Chapter 31

Valuation of Electricity Generation Capacity

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31.1 Introduction

The emerging restructuring of the power industry in the United States and abroad has resulted in change of ownership on a massive scale of electric generation assets through divestiture, merger, and acquisition of physical plants or long-term entitlement to the plants' output. Such ownership transfers are typically done through public auctions. Establishing the market value of generation assets has become an important problem for utility commissions and private organizations buying or selling such assets. The generation capacity of a typical power plant is measured in hundreds of megawatts (MW) and the selling price runs into hundreds of millions of dollars. Hence, even a few percentage points of improvement in the valuation accuracy can have substantial financial consequences. The public interest in such valuation stems from the fact that in most jurisdictions the proceeds from the sales offset ratepayer's liability for stranded costs of uneconomic investments that were made by regulated utilities. Private entities bidding for these assets are obviously interested in establishing their market value, which will guide their bids. Market-based valuation of generation asset is also important for investors in new generation capacity and for financial institutions that are financing such investments.

The uncertain energy prices that prevail in the new competitive electricity markets make the generation asset valuation problem challenging as compared to what it used to be under the old regulated regime, where electricity prices were set by regulators based on a fixed rate of return on investment. Under the rate of return regulation investment decisions in generation capacity were typically based on a discounted cash flow (DCF) method that

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was used to evaluate the expected future cash flow associated with the generation capacity under consideration. This paradigm is being changed by the restructuring of the electricity supply industries and the transition to market-based prices.

It has been recognized in the literature (e.g., [6]) that in the presence of price uncertainty the traditional DCF approach tends to undervalue assets by ignoring the optionality available to the asset owner. In a well-developed financial and physical market for electricity, the payoffs of an electric power plant can be approximated by a series of financial instruments on electricity, and thus financial methods can be applied to value a power plant via valuing the appropriate set of financial instruments. Such an approach is employed in [3] for the valuation of power generation assets. In particular, the authors construct a "spark spread option"–based valuation model for fossil-fuel power plants. They demonstrate that the option-based approach better explains the observed market valuation than does the DCF-based approach. In fact the DCF valuations underestimate, by nearly a factor of four, the sale prices of several power plants divested by a southern California utility. However, the pure option pricing approach tends to oversimplify the valuation problem by ignoring operational costs and constraints on a power plant, such as startup costs, ramp-up constraints, and operating-level-dependent heat rate.

While it is imperative to recognize the embedded optionality to properly value generation capacity, it is of equal importance to recognize that physical operating characteristics of a real asset often impose restrictions on exercising the embedded options. It is therefore important when applying financial option pricing methodology to examine the impact of operational constraints on the capacity valuation. We explicitly incorporate several operating characteristics of a power plant into its valuation and illustrate by way of a numerical example the significance of accounting for operating constraints and costs.

In section 31.2, we highlight several key operating characteristics of a fossil-fuel power generation asset and describe the valuation problem of the power plant in a competitive power market environment. We construct a discrete-time mean-reverting trinomial lattice for the electricity and the generating fuel prices in section 31.3. We then formulate a stochastic dynamic programming (SDP) model based on the lattice price processes for our valuation problem incorporating operational constraints and outline the solution procedure. In section 31.4, we present results from numerical experiments to illustrate the significance of the impact by each of the operating characteristics on the valuation at different operating efficiency levels. Finally, we conclude with observations and remarks.

31.2 Problem description

Real option valuation methods that model generation assets in terms of financial options are becoming increasingly popular. A key concept employed by these approaches is the *heat rate*, which measures the conversion rate from a generating fuel into electricity. In some rough sense, heat rate represents the number of units of the fuel needed for generating one unit of electricity. The economic value of a generation plant of given capacity and known heat rate can then be roughly represented in terms of a spark spread call option, which is an option that yields its holder the positive part of electricity price less the "strike" heat rate-adjusted fuel cost at the option's maturity time. The analogy between a spark spread option and a generation plant stems from the fact that an owner of a merchant power plant (i.e., a power plant that can sell its output into at least one spot market) has the right, but not the obligation, to generate electricity by burning fuel at any point in time during the lifetime of the power plant. Suppose that the owner exercises such operational rights economically over time[1] and there are no operating constraints/lead times in running the power plant; then she would receive the spot price of electricity less the heat rate-adjusted generating fuel cost by selling/purchasing electricity/fuel, respectively, at spot market prices. Thus, the payoff obtainable by a rational merchant power plant owner at time t is the same as that of a spark spread call option with strike heat rate being set to the operating heat rate level of the power plant. The market value of a fossil-fuel power plant can then be obtained by summing up the values of the corresponding set of spark spread call options with an appropriate strike heat rate and the maturity time spanning the lifetime of the plant. Deng, Johnson, and Sogomonian [3] demonstrate that such a spark spread option–based valuation provides a much better approximation to empirically observed market valuations than does a DCF valuation.

However, the financial option valuation approach overlooks the differences between a physical asset and a financial asset. The optionality associated with operating a physical asset at different time epochs is often constrained by specific operating characteristics of the physical asset. These operational constraints may impose significant transaction costs on the exercise of the operational options either directly through setup costs or indirectly through operational lead times. Thus the financial option pricing formula tends to overestimate the option value of a real fossil-fuel generator.

The following implicit assumptions underlie a financial option–based valuation model: (a) a power plant can be instantly turned on or shut down; (b) there are no fixed operating costs but only variable production costs; and (c) the operating efficiency of a power plant is constant. Unfortunately, these assumptions are not very realistic. First, fixed costs are usually incurred whenever a power plant is turned on from the off state. For a steam generating unit, for instance, water in the boiler needs to be boiled before the unit can generate electricity, and the amount of fuel required to boil the water often depends on how long the unit has been shut down. That is, *startup costs* are involved in the process of turning a power generating unit on and the costs could be time-dependent. Second, the output level of a generator cannot be increased instantaneously to the full generation capacity upon turning on a power plant. A certain time period (e.g., the time for the water in the boiler to reach boiling temperature) is needed for a power plant to transit from the off state to the fully operational state. This time lag is often called the ramp-up time. Third, concerning the heat efficiency of a power plant, the rate at which a power plant converts the generating fuel into electricity varies with output levels. Specifically, a power plant is more efficient when it is operated at the rated capacity level than at a low output level.

Our objective is to explicitly incorporate the operating characteristics of a fossil-fuel power plant into the valuation model and illustrate the effects of these constraints on the valuation. In principle, one can formulate the operation of a power plant while incorporating operational characteristics in great detail as a full-fledged dynamic programming problem. Such an approach was employed by [9] for short-term generation asset valuation by solving a stochastic unit commitment problem with constraints on startup and shutdown costs,

[1] To "exercise a right economically" means that a rational power plant owner would exercise an operational right at time t only when the electricity price less generating fuel cost is positive at that time.

minimum run time, and maximum ramp rate. However, the computational complexity makes this approach prohibitively difficult to implement for a long time horizon. The difficulty arises from the fact that operational characteristics affect daily or even hourly decisions, whereas the lifetime of a plant over which its value is determined is of the order of 15 years or more.

Our focus in this paper is on long-term asset valuation. Our primary objective is to demonstrate a computationally feasible approximation method that will capture the essence of the operational constraints in a stochastic dynamic programming framework with realistic stochastic price models. A second objective is to assess the magnitude of the error introduced by ignoring the various operational characteristics of a generation asset and how this error varies with the heat rate of a plant. Our approach compromises on modeling the operational details by using a rather simplistic representation of some key operational aspects. Specifically, we represent the startup cost, ramp-up time, and output-dependent operating heat rate as described below and solve a stochastic dynamic program under the assumption of a discrete-time mean-reverting trinomial stochastic price model for electricity and fuel. A similar approach employing a more elaborate (and more precise) characterization of the underlying price processes is discussed in [5].

Startup cost We assume that there is a constant fixed cost c_{start} associated with the action of turning on a power plant from the off state because the water in the boiler of the generator must be heated before the generator can generate power. In general, the cost for starting up a generating unit depends on how long the unit has been turned off. The longer the unit is off, the more heat is dissipated from its boiler, and thus the higher the cost incurred when reheating the water. Nevertheless, we simplify this effect by assuming that the startup cost, c_{start}, is a constant.

Ramp-up time We approximate the ramp-up time (which introduces a lag in the exercise of the real on/off option) by assuming that whenever a power plant is turned on from the off state, there is a fixed delay time of length D before electricity can be generated. Once a power plant is turned on, it always takes a short period of time (i.e., ramp-up time) for its generating unit to reach certain operating output levels. Similar to the case of startup cost, the length of the ramp-up time also depends on how long the power plant has been off. To reflect this aspect to first order, we assume that there is a constant time lag between the time point at which a generating unit is turned on and the time point at which the generating unit reaches its full output capacity.

Output-dependent operating heat rate While it is known that the dependency between the power output level and the operating heat rate follows a nonlinear functional form (see [10]), we make a simplifying assumption by considering only two possible output levels for a plant: the rated capacity level $\overline{Q}$ per unit of time, called maximum output level, with an operating heat rate of $\overline{Hr}$; and the minimum capacity level $\underline{Q}$ ($\underline{Q} < \overline{Q}$) per unit of time, that is, the minimum output level allowable in order to keep a power plant operational, with a corresponding heat rate of $\underline{Hr}$. The constraint $0 < \overline{Hr} \leq \underline{Hr}$ reflects the fact that a fossil-fuel power plant is more efficient when operated at a high output level than at a low

output level. We also assume that the switching between the maximum capacity level and the minimum capacity level is instantaneous and costless.

31.3 A stochastic dynamic programming approach to capacity valuation

In valuing the power generation capacity, price models for electricity and the generating fuel are key ingredients. We employ a mean-reverting stochastic process to model the prices. Mean reversion has been demonstrated to be a common feature in almost all commodity prices, including energy prices (see [8]). In particular, we construct a discrete-time lattice price process that approximates the continuous-time mean-reverting electricity price model described in [3] and value a power plant based on the price lattice. The lattice (binomial tree) approach to option pricing was rigorously developed by [2]. Our approach is related to [1] and [7], which deal with pricing options on a multinomial lattice when there are multiple state variables.

Consider a finite time horizon of $[0, T]$ for the capacity valuation problem. To set up a discrete-time framework, we divide the interval $[0, T]$ into N subintervals, $[0, t_1]$, $(t_1, t_2]$, ..., $(t_{N-1}, t_N \equiv T]$ of equal length $\Delta t \equiv T/N$. Let the natural logarithm of the prices of electricity and the fuel be the state variables at time t, denoted by (X_t, Y_t). We assume that the state of the price processes changes value only at t_i ($i = 1, 2, \dots, N$) and the state vector (X_t, Y_t) takes on a finite set of values. With the understanding that (X_i, Y_i) denotes (X_{t_i}, Y_{t_i}) ($i = 0, 1, 2, \dots, N$), we rewrite the vector process $\{(X_t, Y_t) : t = t_0, t_1, \dots, t_N\}$ as $\{(X_i, Y_i) : i = 0, 1, \dots, N\}$. We start with the construction of the discrete-time price processes and then present the valuation model formulation.

31.3.1 A discrete-time mean-reverting price process

From here on, the generating fuel is specified to be natural gas. The following continuous-time mean-reversion models are employed in [3] for modeling the returns of electricity price s_t^e and natural gas price s_t^g:

$$\begin{aligned} dX_t &= \kappa_e(\theta_e - X_t)dt + \sigma_e dB_t^1, \\ dY_t &= \kappa_g(\theta_g - Y_t)dt + \rho\sigma_g dB_t^1 + \sqrt{1-\rho^2}\sigma_g dB_t^2, \end{aligned} \tag{31.1}$$

where $X_t = \ln S_t^e$ and $Y_t = \ln S_t^g$; θ_e and θ_g are the long-term means of X_t and Y_t, respectively; κ_e and κ_g are two positive mean-reverting coefficients indicating the rates at which the electricity price and the natural gas price revert to their respective long-term means; σ_e and σ_g are the instantaneous price volatilities of electricity and natural gas, respectively; ρ is the instantaneous correlation coefficient between the electricity and the natural gas price returns; and B_t^1 and B_t^2 are two independent standard Brownian motion processes.

Using the same set of parameters $(\kappa_i, \theta_i, \sigma_i, \rho)$ as those in (31.1), we construct two discrete-time mean-reverting price models for electricity and natural gas on a recombining trinomial lattice as follows. We choose a state space following [7]. Starting from each log-price state vector (X_t, Y_t) at time t ($t = 0, 1, 2, \dots, N-1$), there are three possible states (X_{t+1}^i, Y_{t+1}^i) ($i = 1, 2, 3$) to reach at time $(t+1)$, as illustrated in Figure 31.1.

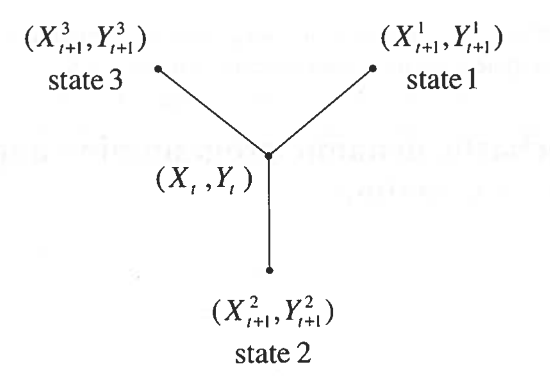

Figure 31.1. *Construction of a trinomial lattice.*

Let the price state movements on the trinomial lattice be

$$X_{n+1} = \begin{cases} X_n + \sigma_e\sqrt{\frac{3}{2}}\sqrt{\Delta t} & \text{(state 1)},\\ X_n & \text{(state 2)},\\ X_n - \sigma_e\sqrt{\frac{3}{2}}\sqrt{\Delta t} & \text{(state 3)}, \end{cases}$$

$$Y_{n+1} = \begin{cases} Y_n + \rho\sigma_g\sqrt{\frac{3}{2}}\sqrt{\Delta t} + \sigma_g\sqrt{1-\rho^2}\sqrt{\frac{1}{2}}\sqrt{\Delta t} & \text{(state 1)},\\ Y_n - \sigma_g\sqrt{1-\rho^2}\frac{2}{\sqrt{2}}\sqrt{\Delta t} & \text{(state 2)},\\ Y_n - \rho\sigma_g\sqrt{\frac{3}{2}}\sqrt{\Delta t} + \sigma_g\sqrt{1-\rho^2}\sqrt{\frac{1}{2}}\sqrt{\Delta t} & \text{(state 3)}, \end{cases} \tag{31.2}$$

where $\Delta t = \frac{T}{N}$, $n = 0, 1, \ldots, n-1$. Define a set of transition probabilities on the trinomial lattice so that the resulting price models for electricity and natural gas are mean-reverting. Let p_i denote the transition probability moving from a given state (X_n, Y_n) to state (X^i_{n+1}, Y^i_{n+1}) $(i = 1, 2, 3)$ and let $P \equiv (p_1, p_2, p_3)'$. If the conditions

$$\begin{cases} -\frac{1}{3} < \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} + \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} < \frac{2}{3},\\ -\frac{2}{3} < \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t} < \frac{1}{3},\\ -\frac{2}{3} < \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} - \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} < \frac{1}{3} \end{cases} \tag{31.3}$$

hold, then

$$P = \begin{cases} p_1: & \frac{1}{3} + \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} + \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t},\\ p_2: & \frac{1}{3} - \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t},\\ p_3: & \frac{1}{3} - \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} - \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t}. \end{cases} \tag{31.4}$$

The conditions in (31.3) are for ensuring that the probabilities p_i $(i = 1, 2, 3)$ are between 0 and 1. If any of the conditions in (31.3) is not satisfied, then P is defined as

$$\text{If } \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t} \geq \frac{1}{3}, \quad P = \begin{cases} p_1: & \frac{1}{2},\\ p_2: & 0,\\ p_3: & \frac{1}{2}. \end{cases}$$

$$\text{If } \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t} \leq -\frac{2}{3}, \quad P = \begin{cases} p_1: & 0,\\ p_2: & 1,\\ p_3: & 0. \end{cases} \tag{31.5}$$

$$\text{If } \begin{cases} -\frac{2}{3} < \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t} < \frac{1}{3},\\ \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} + \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} \leq -\frac{1}{3}, \end{cases} \quad P = \begin{cases} p_1: & 0,\\ p_2: & \frac{1}{3} - \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t},\\ p_3: & \frac{2}{3} + \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t}. \end{cases}$$

$$\text{If } \begin{cases} -\frac{2}{3} < \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t} < \frac{1}{3},\\ \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} + \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} \geq \frac{2}{3}, \end{cases} \quad P = \begin{cases} p_1: & \frac{2}{3} + \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t},\\ p_2: & \frac{1}{3} - \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t},\\ p_3: & 0. \end{cases} \tag{31.6}$$

$$\text{If } \begin{cases} -\frac{1}{3} < \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} + \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} < \frac{2}{3},\\ -\frac{2}{3} < \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t} < \frac{1}{3},\\ \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} - \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} \leq -\frac{2}{3}, \end{cases} \quad P = \begin{cases} p_1: & 0,\\ p_2: & \frac{1}{3} - \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t},\\ p_3: & \frac{2}{3} + \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t}. \end{cases}$$

$$\text{If } \begin{cases} -\frac{1}{3} < \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} + \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} < \frac{2}{3},\\ -\frac{2}{3} < \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t} < \frac{1}{3},\\ \left[\frac{\kappa_e(\theta_e - X_n)}{\sqrt{6}\sigma_e} - \frac{\kappa_g(\theta_g - Y_n)}{2\sqrt{6}\sigma_g}\right]\sqrt{\Delta t} \geq \frac{1}{3}, \end{cases} \quad P = \begin{cases} p_1: & \frac{2}{3} + \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t},\\ p_2: & \frac{1}{3} - \frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}\sqrt{\Delta t},\\ p_3: & 0. \end{cases} \tag{31.7}$$

The transition probabilities defined in (31.4)–(31.7) yield two mean-reverting price processes for electricity and natural gas. For instance, suppose that the conditions in (31.3) are satisfied in the current state (X_n, Y_n); thus (p_1, p_2, p_3) are given by (31.4). In (31.4), if the current Y_n is greater than θ_g, meaning that the current natural gas price is above its long-term mean, then $\frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}$ is a negative number. Therefore from (31.4) we see that p_2, which is the probability of moving toward a decreasing level of Y_{n+1}, is increased. On the other hand, if the current Y_n is a number less than θ_g, meaning that the current natural gas price is below its long-term mean, then $\frac{\kappa_g(\theta_g - Y_n)}{\sqrt{6}\sigma_g}$ becomes a positive number. Therefore from (31.4) we see that p_1 and p_3, which are the probabilities of moving toward increasing levels of Y_{n+1}, are increased. Similarly, when the current X_n is greater/less than its long-term mean θ_e, the probability of moving toward a decreasing/increasing level of X_{n+1} is increased. The transition probabilities defined in (31.5)–(31.7) also have such mean-reverting property with respect to the price state vector (X_n, Y_n).[2]

[2]This discrete-time model may not converge in distribution to the previously mentioned continuous-time mean-reversion model (see also [4]). The parameters need to be estimated directly from the discrete model structure. To guarantee the convergence to the continuous-time price model, a quadnomial rather than a trinomial lattice is needed, as shown in [5].

31.3.2 Valuation of a merchant power plant with operational constraints

Suppose that the log-prices of electricity and natural gas evolve according to the lattice process $\{(X_t, Y_t) : t = 0, 1, \dots, N\}$ constructed in section 31.3.1. Moreover, we make the following assumptions regarding the operational characteristics of a natural gas–fired merchant electric power plant.

Assumption 1 The power plant of interest is subject only to the three operating characteristics described in section 31.2.

Assumption 2 When running the power plant, the operator takes one of the three possible actions at discrete time points. The three possible actions are to shut down the plant, to run the power plant at its minimum capacity level (turn on the plant first if it is currently off), and to run the plant at its maximum capacity level (turn on the plant first if it is currently off). We denote these three actions by **off, on_min**, and **on_max**, respectively.

The operator of the merchant power plant seeks to maximize the expected total profit of the power plant with respect to the random price vector (S_t^e, S_t^g) over the operating time horizon by making optimal decisions regarding whether to turn on or shut down the generating unit as well as how to operate the unit. Under the risk-neutral probabilities, the expected total profit of a power plant over its operating time horizon yields the value of the power plant during that time period.

Before getting into the formulation of the valuation problem, we introduce some additional notation. The operating time horizon T is divided into N periods. The power plant operator makes the operational decisions at the beginning of every m periods, i.e., the operator takes action only in periods $0, m, 2m, 3m, \dots, km, \dots$, etc.

- n index for periods $(1, 2, \dots, N)$;
- X_n state variable indicating the logarithm of the electricity price in period n;
- Y_n state variable indicating the logarithm of the natural gas price in period n;
- w state variable indicating whether the power plant is currently on or off; $w = 0$ means that the power plant is currently off; $w = 1$ means that the plant is currently on;
- β discount factor over one time period;
- a_n action taken by the power plant operator in period n;
- Φ the action space, and $\Phi \equiv \{\text{off, on\_min, on\_max}\}$.

c_{start}, $\underline{Q}$, $\underline{Hr}$, $\overline{Q}$, and $\overline{Hr}$ are defined in section 31.3.1. The ramp-up time is assumed to be one period's delay for simplicity. Let the value function $V_n(X_n, Y_n, w)$ be the expected total profit of the power plant over time periods $n, n+1, n+2, \dots, N$, given the current price state vector (X_n, Y_n) and the current operating state w of the power plant. Then $V_n(X_n, Y_n, w)$ is obtained by solving the following three recursive equations with proper boundary conditions.

If $n \neq k \cdot m$, where $k = 0, 1, 2, \dots$, then the operator takes no action in period n. The value of a power plant is equal to the discounted expected future value of the power plant.

$$V_n(X_n, Y_n, w) = \beta \cdot E_n[V_{n+1}(X_{n+1}, Y_{n+1}, w)], \tag{31.8}$$

where $E_n[\cdot]$ denotes the conditional expectation given information available in period n.

If $n = k \cdot m$, where $k = 0, 1, 2, \dots$ and $w = 0$, then

$$V_n(X_n, Y_n, 0) = \max_{a_n \in \Phi} \begin{cases} \text{on\_max}: & -c_{\text{start}} + \beta \cdot E_n[V_{n+1}(X_{n+1}, Y_{n+1}, 1)], \\ \text{on\_min}: & -c_{\text{start}} + \beta \cdot E_n[V_{n+1}(X_{n+1}, Y_{n+1}, 1)], \\ \text{off}: & \beta \cdot E_n[V_{n+1}(X_{n+1}, Y_{n+1}, 0)]. \end{cases} \tag{31.9}$$

If $n = k \cdot m$, where $k = 0, 1, 2, \dots$ and $w = 1$, then

$$V_n(X_n, Y_n, 1) = \max_{a_n \in \Phi} \begin{cases} \text{on\_max}: & \overline{Q} \cdot [\exp(X_n) - \overline{Hr} \cdot \exp(Y_n)] \\ & \quad + \beta \cdot E_n[V_{n+1}(X_{n+1}, Y_{n+1}, 1)], \\ \text{on\_min}: & \underline{Q} \cdot [\exp(X_n) - \underline{Hr} \cdot \exp(Y_n)] \\ & \quad + \beta \cdot E_n[V_{n+1}(X_{n+1}, Y_{n+1}, 1)], \\ \text{off}: & \beta \cdot E_t[V_{t+1}(X_{t+1}, Y_{t+1}, 0)]. \end{cases} \tag{31.10}$$

The boundary conditions are

$$V_{N+1}(x, y, w) \equiv 0 \quad \forall (x, y) \in R^2, w = 0, 1. \tag{31.11}$$

31.3.3 The solution of the SDP

With the trinomial price model constructed in section 31.3.1, the optimal policies of the SDP have a barrier control form. There exists a "no-action" band on the plane of the natural gas price (plotted on the horizontal axis) and the electricity price (plotted on the vertical axis). If the price vector (S_t^g, S_t^e) consisting of the market prices of natural gas and electricity is inside this band, then it is optimal for the plant operator to maintain the status quo. If the price vector is above the upper boundary of the band, then, depending on the state of the power plant, the operator should increase the output level of the plant from off to on or from minimum capacity to full capacity. If the price vector is below the lower boundary of the band, then it is optimal for the operator to reduce the output level of the power plant to off or minimum capacity depending on the state of the plant.

31.4 Numerical experiments

We have implemented this proposed methodology for valuing a natural gas–fired power plant to examine the impact of operational characteristics on the capacity valuation. We report some numerical results for a hypothetical 100 MW gas-fired power plant over a 720-day period. We assume that the gas power plant incurs a startup cost whenever turned on and that it takes 1 day to ramp up the power plant from the off state to a desired output state but there is no delay in increasing/decreasing output level once the power plant is on. For startup cost, we examine two possible values, \$6000/start and \$12,000/start. The maximum and the minimum capacity levels are assumed to be 100 MW and 50 MW, respectively. Moreover,

Table 31.1. *Parameters for mean-reversion price models.*

κ_1	3	κ_2	2.25
θ_1	3.15	θ_2	0.87
σ_1	0.75	σ_2	0.6
ρ	21.7		

Table 31.2. *Value of a natural gas–fired power plant with/without physical characteristics.*

Heat Rate ($HR_{\max}$ MMBtu/MWh)	8000	9500	12000	14000
Cap. Value (nophy. constr.)	5.211 mill.	3.236 mill.	1.381 mill.	0.679 mill.
Cap. Value (3 phy. constr./stup=\$6k)	5.153 mill.	3.176 mill.	1.335 mill.	0.648 mill.
Pctg. Val. Overstate. (ignoring 3 phy./stup=\$6k)	1.13%	1.88%	3.43%	4.83%
Cap. Value (2 phy. constr./stup=\$0)	5.207 mill.	3.230 mill.	1.378 mill.	0.677 mill.
Pctg. Val. Overstate. (ignoring stup only/stup=\$6k)	1.06%	1.70%	3.21%	4.51%
Cap. Value (3 phy. constr./stup=\$12k)	5.121 mill.	3.144 mill.	1.312 mill.	0.632 mill.
Pctg. Val. Overstate. (ignoring 3 phy./stup=\$12k)	1.75%	2.93%	5.28%	7.45%
Cap. Value (2 phy. constr./stup=\$0)	5.207 mill.	3.230 mill.	1.378 mill.	0.677 mill.
Pctg. Val. Overstate. (ignoring stuponly/stup=\$12k)	1.68%	2.74%	5.05%	7.12%

the ratio between the operating heat rates at the minimum and the maximum capacity levels of the power plant is assumed to be 1.38 : 1. Under the mean-reversion price assumption for electricity and natural gas, the trinomial lattice is built with Δt being 1 day. The operator of the power plant makes operating decisions at all nodes of the lattice, i.e., $m = 1$. The initial prices of electricity and natural gas are assumed to be \$21.70 and \$3.16, respectively, which are sampled from the historical market prices.

The parameters used to construct the mean-reverting trinomial lattice are given in Table 31.1.

The value of the underlying power plant is calculated for each of the three cases: considering all three physical operating characteristics, ignoring the three operating characteristics, and ignoring the startup cost only. The numerical results are presented in Table 31.2.

We plot the value of the power plant accounting for all three operating characteristics for different heat rates in Figure 31.2. The x-axis represents different heat rates. The solid curves with crosses and circles plot the capacity value per year with startup cost being \$6000/startup and \$12,000/startup, respectively. The capacity value ignoring the three operating characteristics for different heat rates is plotted by the plain solid curve. All three curves are plotted against the capacity value axis on the left. The dashed curves

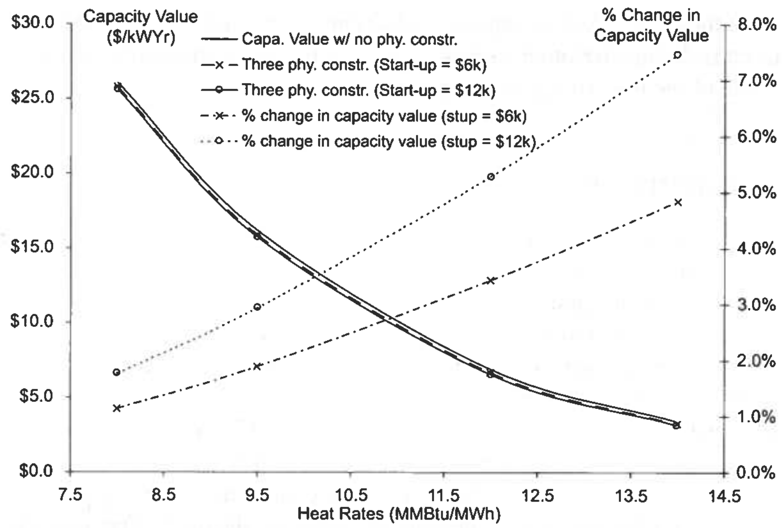

Figure 31.2. *Valuation of a power plant with/without physical characteristics.*

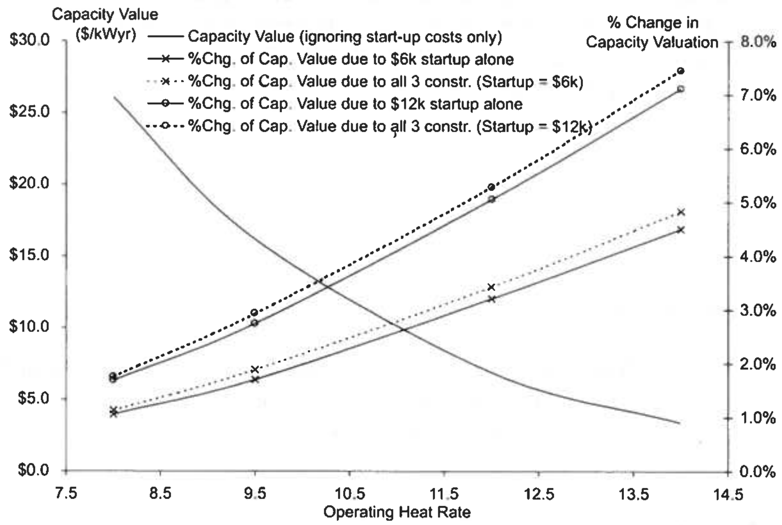

Figure 31.3. *Valuation of a power plant with/without the startup cost.*

with circles and crosses plot the percentage by which the capacity value is overstated due to ignoring the physical operating characteristics with the startup cost being \$6000 and \$12,000, respectively. The percentage for which the capacity value is overstated due to ignoring the operating characteristics ranges from 1.13% for the most efficient plant with a low startup cost to 7.45% for the least efficient plant with a high startup cost.

Figure 31.3 plots the values of the power plant with and without the startup cost only.

The impact of the startup cost on capacity valuation is very significant. Ignoring the startup cost while considering the other aspects accounts for more than 90% of the overstated capacity value of the underlying power plant.

31.5 Conclusion

We conclude from the numerical results that the operational characteristics affect the valuation of a merchant power plant to different extents depending on the operating efficiency of the power plant and the assumptions about the electricity and the generating fuel prices. In general, the impact of physical operating characteristics on power plant valuation can be very significant under the mean-reversion price models. Moreover, the more efficient a power plant is, the less affected its valuation is by the operational constraints and vice versa. The impact on capacity valuation ranges from 1.13% overvaluation for the most efficient plant with a low startup cost to 7.45% overvaluation for the least efficient plant with a high startup cost. Among the three operating characteristics of a power plant which we consider here, startup cost affects the capacity valuation the most. The reason is twofold. The first-order effect of the startup cost on capacity valuation is that it directly imposes a transaction cost on exercising the embedded spark spread options in a fossil-fuel power plant when the electricity price is greater than the fuel cost. The second-order effect of the startup cost is that it forces the power plant to keep operating at a loss or to forego a profit when the startup cost cannot be justified by the expected loss-saving or the expected profit that would result from turning the power plant off or on. In other words, the startup cost reduces the option value of a power plant. Our sensitivity analysis reveals that, under the mean-reversion price models, ignoring the startup cost alone can explain more than 90% of the overstated capacity value of a power plant (as compared to the overstated value when all three operational characteristics are ignored).

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