	50.8
Q (MW) (α:1.5)	250.0	75.0	125.0	241.7	133.3	308.3
P ($)	24.3	25.5	28.5	47.8	53.9	50.8
Q (MW) (α:1.0)	250.0	75.0	125.0	185.4	189.6	195.8
P ($)	25.5	25.5	28.5	50.8	50.8	50.8
Q (MW) (α:0.7)	200.5	124.5	125.0	129.8	147.5	182.3
P ($)	28.5	28.5	28.5	47.1	47.1	47.1
Q (MW) (α:0.5)	143.2	105.3	48.5	51.2	105.4	145.8

applying the strong duality result between the primal LP problem (6) and the dual problem (8).

When the nodal clearing price at a node in the virtual energy auction differs from the expected nodal price at that node under the various transmission contingencies and load scenarios, the resulting FTR clearing prices for FTRs involving that node also differs from their expected payoffs. In the following section we demonstrate this phenomenon by means of numerical examples.

3. Numerical examples

Proposition 1 proves theoretically that the FTR auction clearing prices deviate systematically from the ex ante FTR bids whenever the energy injection/ejection quantity bounds in Eq. (6), as implied by the FTR bid quantity bounds in Eq. (1), become binding. We now use two numerical examples to illustrate the impacts by the FTR quantity bounds on the deviation of the FTR market clearing prices. To compute

Table 8
FTR price comparison under both load and transmission contingencies.

α	1.5	1	0.7	0.5	FTR Bids (exante)
FTR-12	1.21	1.21	0	0	1.21
FTR-13	4.18	4.18	2.97	0	4.18
FTR-14	22.82	23.43	25.31	18.64	22.82
FTR-15	29.60	29.60	25.31	18.64	29.60
FTR-16	26.52	26.52	25.31	18.64	26.52
FTR-23	2.97	2.97	2.97	0	2.97
FTR-24	21.60	22.22	25.31	18.64	21.60
FTR-25	28.39	28.39	25.31	18.64	28.39
FTR-26	25.30	25.30	25.31	18.64	25.30
FTR-34	18.64	19.25	22.34	18.64	18.64
FTR-35	25.42	25.42	22.34	18.64	25.42
FTR-36	22.34	22.34	22.34	18.64	22.34
FTR-45	6.79	6.17	0	0	6.79
FTR-46	3.70	3.09	0	0	3.70
FTR-56	−3.09	−3.09	0	0	−3.09

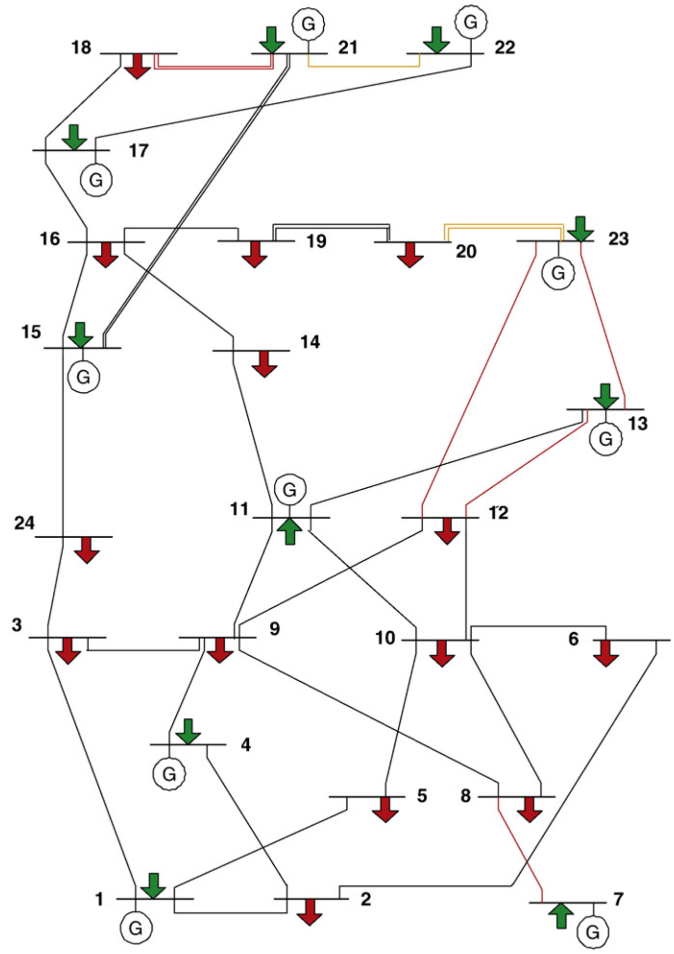

Fig. 2. IEEE 24-bus reliability test system.

the outcomes of an energy auction, the energy bid quantity bounds in Eq. (6) need to be specified. For the ease of exposition, we assume that the bid quantity bound for each FTR type is given by a constant α times its expected transaction volume between the corresponding points. Consequently, the quantity of an energy bid at each node is bounded by the corresponding component of $\alpha\cdot\hat{Q}$where $\hat{Q}\equiv(\hat{q}_1,\hat{q}_2,\cdots,\hat{q}_n)$ denote the expected quantities of energy transactions implied by the aggregate FTR transactions. Namely, $\overline{q}_i=\alpha\cdot\hat{q}_i$ and $\overline{q}_i=-\alpha\cdot\hat{q}_i$ $(\forall i\in N)$ in Eq. (6). This characterization of the quantity bound enables simple sensitivity analysis by varying the multiplier α. Two test systems are considered in our simulation experiments. One is a 6-bus system and the other is the IEEE 24-bus Reliability Test System (RTS).

3.1. A 6-bus example

First consider a 6-bus network example used in Chao and Peck (1998) and Chao et al. (2000) (see Fig. 1). Buses 1, 2 and 4 are generation nodes while buses 3, 5 and 6 are load nodes. The supply and demand functions at the 6 nodes are assumed to be linear in quantity q with parameters given in Table 1.

The transmission line capacities (MW) and admittances (p.u.) are shown in Fig. 1. Bus-6 is designated as the swing bus. We choose a set of 5 transmission reliability scenarios that are accounted for in the FTR auction: no line outage, line-13 out, line-45 out, line-16 out, and line-25 out.

3.1.1. Case 1: transmission line contingency but no load variation

We use the same supply and demand bid functions as in Chao et al. (2000). The *ex post* nodal prices in each of the 5 contingencies are

Table 9
IEEE 24-bus RTS: generation and load bid functions.

Bus-ID	Supply bids	Bus-ID	Demand bids
1	$15.483+0.0150q$	2	$65.000-0.0820q$
4	$20.000+0.0161q$	3	$75.517-0.1129q$
7	$12.555+0.0352q$	5	$63.000-0.0925q$
11	$29.000+0.0362q$	6	$42.289-0.0847q$
13	$39.859+0.1012q$	8	$62.517-0.1016q$
15	$29.678+0.0220q$	9	$50.517-0.0876q$
17	$23.180+0.0295q$	10	$59.517-0.0502q$
21	$30.031+0.0270q$	12	$45.289-0.0733q$
22	$20.966+0.0268q$	14	$64.517-0.0851q$
23	$35.330+0.0552q$	16	$58.289-0.1146q$
		18	$76.547-0.0792q$
		19	$72.517-0.0682q$
		20	$63.289-0.1033q$
		24	$72.289-0.0733q$

given in Table 2 (The quantity inside the parenthesis in the first column indicates the line on outage). The assumed probabilities of the contingencies are [0.6 0.1 0.1 0.1 0.1]. The expected nodal prices ($E[P]$) are given in the last row of Table 2.

Suppose the FTR market participants submit FTR bids that are equal to the expected payoffs over all contingencies. These bids are the differences in the expected nodal prices given in Table 2. Then the corresponding nodal price bids c_i's in the equivalent virtual energy auction can be set to the expected nodal prices given at the bottom of Table 2. The FTR bid quantity bounds are given by $-\alpha\cdot\hat{Q}$ and $\alpha\cdot\hat{Q}$ where the expected dispatch quantities $\hat{Q}$ obtained over all five reliability contingencies at all nodes are $\hat{Q}=(308.053MW, 213.733MW, 204.837MW, 243.855MW, 252.535MW, 308.320MW)$[5].

From this data we compute the resulting market clearing nodal prices P_i's to examine whether $c_i=P_i$, $\forall i=1,2,\cdots,6$. We vary the bounds for FTR quantity bids by varying the value of α. When $\alpha=1$, none of the FTR bid quantity bounds is binding and the resulting P_i's, as reported in the second column of Table 3, are the same as the c_i's (last column of Table 3). When $\alpha=0.7$ or $\alpha=0.5$, some of the FTR bid quantity bounds reach the upper bounds thus resulting in market clearing prices P_i's (see Table 3) that are different from the bid prices c_i's. In particular, Gen-1, Gen-4 and Load-5 reach their respective upper bounds when $\alpha=0.7$ while Gen-1, Gen-2, Load-5 and Load-6 reach the upper bounds when $\alpha=0.5$. The market clearing nodal energy prices for different α's are shown in Table 3.

Table 4 shows the sensitivity of FTR auction market clearing prices to bid quantities under the assumption that bid quantities are constant multiples of the average transaction volume between any two points. It provides a comparison of the FTR values for three different values of the multiplier α. The last column reports the *ex ante* FTR price bids.

3.1.2. Case 2: both transmission line and load contingencies

We then assume that under each transmission contingency there are three equally likely scenarios for loads: no change in loads, 25% more loads, and 25% less loads. Table 5 lists the load curves in all three scenarios at nodes 3, 5 and 6.

The assumed joint probability distribution of the load and transmission line contingencies is given in Table 6.

The computational results on market clearing nodal energy prices, energy quantities, and auction-clearing FTR prices are given in Tables 7 and 8. Specifically, Table 7 shows the nodal clearing prices and the dispatch quantities in the virtual energy auction as functions of the multiplier α, which is the ratio of the energy bid quantity bound to the expected dispatch quantity at each node. The first row in Table 7 contains the expected nodal energy prices and the expected dispatch quantities at the 6 buses over the 15 combined load and transmission line contingencies. We then assume that the FTR auction is conducted based on the price bids being set by the expected nodal energy prices (upper numbers in the first row) and the quantities of bids being bounded by α times the expected dispatch quantities (lower numbers in the first row), which corresponds to an FTR auction under the assumption of perfect price discovery. The rest of Table 7 contains the resulting nodal prices and the dispatch quantities at the 6 buses for α being 1.5, 1.0, 0.7, and 0.5.

Comparisons of the FTR values in the 4 cases of different α's are shown in Table 8.

[5] $\hat{Q}$ is calculated in the same way as $E[P]$ is in Table 2. Its entries are equal to the probability weighted energy transaction quantities corresponding to the 5 reliability scenarios at all buses.

Table 10
IEEE 24-bus with line contingency only: FTR auction market clearing nodal prices.

Bus	$\alpha=1$	$\alpha=3$	$\alpha=8$	$\alpha=30$	FTR Bids
1	29.9	21.5	21.5	21.5	21.5
2	40.8	40.8	40.8	40.8	40.8
3	39.2	43.8	43.8	43.8	43.8
4	25.2	25.2	25.2	25.2	25.2
5	40.1	40.1	40.1	40.1	40.1
6	40.4	40.6	40.7	41.3	41.3
7	18.7	18.7	18.7	18.7	18.7
8	40.2	40.6	42.4	42.4	42.4
9	41.4	42.2	41.8	43.3	43.3
10	40.5	41.4	41.4	41.4	41.4
11	41.6	41.6	41.6	41.6	41.6
12	40.5	41.1	41.0	41.6	41.6
13	41.4	41.4	41.4	41.4	41.4
14	39.1	40.1	40.9	40.9	40.9
15	40.2	39.7	39.7	39.7	39.7
16	40.0	39.9	40.0	40.0	40.0
17	40.2	40.1	40.1	40.1	40.1
18	40.1	40.1	40.1	40.1	40.1
19	40.1	40.1	40.1	40.1	40.1
20	40.1	40.3	40.3	40.3	40.3
21	40.3	40.3	40.1	40.1	40.1
22	40.2	40.1	40.1	40.1	40.1
23	40.5	40.7	40.7	40.5	40.5
24	39.8	46.9	46.9	46.9	46.9

Table 11
IEEE 24-bus with line contingency and load variation: FTR auction market clearing nodal prices.

Bus	$\alpha=1$	$\alpha=3$	$\alpha=8$	$\alpha=30$	FTR Bids
1	21.6	21.6	21.6	21.6	21.6
2	42.7	41.4	42.7	42.7	42.7
3	38.3	45.8	45.8	45.8	45.8
4	24.9	24.9	24.9	24.9	24.9
5	34.0	41.8	41.8	41.8	41.8
6	40.7	40.9	41.2	41.2	41.4
7	18.7	18.7	18.7	18.7	18.7
8	40.1	40.7	42.7	42.7	42.7
9	41.5	42.5	40.9	43.5	43.5
10	39.7	42.0	42.0	42.0	42.0
11	41.7	41.7	41.7	41.7	41.7
12	40.3	40.2	40.8	41.6	41.6
13	43.2	43.2	43.2	43.2	43.2
14	37.1	41.1	41.1	41.1	41.1
15	40.1	39.4	39.4	39.4	39.4
16	39.8	40.0	40.0	40.0	40.0
17	39.8	38.8	38.8	38.8	38.8
18	39.3	39.3	39.3	39.3	39.3
19	40.0	40.0	40.0	40.0	40.0
20	39.9	39.9	39.9	39.9	39.9
21	40.6	40.8	39.5	39.5	39.5
22	40.3	39.1	39.1	39.1	39.1
23	40.6	40.6	40.6	40.6	40.6
24	39.4	49.5	49.5	49.5	49.5

3.2. An IEEE 24-bus RTS Example

We next consider the IEEE 24-bus RTS with system topology shown in Fig. 2. Generators are located at buses 1, 4, 7, 11, 13, 15, 17, 21, 22 and 23. The rest of the buses are loads. Generation and load are represented by linear supply and demand functions, respectively.

In the base case (or, the no-contingency case), the supply and demand bid functions are given in Table 9.

3.2.1. Case 1: transmission line contingency but no load variation

Following the same procedure as the one outlined in the 6-bus example, we first consider the transmission line outages over links connecting buses 10 and 11, 14 and 16, 15 and 21, as well as 19 and 20 in computing FTR price bids. The outage probability of each of the 4 lines is 0.1. We then compute the market clearing prices of FTRs with different multiple α. Table 10 provides a comparison of the FTR values for 4 different α values. The last column reports the *ex ante* FTR price bids. We observe that there are notable differences between the market clearing FTR prices and the FTR bids over buses 6, 9, 12 and 23 even when the multiple α is 8. The auction clearing FTR prices converge to the bids (which reflect correct expected settlement values) when α reaches a large value of 30.

3.2.2. Case 2: both transmission line and load contingencies

As we incorporate load variation besides the line contingency in computing the *ex ante* FTR bids and then compute the FTR market clearing prices, we still find that the multiple α needs to be increased to 30 in order to achieve the convergence between the FTR auction clearing prices and the corresponding expected settlement values reflected by the bids (see Table 11). Again, Table 11 contains the market clearing FTR prices for 4 different α values and the FTR bids (the last column). A joint probability distribution (similar to the one defined by Table 6 in the 6-bus example) on load variation (25% up or down) and line outages is assumed in computing the prices in Table 11.

4. Conclusion

We demonstrated that FTR auctions enforcing the simultaneous feasibility constraints have inherent properties that result in a fundamental inefficiency in the FTR market. Specifically, the auction clearing prices do not converge to the expected payoffs of the auctioned instruments. Our analysis indicates that such divergence, which has been proved theoretically and demonstrated empirically, cannot be attributed just to lags in price discovery. It is indeed a convoluted effect of the current FTR auction clearing mechanism design and the bounded FTR bid quantities at all the network nodes[6]. We show that even when bidders are risk neutral and have perfect foresight of expected payoffs (which they bid) the FTR auction would produce clearing prices that differ from the expected FTR payoffs.

Based on our analysis, it is evident that the clearing prices depend on the natural quantity bounds of submitted FTR bids. When the FTRs serve primarily as hedging instruments, bid quantities for FTRs tend to track expected transaction volumes and FTR bids are spread over large number of node pairs. Such spread, however, has the effect of imposing quantity limits on certain FTR awards causing the clearing prices to deviate from the initial bid prices. In a more speculative market where FTR bid quantities exceed hedging needs, larger quantities of fewer FTR types would be awarded and auction clearing prices are likely to better match their expected *ex ante* valuations.

We conclude that price discovery alone does not remedy the discrepancy between the auction prices and the realized values of the FTRs. Such convergence is essential if the FTRs are to fulfill the need for efficient risk management and provision of correct price signal for transmission usage and investment. More liquidity in the FTR market through frequent reconfiguration auctions and the introduction of flowgate rights that can be traded in secondary markets are ways through which better convergence between forward prices and spot realization of the congestion rents can be achieved. Characteristics of a liquid secondary market for FTRs include the presence of a set of standardized trading quantities for FTRs and finer granularity in trading time intervals such as hours and days. Other approaches to increase the trading liquidity of FTRS include securitization of FTRs thus encouraging a broader base for the FTR market participation.

To facilitate the computational analysis on the sensitivity of the price divergence with respect to the FTR bid quantity bounds, it is assumed that the bid quantities are fixed multiples of expected transaction volume. In reality the ratio of bid quantity to average transaction volume can vary across FTRs. Our theoretical analysis ensures that the qualitative conclusion is valid as long as the bid quantities are a relatively low multiple of the expected volume which is the case when FTRs are allocated or auctioned off as hedging instruments.

Another stylized assumption made in this analysis is that FTR bidders have perfect foresight of the expected FTR payoffs. Since in practice bidders may form wrong expectations of the FTR payoffs, this fact introduces an additional source of divergence between the auction clearing prices and the expected *ex post* FTR payoffs. The perfect foresight assumption, however, should not be viewed as a weakness of our model. It is rather for setting up a hypothetically controlled experiment that is designed to single out the potential impact by market design on the divergence between FTR auction prices and realized payoffs.

Finally, the above conclusions also suggest that from a property rights perspective it might be more appropriate to allocate the FTRs themselves based on historical entitlements leaving it to the recipients to re-trade these rights as opposed to auctioning the FTRs and allocating the auction revenues.

Acknowledgement

This research is supported in part by NSF Grant ECS-0134210 (Deng) and by the Power System Engineering Research Center (PSerc) (all three authors). The research assistance by Haibin Sun is gratefully acknowledged. Comments from two anonymous referees and the associate editor are greatly appreciated.

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[6] An anonymous referee pointed out that some bidders might adjust their bidding behaviors to offset this effect. However, the empirical results reported here indicate that such offsetting cannot have been complete.