Optimal Transmission Switching – Sensitivity Analysis and Extensions

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***Abstract*--In this paper, we continue to analyze optimal dispatch of generation and transmission topology to meet load as a mixed integer program (MIP) with binary variables representing the state of the transmission element (line or transformer). Previous research showed a 25% savings by dispatching the IEEE 118 bus test case. This paper is an extension of that work. It presents how changing the topology affects nodal prices, load payment, generation revenues, cost, and rents, congestion rents, and flowgate prices. Results indicate that changing the topology to cut costs typically results in lower load payments and higher generation rents for this network. Computational issues are also discussed.**

***Index Terms*—Mixed integer programming, Power generation dispatch, Power system economics, Power transmission control, Power transmission economics**

I. Nomenclature

Indices
n, m: nodes
k: transmission element (line or transformer)
g: generator
d: load

Variables
θ_n: voltage angle at node n
P_{nmk}: real power flow from node m to n for transmission element k
P_{ng}: real power supply from generator g at node n
P_{nd}: real power load at node n
z_k: binary variable for transmission element k (0 open; 1 closed)
TC_J: Total system cost with J open elements

Parameters
P_k^{max}, P_k^{min}: max and min capacity of transmission element k
P_g^{max}, P_g^{min}: max and min capacity of generator g
θ_n^{max}, θ_n^{min}: max and min voltage angle at node n
c_{ng}: cost of production for generator g at node n
B_k: electrical susceptance of transmission element k
J: number of open transmission elements

II. Introduction

Transmission is traditionally characterized as a static system with random outages over which the system operator dispatches generators to minimize cost. However, it is acknowledged, both formally and informally, that system operators can, and do, change the topology of systems to improve voltage profiles or increase transfer capacity.[1] These decisions are made at the discretion of the operators, rather than in an automated or systematic way. The concept of transmission dispatch was introduced by O'Neill et al. [1] in a market context, in which the dynamic operation and compensation of transmission elements is examined. Transmission dispatch was formulated by Fisher et al. [2] and tested on a standard engineering test case. In this paper, we examine some of the economic impacts and other sensitivities of the formulation and network from [2].

We formulate the problem as a mixed-integer program (MIP), based on the traditional DC optimal power flow (DCOPF) used to dispatch generators to meet load in an efficient manner. We then use this formulation to examine the potential for improving generation dispatches by optimizing transmission topology for a well-known IEEE test case.

We do not ignore the importance of reliability, nor are we suggesting dispatching transmission at the expense of reliable network operations. We are simply examining the potential for automating actions operators currently take, such as implementing special protection schemes (SPSs), and improving network operation by making use of controllable components. Transmission elements that are open in the optimal dispatch of a network may be available to be switched back into the system as needed, as in PJM's SPSs. In cases where this may not be possible, transmission switching can be conducted in conjunction with contingency analysis in order to maintain reliability levels while taking advantage of improved topology. However reliability is maintained, transmission dispatch is not by definition incompatible with reliable operation of the grid.

The paper is organized as follows. Section III presents the MIP formulation for the transmission switching problem. Section IV provides the main results and analysis. Section V discusses the computational statistics. Section VI covers sensitivity studies and discusses issues regarding practical implementation of this model. Section VII contains a brief discussion of policy implications including the impacts on market participants and revenue adequacy of financial transmission rights (FTRs). Section VIII discusses current and future work; section IX concludes this paper.

III. MIP Formulation

This transmission switching formulation is the same as in [2]. Generation cost is minimized, subject to physical constraints of the system and Kirchhoff's laws governing power flow. The chosen min and max bus angle values are ±0.6 radians. Equation (1) represents the bus angle constraint; (2a) and (2b) represent the lower and upper bound constraints for the generators and transmission elements. z_k is the binary variable representing the state of the transmission element. The

[1] Personal communication with Andy Ott, Vice President PJM.

capacities on the left-hand side and the right-hand side of (2b) are multiplied by z_k so that P_{nmk} is zero when z_k is zero. M_k, listed in (4a) and (4b), is referred to as the "big M" value. When the binary variable z_k is one, the value of M_k does not matter; when the binary variable is zero, the value of M_k is used to ensure that (4a) and (4b) are satisfied regardless of the difference in the bus angles. P_{nmk} is zero when z_k is zero so for this to work, M_k must be a large number greater than or equal to $B_k\left(\theta_n^{\max} - \theta_m^{\min}\right)$. Without this adjustment to the power flow equations, the buses that are connected to this transmission element would be forced to have the same bus angle when z_k is zero. Forcing the buses' angles to be the same is incorrect as the element is no longer present. For the situation where there are two parallel transmission elements, if one were removed by the program, the other would be forced to have a zero power flow without this adjustment to the power flow equations. However, using (4a) and (4b), the DCOPF provides the solution corresponding to the case when the opened transmission element is not present in the network.

Equation (5) specifies the number of open transmission elements in the altered topology. We are not advocating introducing (5) to solve practical problems; this constraint is only used to gain understanding about the effects of changing the network topology for various solutions. To solve the transmission dispatch problem to optimality, (5) would not be present. The formulation below is a basic direct current optimal power flow (DCOPF) problem along with the transmission switching formulation. Variable admittance devices, such as phase shifters, are not modeled within this study and transformers are modeled as transmission lines. Injections into a bus are positive (generator supply, power flow into a bus) and withdrawals are negative (load, power flow out of a bus).

Minimize: $\mathrm{TC_J} = \sum_g c_{ng} P_{ng}$

s.t.:

(1) $\theta_n^{\min} \le \theta_n \le \theta_n^{\max} \qquad \forall n$

(2a) $P_g^{\min} \le P_{ng} \le P_g^{\max} \qquad \forall g$

(2b) $P_k^{\min} z_k \le P_{nmk} \le P_k^{\max} z_k \qquad \forall k$

(3) $\sum_k P_{nmk} + \sum_g P_{ng} + P_{nd} = 0 \qquad \forall n$

(4a) $B_k(\theta_n - \theta_m) - P_{nmk} + (1 - z_k) M_k \ge 0 \qquad \forall k$

(4b) $B_k(\theta_n - \theta_m) - P_{nmk} - (1 - z_k) M_k \le 0 \qquad \forall k$

(5) $\sum_k (1 - z_k) = J \qquad \forall k$

IV. Results and Analysis

In [2], we examined overall system cost with changes in topology. In this paper, we analyze other economic indicators, including payments to generators and payments from loads, based on a nodal marginal price settlement.

The nodal price, or locational marginal price (LMP), is the marginal value of energy at a given location in the network, and is calculated as the dual variable of the power balance constraint (3). Total system cost is the sum of all the costs in the system to meet the load, and is referred to as generation cost within this paper. In the present model, this comprises variable generator cost. Generator revenue is the amount generators are paid based on nodal pricing, LMP times amount produced, common in several restructured markets in the US. Generation revenue is the sum of all generator revenues. Generator rent, then, is the difference between revenue and cost for an individual generator and generation rent is the sum of all generator rents. Load payment is the sum of all individual load payments, which is the nodal payment or LMP times amount consumed.

In this paper, we are defining congestion rent as the difference in LMPs across a transmission element times its power flow (6). Within (6), *n* represents the *to bus* and *m* represents the *from bus*. This form of pricing has been shown to capture the marginal value of a transmission element's capacity, or the flowgate marginal price, and the marginal value of the electrical properties, which has been called admittance pricing in the literature by Gribik et al. in [3], Baldick et al. in [4], and O'Neill et al. in [5].

The flowgate marginal price (FMP) is the shadow price on the capacity of a transmission element, or the marginal value of increasing the thermal capacity. The admittance price is equal to the FMP minus the LMP difference. When a transmission element is not capacity constrained, the FMP is zero; therefore, the marginal value of the admittance, or admittance price, is equal to the negative LMP difference since the FMP is zero. A transmission element can have a non-zero congestion rent if it is thermally constrained or admittance constrained.[2] A transmission element is admittance constrained when more power can be sent across the element if the admittance is changed. For more information on flowgate and admittance pricing, see [3], [4], and [5].

(6) $\text{Congestion Rent} = (LMP_n - LMP_m) P_{nmk}$

A. System Cost, Revenues, Rents, and Load Payment

The IEEE 118 bus test case was used to test and analyze the transmission dispatch formulation. The transmission switching problem was written in AMPL, and solved with CPLEX version 10.1. Data for the IEEE 118 bus test case was downloaded from the University of Washington Power System Test Case Archive [6]; transmission element characteristics and generator variable costs were taken from the network as reported by Blumsack in [7].

The system consists of 118 buses, 186 transmission elements, 19 committed generators with a total capacity of 5,859 MW, and 99 load buses with a total load of 4,519 MW. Table I provides an overview of the components that are modeled within the IEEE 118 bus test case. All generators have a minimum operating capacity of zero MW.

[2] Losses are not modeled so an LMP gap exists only when the transmission element is capacity constrained or admittance constrained.

Table I. IEEE 118 Bus Test Case Data

		Capacity (MW)			Cost ($/MWh)	
	No.	Total	Min	Max	Min	Max
Transmission	186	49,720	220	1,100		
Generators	19	5,859	100	805	0.1897	10
Load	99	4,519	2	440		

Most of the generator costs, listed in [7], are around $0.50/MWh. A few generators have costs above $2.00/MWh. It is important to realize that the DCOPF solution with the original network is $2054/h for a 4519MW load. This places the average cost of energy at $0.455/MWh, which is about 1/50th of typical average costs for systems. The generator costs could all be scaled up to reflect a more typical average cost, thereby producing more significant dollar savings from transmission switching. The percent change, however, would be the same, which is the reason that the focus in this paper is on percent changes. In order to use a published source, we did not adjust the generator costs.

As previously stated, the objective is to minimize generation cost. The optimization problem was solved multiple times allowing for different number of transmission elements to be taken out. In particular, optimal solutions were found for J={0,…,10}, where J is the number of elements allowed to be open, enforced by (5). There is no guarantee that the generation cost will not increase as J increases since (5) is an equality constraint. Setting (5) to be an inequality (less than or equal to) constraint would ensure that the generation cost does not increase; however, an equality constraint reduces computation time and happened to produce the same results.

In the J=0 case, in which no transmission elements are opened, the system cost of meeting this load is $2,054/h. Two of the 186 transmission elements are fully loaded, or thermally constrained. The problem was run for J unrestricted as well, but it did not solve to optimality. The best found solution to the unrestricted J problem, which we reference in figures as "best," has a cost of $1542/hr with 38 transmission elements removed [2]. Solving the transmission switching problem to optimality is discussed in section V.

Figure 1 displays the fluctuations in generation cost, generation revenue, generation rent, congestion rent, and load payment for various solutions to the transmission switching problem for varying values of J (J={0,…10}) as well as two sensitivity cases discussed in later sections and the best found solution. Case 1 and case 2 are discussed in sections VI.C and VI.E respectively. Case 1 reflects a solution when a stopping time of 120 minutes is enforced while restricting the number of open lines to be less than or equal to 40. Case 2 reflects the solution when a heuristic approach is used to find a good solution fast and is represented by "iteration 3" in section VI.E. These sensitivity cases are presented here to increase the data for comparison. The values in the figure are displayed as percentages of results from the J=0 case, i.e. the percent values reflect the specific case's value divided by the J=0 case value. For example, generation rent is $1795/hr for J=0 and 122% of that, or $2192/hr, for J=3. Because these are percentages, the values shown do not add up in the way the actual values do; thus, the percentage value for generation rent plus cost does not add up to the percentage value for the generation revenue.

The case where J=0 has a generation cost of $2054/hr, generation revenue of $3850/hr, generation rent of $1795/hr, congestion rent of $3907/hr, and load payment of $7757/hr. Note that the congestion rent is unusually high; typically congestion rent is 5 to 10 percent of generation cost. The best found solution reduces operating cost to $1542/hr, which is 75.1% of the J=0 cost of $2054/hr.

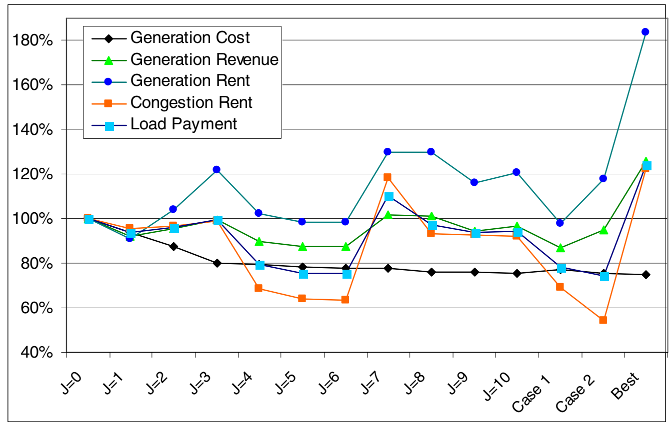

Figure 1. Generation Cost, Generation Revenue, Generation Rent, Congestion Rent, and Load Payment for Various Solutions

Results from Figure 1 indicate that for the majority of cases both the generators and the consumers are benefiting in comparison to the case with no open transmission elements. The generation rent is typically higher while the load payment is almost always lower than in the J=0 case. The load payment only increases in the J=7 case and in the best found solution. Congestion rent, in contrast, is generally lower than those calculated in the J=0 case. One key result is that generation revenue, congestion rent, load payment, and generation rent fluctuate dramatically as J changes. Figure 2 displays the differences, from all cases as described above, in generation cost, generation rent, and congestion rent; the load payment equals the sum of those three terms.

As was noted in [2], although the best found solution occurs when 38 transmission elements are opened, the majority of savings in generation cost can be realized when restricting J to a very small value, J=3. This suggests that a solution with significant savings can be obtained in a short amount of computational time. However, the preferred stopping criteria would be to allow the transmission dispatch program to run for the entire amount of time available. This is a potentially controversial choice precisely because of the fluctuations in payments. Take, for example, the significant differences in wealth transfers between cases J=6 and J=7 as shown by Figure 2. Even though there are only minor differences in the generation cost between solutions, the other results, i.e. load payment, generation revenue, and congestion rent, are very different. Unhedged loads would prefer J=6 to J=7, because they save nearly $2500/hr, yet generators would prefer J=7 to J=6, since the generation rent is higher.

Likewise, there are significant differences in wealth transfers between case 2 and the best found solution. Case 2 has a much lower load payment than the best found solution while the best found solution has only a $4/hr lower generation cost. Case 2 also has only 15 open transmission elements as compared to the 38 open elements for the best found solution.

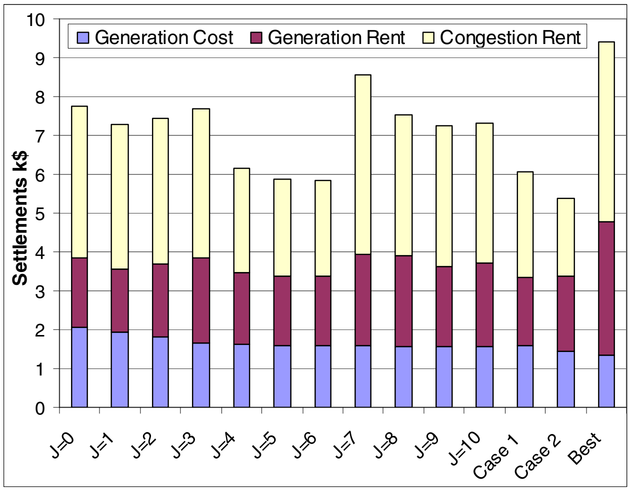

Figure 2. Load Payments and Wealth Transfers

B. LMP Results and Analysis

Figure 3 shows the average and extreme percent changes for the LMPs for J=1 through J=10 as compared to J=0. For J=7, the LMP for bus 77 changed from \$0.16/MWh to \$3.69/MWh, a 2159% change in LMP that is not shown in the figure. The best found solution had a 2271% change in LMP for bus 77 and this result is not shown in Figure 3 as well. Bus 77 has a load of 61 MW. Negative percent changes occur when the LMP decreases but also when there is a change in the sign. Negative percent changes greater than 100% reflect instances when the LMP sign changes; the percent change is calculated as the difference between the new LMP and the base (J=0) LMP divided by the base LMP.

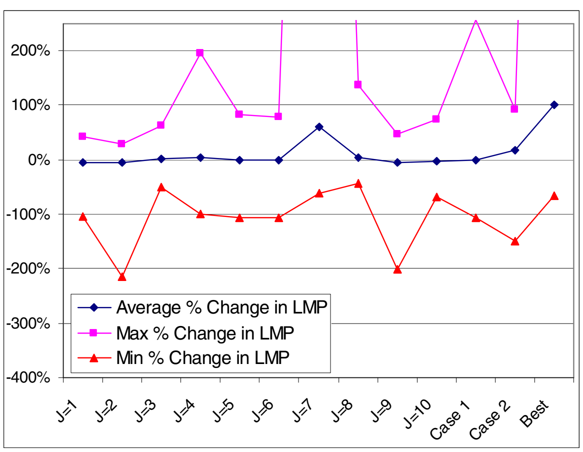

Figure 3. Average, Max, and Min Percent Change in LMP

Figure 4 shows the dispatched generator and load bus with the largest variance in LMP. The MW size of the load and generator are listed. The generator has an LMP of \$0.95/MWh in the base case but has an LMP that is more than 250% higher for J=7. This generator is the second largest generator and is fully dispatched, making the variation in LMP even more significant. The LMP at the load bus fluctuates from over \$6/MWh to -\$0.37/MWh. An LMP of \$6/MWh is high for this network, so this load may go from being charged one of the higher LMPs to being paid to consume.

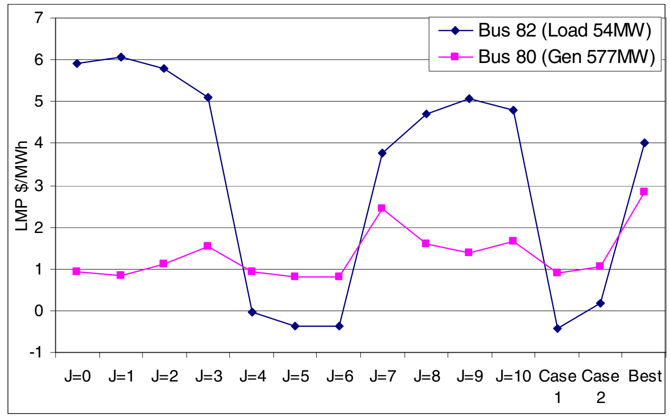

Figure 4. Dispatched Generator and Load Bus with Largest LMP Variance[3]

Figure 5 shows the LMPs for the buses that have some of the highest variances in LMP over values of J. Bus 83 experiences a \$3.16/MWh increase in LMP for J=6 but has a \$1.84/MWh decrease in LMP for J=7; these two solutions differ by \$5/MWh. Thus, bus 83, which is a load bus, faces a difference of \$5/MWh, and a difference in total payment of \$100/h, between these two solutions.

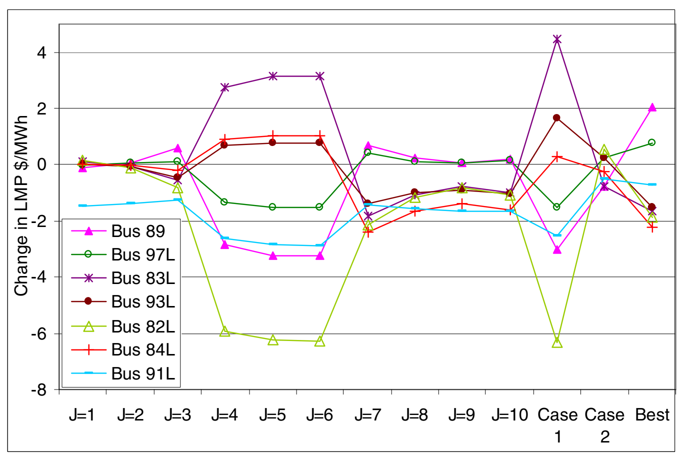

Figure 5. Change in LMP Values

C. Congestion Rent Results and Analysis

There are many dramatic changes in congestion rent for individual transmission elements for the various solutions. Some transmission elements go from having almost no congestion rent with the static network to having congestion rent with a transmission switching solution and vice versa. For example, there were transmission elements that had a percent increase or decrease in congestion rent that exceeded 10,000% due to the congestion rent being very small in the static network solution. Table II lists the number of capacity-constrained, i.e. fully loaded, transmission elements and the number of elements with non-zero congestion rent.

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[3] The buses with the largest variance (mean square deviations) in LMP for these two categories are displayed.

Table II. Number of Thermally Congested Transmission Elements and Elements with Congestion Rent

Elements	J=0	J=1	J=2	J=3	J=4
With Congestion Rent	161	155	159	155	154
Capacity Constrained	2	2	2	3	3
	J=5	J=6	J=7	J=8	J=9
With Congestion Rent	152	149	151	147	137
Capacity Constrained	2	2	4	3	3
	J=10	Case 1	Case 2	Best	
With Congestion Rent	133	108	102	97	
Capacity Constrained	4	2	4	4	

Figure 6 shows congestion rent for transmission elements with large variations for the various solutions. Negative congestion rent values reflect a power flow that is flowing from an expensive bus to a cheaper bus. Congestion rent on the transmission element between bus 77 and 82 changes from positive to negative. The power flow direction is the same for J=0 and J=6, in the direction of bus 77 to 82; however, the high LMP is at bus 82 for J=0 and at bus 77 for J=6. Thus, the flow is from the cheap bus to the expensive bus in J=0 and from expensive to cheap in case J=6. The transmission element connecting bus 92 and 89 has its congestion rents ranging from 23% to 133% of the base case throughout the solutions. This further demonstrates the uncertainty an unhedged market participant may face once the topology is changed, thereby increasing the incentive to hedge.

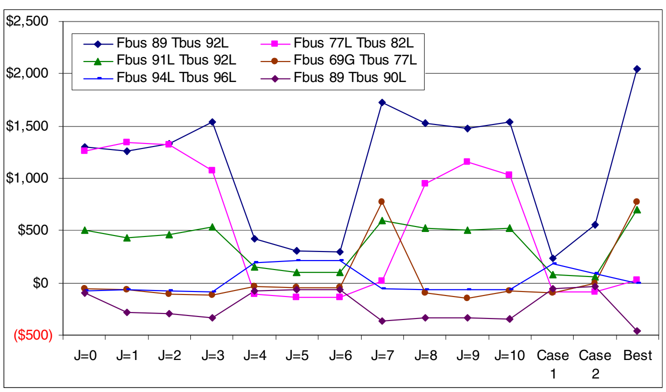

Figure 6. Congestion Rent Fluctuations

Market participants that hedge by holding FTRs are not affected by transmission switching as the hedge is maintained as long as there is revenue adequacy. The FTR purchase price may change due to the change in congestion rent; however, the hedged position is still maintained. Even though the congestion rent fluctuates as shown by Figure 6, all solutions have been tested and revenue adequacy has been maintained though a formal proof of revenue adequacy has not been developed yet.

D. Flowgate Results and Analysis

Unlike congestion rent, which can accrue to a transmission element whether it is thermally constrained or not, a flowgate marginal price (FMP) is non-zero only when the thermal constraint is binding. Despite the large changes in LMPs and congestion rents, only seven unique transmission elements are congested in all cases J={0…10}.

In cases J=0 through J=3 the transmission element connecting bus 77 to bus 82 has a relatively high FMP, then the transmission element is uncongested for J=4 through J=6, and is congested again for J=7 and higher. Another interesting result is the change in FMP for the transmission element connecting buses 92 and 89. The FMP stays relatively high at first but drops significantly for J=4 through J=6 and then becomes one of the larger FMPs again from J=7 on. These examples demonstrate the uncertainty in FMPs when the topology is changed as well as the variation in results from one minor change in topology to the next.

E. Why is Transmission Switching Beneficial?

It may not be perfectly clear why transmission switching can provide savings with well planned networks. First, we used the IEEE 118 bus test case as it is a standard IEEE test case. It may not be as well planned as practical networks but we have tested our model on a 5000 bus ISONE network and have found significant results with this well planned, practical network. Second, it is important to understand that the OPF equations ensure that load is always satisfied no matter the configuration of the network; thus, there is no load shedding. Last, and more importantly, transmission planning is supposed to determine the best network configuration by examining the total benefits over many future years. This is a long run problem, which is different from transmission switching, which deals with the short run problem of finding the best configuration for a specific hour. This is one key reason why transmission switching can provide benefits even in well planned networks. Transmission switching can also be beneficial since transmission planning is very difficult based on the great uncertainty of future network conditions.

V. Computational Statistics

Generally, large production systems cannot be solved to optimality or optimality cannot be proven even if it is found. In theory, transmission switching is NP hard and no special structures or techniques have yet been developed to solve it quickly to optimality. The IEEE 118 bus test case could have 2^{186} (or approximately 10^{56}) alternate topologies. Even if each problem could be solved within one pico second, it would take 10^{27} billion years to solve the IEEE 118 bus test case to optimality by complete enumeration. Nevertheless, solutions that improve on the static case can be found in reasonable time. The practical implementation of transmission switching would be to allow the solver to find the best solution within the full time available. Since finding and especially proving optimality is unlikely, this creates the need to analyze the intermediate, suboptimal results that improve on the static topology.

Table III lists the number and type of variables and constraints within this problem. The J=0 case is a linear program (LP) while the rest of the test cases, J=1 through J=10, have the same number of variables and constraints and are mixed integer programs (MIP). Redundant variables and constraints are eliminated during the presolve phase of the problem, conducted automatically by AMPL. The residual variables and constraints are identified in Table III by "post presolve." The computer specifications are listed in Table IV.

Table III. LP and MIP Variables and Constraints

IEEE 118	LP	MIP
Total Variables:	323	509
Binary Variables:	0	186
Total Linear Constraints:	627	1000
Upper or Lower Bound Constraints:	323	509
Total Variables (Post Presolve):	315	492
Binary Variables (Post Presolve):	0	177
Linear Constraints (Post Presolve):	482	833

Table IV. CPU Specifications

	Type 1	Type 2	Type 3
No. processors	2	2	4
CPU speed	3.4 GHz	2.8 GHz	2.8 GHz
Total memory	1.0 GB	2.1 GB	2.1 GB

Figure 7 displays the computational statistics, which includes solution time, the total number of simplex iterations, and the number of branch and bound nodes for solving J=0 through J=10 to optimality.[4] The computational statistics for the best found solution are listed in [2]. J=0 is a LP so the number of branch and bound nodes is zero; this value is not shown on Figure 7 as it is in a log scale. CPLEX has a default relative MIP gap of 0.01%. The relative MIP gap is an optimality gap and without adjusting this parameter within CPLEX, the program terminates once the gap is less than 0.01%. J=0 through J=10 were solved both with the gap set at 0.01% and 0.0%, i.e. they were solved to optimality. The solutions found with a gap of 0.01% were the same as the optimal solutions but optimality had not been proven.

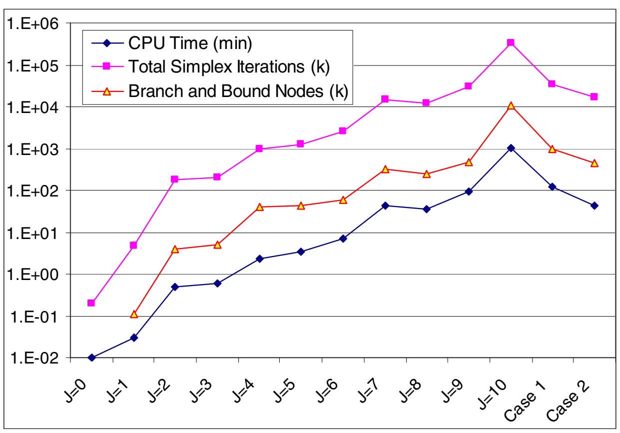

Figure 7. Computational Statistics on Computation Time, Simplex Iterations, and Branch and Bound Nodes

VI. Sensitivity Results

Various sensitivities were tested on this model. In the first sensitivity, the model formulation itself is examined; we test the formulation of the bus angle constraints to see if it has an affect on the results. We then look at the sensitivity of the model results to the particular data being used by running the OPF with different generator costs. Next, we examine possible strategies for improving the trade-off between run time and the objective function. Last, we analyze different load levels. The solutions within the sensitivity studies are stopped once they reach a relative MIP gap of 0.01%.

A. Bus Angle Difference Constraint

The initial model implements a constraint that limits the magnitude of the bus angle to be less than 0.6 radians.[5] In this sensitivity, the previous absolute angle constraint is removed and there is a new constraint that limits the difference of bus angles between connected buses to be within the range of ±0.6 radians. The J=0 case serves as the base case for this sensitivity, with a generation cost of $2053/hr, generation revenue of $3684/hr, generation rent of $1630/hr, congestion rent of $3855/hr, and load payment of $7539/hr. Figure 8 shows the values for J={1…10} as a percentage of the values obtained from J=0 for this sensitivity.

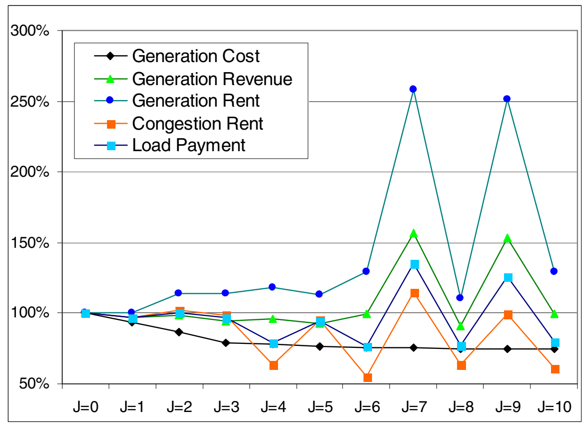

Figure 8. Value Propagations with J for Bus Angle Sensitivity

By placing the constraints on bus angle differences instead of on the overall bus angle, the DCOPF solution for the static network is lower. At first, there is no significant difference between the original model's results discussed in section IV and this model's results; however, there are significant differences for the middle solutions, J=5 through J=7. Just as before, there is no clear trajectory for the various values. J=7 demonstrates a large jump in load payment while J=8 has a small value and J=9 has a significant increase like J=7. Again, the different topologies will benefit certain unhedged market participants and hurt others and, at the same time, one cannot predict who will benefit and who will not. Last, it is important to note that none of the bus angle difference constraints were active for any of the solutions. Thus, these solutions are the same as solutions that would be obtained if the bus angle constraints were removed from the formulation. It is atypical to have no active bus angle constraints. Thus, one should consider whether the solution is a good approximation of the ACOPF or whether this is a unique result for this specific test case.

B. Expensive Generators

Next, we examine the effect of the five most expensive generators. The majority of generators have costs that are less than $1/MWh but there are five generators with costs from $2

[4] Note that these results are run on various shared computers that face different loading levels at different times so the CPU time may not be the best indicator of the difficulty of the problem; total simplex iterations and branch and bound nodes are also indicators of the difficulty of the problem.

[5] Note that the initial model from section IV has binding bus angle constraints.

to \$10/MWh. The costs of these five expensive generators were halved to examine whether the large generator costs are the main driving force for the large generation cost savings as was seen in section IV. The J=0 solution for this sensitivity provided a total generation cost of \$1669/hr, generation revenue of \$3469/hr, generation rent of \$1800/hr, congestion rent of \$1690/hr, and load payment of \$5159/hr. By comparing Figure 9 and Figure 1, it is clear that the percent saving in generation cost for this sensitivity is not significantly smaller than the saving in the original study. This suggests that the existence of the expensive generators is not the driving force behind the large percent savings. However, additional research would be needed to determine whether dramatic savings are a feature of dispatchable transmission in general.

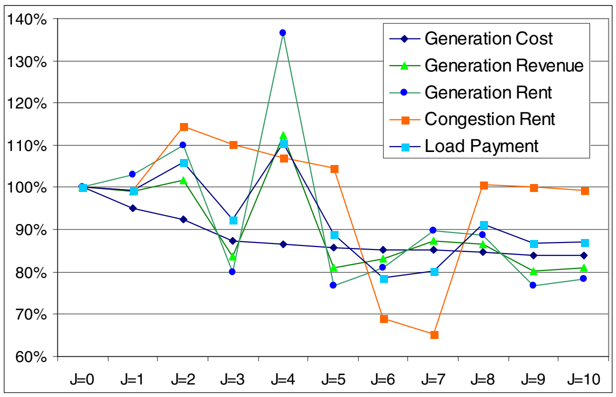

Figure 9. Value Propagations with J for Expensive Generators Sensitivity

C. *Stopping Times*

This sensitivity study examines the practical implementation of this model. In production systems, the system operator would look for a solution within a certain timeframe. Thus, the model was simulated for different stopping times to analyze how the results may differ.[6] From the original model, J=3 provides an objective that is less than \$1650 within one minute. A bound of \$1650 is placed on the objective. This cuts the computation time as it eliminates the exploration of inferior branches. The results are displayed in Figure 10. The original results for the model presented in section IV are listed in Figure 10 as Base Case J=0. The number of transmission elements chosen to be opened in the current solution that were also opened in the previous solution is displayed in Table V along with the total number of open transmission elements in parentheses. The 120 minute solution was listed as case 1 in sections IV and V.

The solution decreases substantially in the first five minutes and then plateaus. The solution for 15 minutes and 30 minutes are the same. Thus, it appears that while a good solution is found quickly, more solution time does not result in significant improvements to the solution.

Allowing the program to run for 2 hours provides an inferior solution to the result from J=9, which has about the same solution time but a lower generation cost. The goal is to find a good solution as fast as possible, but additionally we may want to minimize the number of open transmission elements for reliability reasons. In other words, if two solutions have near-identical objective functions the one with fewer open transmission elements may be preferred. These factors suggest that letting the algorithm run for a specific amount of time while not limiting the number of open transmission elements may not be the best approach. The best approach may need to be tailored to specific networks, a possible area of future research into the best practical approach for this problem. Section VI.E. discusses a simple heuristic approach that can not guarantee optimality; however, it provides a near optimal solution within a short time.

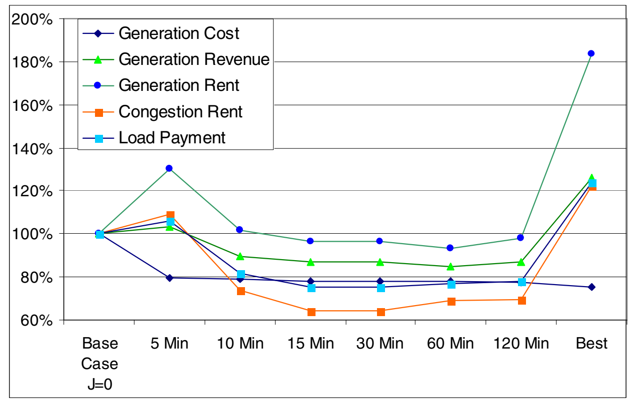

Figure 10. Value Propagations for Various Stopping Times Compared to Original Base Case J=0 for Stopping Time Sensitivity

Table V. Number of Open Transmission Elements in Previous and Current Solution (Total Open Elements)

5 min	10 min	15 min	30 min	60 min	120 min
0 (36)	12 (34)	24 (36)	36 (36)	17 (35)	13 (35)

D. *Open One Transmission Element at a Time*

In [2], it was shown that the set of open transmission elements for smaller J values are not subsets of the solutions for cases where J is larger. Therefore, an approach to solve for the next best element to open cannot guarantee optimality but may provide good solutions fast. This can be seen in Table VI, which shows changes in transmission element status for the results from section IV, J={1…10}. Transmission element out (or in) reflects an open (closed) element in the present solution that was a closed (open) element in the previous solution.

Table VI. Change in Transmission Element Status from Previous Case: Element Out (Element In)

	J=1	J=2	J=3	J=4	J=5
Elements	153	132	136	162	64
	J=6	**J=7**	**J=8**	**J=9**	**J=10**
Elements	70	86, 146 161 (70, 162)	35, 38, (86)	72	34, 67, 68, (35, 72)

E. *Open Five Transmission Elements at a Time*

As previously stated, optimality cannot be guaranteed by taking a previous solution and building on from it by allowing more transmission elements to be opened. For practical implementation, the goal is to obtain a good solution within the time available. This section presents the results of using an iterative approach by taking the previous solution as fixed and solving for the optimal topology based on allowing a certain number of additional open transmission elements. The chosen

[6] To reduce computational burden, no more than 40 transmission elements were allowed to be open.

number of elements to be removed for each iteration would depend on the tradeoff between solution time and the generation cost savings. For this test case, we solve for the optimal topology when 5 transmission elements are allowed to be opened, fix the solution's chosen 5 transmission elements as open, and then repeat the process by allowing an additional 5 transmission elements to be opened.

Figure 11 presents the results as percentages of the original J=0 case presented in section IV, identified as Base Case J=0 in the figure. After three iterations, this method produced a generation cost of $1549/hr, generation revenue of $3662/hr, generation rent of $2113/hr, congestion rent of $2111/hr, and load payment of $5773/hr. The solution "iteration 3" is referred to as "case 2" in sections IV and V.

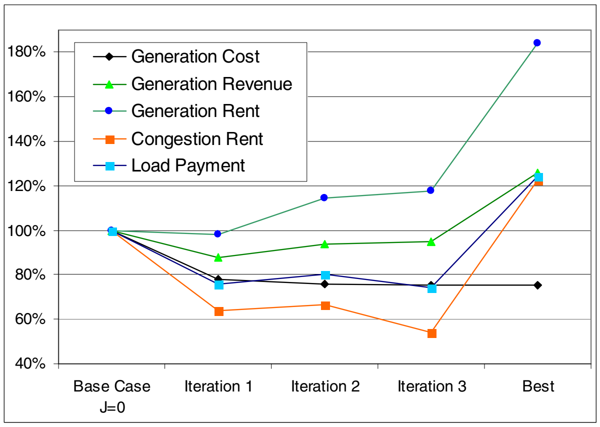

Figure 11. Value Propagations for Base Case and Heuristic Approach Sensitivity Study

The first iteration is the same as the original solution for J=5 in section IV. The second iteration takes the chosen open transmission elements from iteration one, fixes them, and allows five additional transmission elements to be opened, making the total open transmission elements to be ten. Note that this second solution is not the same as J=10 in section IV. This process is repeated for every iteration. After only three iterations, this method has achieved 24.6% savings and it is within $4 of the best found solution. This was accomplished within 42 minutes, as opposed to taking more than 2 hours to achieve similar results based on the original method. The fourth iteration was infeasible after 20 hours with over 10 million branch and bound nodes. This suggests that the solution from this method cannot be improved after three iterations, or that the additional benefit is not worth the solution time. This is not a generic result for this approach but a specific result for using this approach on this specific network.

This simple heuristic approach demonstrates the practical possibilities of better heuristics obtaining very good solutions within a moderate amount of time. The data in Table VII shows that the chosen transmission elements are not necessarily the same as the open transmission elements of the current best found solution reported in [2], but that the approach still provides significant savings. To get such results in a short period of time took some experimenting by us. We do not claim that this is the best heuristic or that such an approach would work for practical problems. The purpose of this sensitivity is to demonstrate the value of good heuristic methods as well as the need to investigate such valuable approaches.

Table VII. New Open Transmission Elements from Previous Iteration

Iteration No.	1 (J=5)	2	3
Transmission Elements	64, 132, 136, 153, 162	35, 38, 86, 148, 161	70, 73, 85, 91, 126

F. Load Analysis

Sensitivities at 10% below and 10% above the base load for the IEEE 118 bus test case were presented in [2]. The off-peak load produced a 17% savings with 4 transmission elements opened and the peak load produced 12% savings with 5 transmission elements opened. If the load was decreased by 20%, the IEEE 118 bus test case produces a DCOPF solution of $875.61/hr. This solution is only $4/hr higher than the unconstrained economic dispatch solution for this load level, which is $871.63/hr. Transmission switching adds flexibility to the optimization problem in order to decrease the cost but the lower bound is the unconstrained economic dispatch solution. Transmission switching brings the network to the unconstrained economic dispatch solution by opening just two transmission elements and saving 0.5% of the DCOPF solution when the load is decreased by 20%.

For this lower load level, it seems that transmission switching has little value. We, however, contest that this is a result based on the design of the IEEE 118 bus test case, as most networks do not have such flexibility that the unconstrained economic dispatch solution is only 0.5% below the DCOPF solution when the load is at 80% of the annual peak load. We are currently investigating transmission switching applied to a 4896 bus model of the ISONE network and surrounding areas. The DCOPF solution when the load is 80% of the peak load is still much higher than the unconstrained economic dispatch solution, meaning that there is still the possibility for transmission switching to be beneficial. For this ISONE network model, when the load is set to be 80% of the peak load, the unconstrained economic dispatch cost is 55% of the DCOPF cost. As previously mentioned, for the IEEE 118 bus test case, the unconstrained economic dispatch is 99.5% of the DCOPF leaving little room for transmission switching to be beneficial.

VII. Policy Implications

The results throughout this paper demonstrate the uncertainty and potential variability in the total and individual values of generator revenues and rents, congestion rents, and load payments, even when the total generation costs differ only slightly.[7] LMPs are sensitive to the particular topology selected and the selected topology depends on the stopping criteria and solution heuristics.

Since individual bus LMPs may change dramatically between solutions, the chosen network dispatch may cause significant fluctuations for unhedged market participants. Determining the optimal topology for a practical network is

[7] These results are consistent with those found by Sioshansi et al. in [8].

likely to be extremely time consuming, if not impossible; thus, a certain amount of discretion will exist in choosing the solution, e.g. when to stop, how many transmission elements to open, etc. The opportunity exists to change the topology of the grid strategically to make one participant better off while making others worse off. These topology decisions, therefore, should be made by an independent party with no financial interest in the settlement.

Even still, determining a stopping criterion may be a very sensitive topic. An operator may have a choice between solutions with system costs that differ by a trivial amount but have significant wealth transfers, as was evident in Figure 2 with case 2 as compared to the best found solution. This hypothetical situation raises the question as to whether an operator should care only about minimizing generation cost. Perhaps there should be additional objectives, such as minimizing the number of open transmission elements or the load payment. Identifying additional objectives is a job for policy-makers and would depend on societal values or objectives for the market.

Another implication of these potentially volatile and unpredictable prices is the need for a forward market in which to hedge the real-time prices. Real-time prices are useful as marginal indicators, sending financial signals to users and suppliers to alter behavior based on real-time conditions. However, it is likely that most risk-averse market participants will want to hedge the risk of volatile real-time prices by trading in longer-term forward markets in which the negotiated or clearing price will be less uncertain. Forward contracts can also help to suppress the ability and incentive to exercise market power, Allaz and Vila [9].

Point-to-point financial transmission rights are common in many restructured markets, and allow market participants to hedge forward contracts or speculate on price differences. Typically in markets that employ FTRs, the system operator auctions the rights before the real-time dispatch of the network is determined. Revenue inadequacy can be caused by inaccurate PTDFs and derated thermal capacities. Inaccurate PTDFs can be caused by variable impedance devices like phase shifters, an unexpected transmission element outage, etc [10]. To achieve revenue adequacy for transmission rights, the system operator may need to implement a rationing/wealth transfer rule, such as pro-rating FTR payments, which can be controversial because it can affect the financial positions of generators, consumers, and FTR holders.

The implementation of transmission dispatch should not affect the normal FTR mechanism, since the presence or absence of a transmission element does not eliminate the existence of point-to-point differences in prices. Market participants that hold FTRs for hedging face risk due to the policy on revenue inadequacy of transmission rights. Empirical evidence suggests that, as long as revenue adequacy is maintained for the static network, there will be revenue adequacy for the transmission switching solution that produces a higher social surplus. The empirical evidence therefore suggests that transmission switching does not increase risk for hedgers though a formal proof of revenue adequacy has not been developed yet. Speculators holding FTRs, on the other hand, may be exposed to additional risk due to added uncertainty of LMPs.

Volatile LMPs may not be altogether negative. Not only do they create an incentive for market participants to hedge and sign long-term contracts, they may also make strategic behavior more difficult. In a sense, transmission dispatch allows transmission elements to compete in the market dominated by generators. The introduction of additional competitors can reduce the influence of existing competitors, thus limiting their ability to control prices.

Ultimately, optimizing the transmission network can result in a more economically efficient system, otherwise the network will not be altered. A more efficient system dispatch produces more surplus, and with more surplus, it is possible for the ISO to implement wealth transfers that result in Pareto improvements for all market participants. While questions of surplus allocation are not a concern for completely vertically integrated utilities, since in a sense all surplus accrues to the utility, transmission dispatch would still be beneficial.

VIII. Current and Future Work

We are currently working on an N-1 DCOPF formulation with transmission switching, Hedman et al. [11], for the IEEE 118 bus test case and the Reliability Test System 1996 [6]. We are also working on a paper that is applying the DCOPF transmission switching problem to ISO networks; we also discuss simple heuristic techniques that can be used to find good solutions fast and include a short discussion on MIP versus LaGrangian relaxation (LR), Hedman et al. [12].

Future work should consider the impact on reliability when changing the topology of the network. An open transmission element may help or hurt reliability. A security constrained optimal power flow may be needed to determine which transmission elements can be opened. The results show that there can be significant savings by having only a few open transmission elements and since solution time would be an issue for large networks, it may be possible to focus only on key transmission elements that do not affect reliability.

Multiple solutions can provide similar objectives even though the number of open transmission elements can be dramatically different. Research is needed to determine whether it is appropriate to not only minimize generation cost but also minimize the number of open transmission elements or some other objective that would consider the impact on reliability among these similar solutions.

Stability studies should also be investigated to determine the impact of having a significant number of open transmission elements and having to close them during a disturbance. Likewise, it is important to analyze the effects on reactive power, voltage stability, etc at varying load levels. The capacitive component of the transmission element is predominant at lower load levels and the reactive component is predominant at higher load levels, thereby causing different effects on the network when the element is removed during different load levels. Future work should also include the development of an ACOPF transmission switching model, incorporate generation unit commitment, and analyze the effects of transmission switching over multiple periods.

IX. Conclusion

This paper has demonstrated the uncertainty market participants may be subject to when the topology of the network is modified by choosing certain transmission elements to be open in order to achieve a better dispatch. Analysis of the IEEE 118 bus test case results in higher generator payments and lower load payments, in general, when the transmission network is optimized.

An important finding of this paper, and one that is consistent with [8], is that nodal prices can vary dramatically between topologies, even topologies that have similar system costs. Thus, the economic impact on market participants relies heavily on topology and can be unpredictable in the real-time or short-term market. This implies that (1) an unbiased and independent actor with no financial interest in market settlement should be in charge of determining topology to avoid intentional manipulation, and (2) hedging will be more important for market participants. We also find that heuristics may play an important role in solving the transmission switching problem.

Since the problem is, in theory, NP hard, optimality would most likely not be achieved in a production setting; however, significant savings can still be achieved in reasonable amounts of time as has been shown by the results presented in this paper. If an optimal solution is not found, allowing the solver to use all the time available appears to be the best strategy. These solutions, while improving system cost, can have drastically different implications for varying market participants; this may make determining the appropriate stopping criteria a controversial policy decision.

There are many potential policy implications of transmission dispatch. Market participants would need to be informed of the potential for volatile real-time prices so they can prepare by signing forward contracts or other long-term agreements. Criteria for choosing an optimal topology would need to be clear and unbiased. Some potential benefits, in addition to increased market surplus, may include reduced exercise of market power, due to an increase in long-term contracts and uncertainty in real-time prices.

Overall, we have discussed benefits of transmission switching, the potential financial impacts on market participants, the new decisions and questions that a system operator may face given a change in topology, and whether such an approach can be applied in a practical setting. This paper has also identified future areas that need to be studied to determine the overall impact of transmission switching.

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