Economic Consequences of Alternative Solution Methods for Centralized Unit Commitment in Day-Ahead Electricity Markets

Ramteen Sioshansi, Richard O'Neill, and Shmuel S. Oren, *Fellow, IEEE*

***Abstract*—Many wholesale electricity markets call on the independent system operator (ISO) to determine day-ahead schedules for generators based on a centralized unit commitment. Up until recently, the Lagrangian relaxation (LR) algorithm was the only practical means of solving an ISO-scale unit commitment problem, and it was the solution technique used by most ISOs. Johnson *et al.* [1] demonstrate, however, that equity, incentive, and efficiency issues will arise from use of LR solutions, because different commitments that are similar in terms of total system costs can result in different surpluses to individual units. Recent advances in computing capabilities and optimization algorithms now make solution of the mixed-integer programming (MIP) formulation by means of branch and bound (B&B) tractable, often with optimality gaps smaller than those of LR algorithms, which has led some ISOs to adopt B&B algorithms and others proposing to do so.**

With the move towards B&B, one obvious question is whether the use of MIP will eliminate or reduce the issues with LR raised by Johnson *et al.* Using actual market data from an ISO, we demonstrate that both LR and MIP solutions will suffer the same equity issues, unless the ISO unit commitment problems can be solved to complete optimality within the allotted timeframe—which is beyond current computational capabilities. Our results further demonstrate that the size of the payoff deviations are not monotone in the size of the optimality gap, meaning smaller optimality gaps from B&B will not necessarily mitigate the issues Johnson *et al.* raise. We show that the use of "make-whole" payments, which ensure units recover any startup and no-load costs not recovered by inframarginal energy rents, can help to reduce surplus volatility and differences to some extent.

***Index Terms*—Electricity market design, power systems economics, unit commitment.**

I. Introduction

With many jurisdictions moving towards competitive wholesale electricity markets, an important but contentious issue has been the proper role of the independent system operator (ISO) in determining unit commitments. In many restructured systems, an ISO operates central energy markets and has the authority to commit and schedule generators based on load forecasts and multipart offers, including nonconvex costs and unit operating constraints. Until recently, the Lagrangian relaxation (LR) algorithm was the practical means of solving a commercial-scale unit commitment. Johnson *et al.* [1] demonstrate, however, that centralized scheduling of resources owned by multiple parties by means of an LR algorithm may face difficulties that do not arise when resources are centrally owned. A case study based on load data and a stylized generator set from Pacific Gas and Electric Company shows that variations in near-optimal unit commitments that have negligible effects on total system costs could yield significantly different payoffs to individual resources—meaning the details underlying the solution methodology could impact which generators are "winners" and "losers" in dispatch determination. Johnson *et al.* [1] further raise an incentive issue: generators, knowing the dispatches and payoffs resulting from LR solutions are sensitive to the specific solution found, may profitably misrepresent their cost and constraint parameters to affect the outcome of the market. Indeed, Newbery [2] notes that one of the criticisms of the original British Electricity Pool was that generators were able to manipulate their offers to maximize their payoffs. The potential for misrepresented bids would, of course, call into question the efficiency of the resulting commitment, since it may potentially be based on incorrect cost and constraint parameters. Another issue complicating a market with nonconvex costs is that linear price payments that compensate generators only for energy produced may be confiscatory, a prime example of which is a near-marginal unit which does not receive sufficient inframarginal rents from linear prices to recover its offer-based fixed costs. These issues raise concerns regarding the feasibility of proper mechanisms to oversee an equitable centrally committed market and generators' incentives to submit truthful offers in such a market—calling into question the efficiency of the underlying unit commitment solution.

One of the issues that has traditionally plagued the use of mixed-integer programming (MIP) in solving unit commitment problems has been the inability of branch and bound (B&B) algorithms to provide a solution within a reasonable amount of time. Because many markets frequently resolve commitment and dispatch problems with limited solution times, ISOs rely on software to provide a feasible and near-optimal commitment within a short time frame. Streiffert *et al.* [3] demonstrate, however, that recent advances in computing capabilities and improvements in optimization algorithms now allow MIP to be a

Manuscript received April 11, 2007; revised September 13, 2007. This work was supported in part by the NSF under Grant ECS0119301, in part by the Power System Engineering Research Center, and in part by FONDECYT under Grant 11060347. Any opinions and conclusions expressed in this paper are those of the authors and do not necessarily represent those of ISO New England or any other entity. Paper no. TPWRS-00271-2007.

R. Sioshansi and S. S. Oren are with the Industrial Engineering and Operations Research Department, University of California, Berkeley, Berkeley, CA 94720-1777 USA.

R. O'Neill is with the Office of Energy Markets and Reliability, Federal Energy Regulatory Commission, Washington, DC 20426 USA.

Digital Object Identifier 10.1109/TPWRS.2008.919246

viable alternative to LR. Even if the B&B algorithm times-out before finding an optimum, one is still left with a primal-feasible solution and a bound on the optimality gap.[1] These intermediate solutions are often found within the same amount of time an LR-based algorithm takes, and they typically have optimality gaps of the same size or smaller than LR commitments. Moreover, Streiffert *et al.* [3] and Li and Shahidehpour [4] note that B&B benefits more in comparison to LR from having additional solution time, as the B&B algorithm is able to find better solutions or tighten the optimality gap with additional time. Streiffert *et al.* [3] also note that a MIP-based algorithm can represent complex units such as combined-cycle combustion turbines (CCCTs), pumped storage, and cascaded watershed hydro systems better than LR can. The units within a watershed hydro system, for instance, violate the problem decomposition assumptions underlying LR, meaning approximations such as peak-shaving are typically employed to use an LR algorithm in systems with these types of units. CCCT units also violate the decoupling assumption and require special algorithms, such as that proposed by Lu and Shahidehpour [5]. A MIP-based implementation does not require such special algorithms and can typically more fully represent complex units. Finally, a MIP-based solution algorithm allows ISOs to easily introduce new types of unit-operating and system constraints to the formulation of the problem, whereas LR-based techniques generally require extensive reprogramming of the feasibility heuristics to ensure that the final commitment satisfies all the necessary conditions. These overwhelming advantages and the tractability of MIP algorithms have led several ISOs, such as PJM, to implement MIP-based solution methods as opposed to LR. Moreover, the California ISO's Market Redesign and Technology Update and the new ERCOT nodal market will feature centralized commitment solved using MIP, and ISO New England (ISONE) is similarly exploring the switch from LR to MIP.

Due to the computational complexity of unit commitment problems and limited solution times, ISOs which implement B&B-based algorithms do not solve their unit commitment problems to complete optimality or prove that the best solution found is optimal. PJM, for instance, allows its MIP optimizer to run within a certain period of time or until the optimality gap is below some maximal threshold, and uses whatever intermediate integer-feasible solution the solver has found. An obvious issue raised in using MIP to solve the commitment is whether its use of the B&B algorithm will mitigate or eliminate the issues raised by Johnson *et al.* [1], especially in light of the fact that ISO commitments are not solved to complete optimality using MIP.

We revisit these issues raised in the design of a central unit commitment market by studying test problems from an ISONE data set. We examine properties of a simple unit commitment problem, which is meant to be comparable to the problem studied by Johnson *et al.* [1], by comparing LR solutions to the actual MIP optimum. Our results are consistent with those in Johnson *et al.* [1], showing that LR solutions with near-optimal system costs can result in vastly different payoffs for the individual units, both when the LR solutions are compared to one another and to the MIP optimum. We further show these payoff differences are due not only to units being committed and dispatched suboptimally in the LR solutions but also because of different energy prices resulting from the suboptimal LR dispatches.

We show that these market design issues can be reduced in two ways. First, the use of a B&B algorithm which can solve a unit commitment to optimality will guarantee units receive their optimal dispatches and payoffs. We demonstrate, however, that this relies crucially on solving the problem to complete optimality. If the B&B algorithm times-out or is interrupted before reaching optimality and is just a minuscule fraction of a percent away from optimal, the payoffs to individual units and energy prices can vary significantly when compared to the MIP optimum.[2]

Secondly, due to the possible confiscatory nature of linear energy-only price payments, most ISOs which operate a central commitment market make supplemental "make-whole" payments, which guarantee that units recover their offer-based costs over the course of the day. We argue that such a make-whole provision has an added benefit of helping to "smooth out" the payoffs of near-marginal units which may not receive their MIP-optimal commitment in a suboptimal LR or intermediate MIP solution. Because such a unit would be close to marginal, it would likely require a make-whole payment, implying that the unit breaks even (in terms of offer-based costs), regardless of whether or not it is dispatched in the final commitment. We show that while such make-whole provisions reduce payoff deviations between the MIP optimum and suboptimal commitments, such deviations will not generally be completely eliminated.

II. Properties of LR and B&B Solutions

At its heart, the unit commitment problem finds the least-cost commitment and dispatch of a set of generating units to meet expected load over a time horizon consisting of a fixed number of periods, which we take in our computations to be 24 single-hour periods. The problem can be formulated as an MIP in which the operating status of each unit (online or offline) in each planning period is characterized by a set of binary variables, and a set of continuous variables indicate the generating output of each unit in each planning period. In addition to a load balance constraint, which ensures that expected demand is met in each period, unit commitment formulations will typically also have ancillary service requirements, upper- and lower-generating capacities for each unit, ramping constraints, minimum up and down times when units are started and stopped, startup costs which are dependent on the length of time a unit has been offline, and transmission network constraints.

Historically, solving a commercial-scale problem with hundreds of generating units was impractical using a B&B algorithm. As such, LR techniques were employed in which

[1]In theory, the B&B algorithm may time out before finding an integer-feasible solution, in which case heuristics or an alternative solution method would have to be employed. Nonetheless, PJM currently relies on B&B, and the California ISO and ERCOT are planning to use MIP in their market redesigns, which are to be implemented in 2008, since this issue does not tend to arise in practice.

[2]This can also be true for alternative optimal MIP solutions, if the optimum is non-unique.

a Lagrangian dual is obtained by relaxing the load balance, reserve, and any other "coupling" constraints and penalizing violations in the objective. When these constraints are relaxed, the problem decouples in the sense that there is no longer an interdependency between the generating units, making the dual problem relatively simple to solve. The LR algorithm then works iteratively to try and find a set of energy and reserve capacity "prices" (the objective function penalty coefficients), which incent an optimal commitment and dispatch of the units. As Wolsey [6] notes, the LR algorithm finds a dual-feasible/primal-infeasible solution which is a local (but not necessarily global) optimum, with additional processing of the LR solution and heuristics needed to restore primal-feasibility.

Our results comparing LR and MIP solutions are based on a simplified model of the ISONE commitment problem, which includes minimum up and down times, ramping constraints, hourly load balance constraints, and a single type of load-based ancillary service requirement. To simplify the model, marginal generating costs are assumed to be constant, startup costs are not time dependent, and transmission constraints are ignored—making our model similar to that studied by Johnson *et al.* [1]. The specific formulation studied is given in the Appendix.

Johnson *et al.* [1] further simplify their model by eliminating never-used resources and units which are not economically dispatchable from the generator set and only considering 17 units in their commitment problem. Our model, by contrast, includes the full ISONE generating set of 276 dispatchable units. This both reflects our desire to study a commercial-scale unit commitment problem and to highlight drastic improvements in the capabilities of MIP solvers, which could not solve a unit commitment problem of this size a decade ago.[3] The data used are based on actual cost and operating constraint offers, which are submitted by generators to ISONE on a daily basis. Because the cost parameters used in the unit commitment consists of offered costs, they may not reflect a unit's actual operating costs. As such, our calculations of a unit's net payoffs will not necessarily reflect revenues less costs, but rather its "offer-based surplus."[4]

A. Comparisons of LR Solutions

The LR algorithm we employ is a "textbook" subgradient algorithm which utilizes a geometric step-size sequence, as described by Wolsey [6]. A myopic recommitment heuristic is then used to obtain a primal-feasible commitment from the dual solution, and finally, a linear program is used to determine a least-cost dispatch based on the fixed primal-feasible commitment of the units. Although our LR algorithm is not finely tuned, the results we report are for solutions which are all within 1.82% of the MIP optimum. The different solutions are obtained by adjusting the rate of convergence of the step-size sequence used in the subgradient algorithm step. Our calculations of unit payoffs assume the market settles with a uniform energy price in each hour based on the dual variable associated with the load-balance constraint. Pricing energy based on these dual variables captures the correct marginal cost of serving the system's loads, and it has the property that it "smooths out" prices between periods in which intertemporal constraints are binding. In practice, however, most ISOs settle based on prices resulting from the day-ahead optimal power flow (OPF) model, which generally do not capture these intertemporal constraints and as such will not correctly account for binding ramp constraints in determining prices.[5] In the context of our simple unit commitment, these OPF prices would be equivalent to setting the hourly energy price based on the highest marginal cost unit which is running in that hour but not held at minimum load. Unit payoffs are nearly identical with both pricing schemes, and as such, we only report calculations with energy prices based on the unit commitment dual variables.

TABLE I
COMPARISON OF LR SOLUTIONS AND MIP OPTIMUM, WITH LINEAR ENERGY PAYMENTS ONLY

Solution	Total Cost ($)	Optimality Gap (%)	Units Affected
MIP	8,074,002.55		
1	8,181,665.93	1.33	125
2	8,212,269.01	1.71	128
3	8,202,929.15	1.60	132
4	8,171,416.63	1.21	141
5	8,125,904.86	0.64	109
6	8,220,547.52	1.82	128
7	8,211,208.66	1.70	132
8	8,124,845.51	0.63	112
9	8,180,528.20	1.32	129
10	8,189,867.05	1.44	125
11	8,116,566.01	0.53	112

Table I summarizes the 11 near-optimal LR solutions used in our analysis, showing the size of the final optimality gap (with primal-feasibility of the dual solution restored) and the number of units affected by not receiving their MIP-optimal offer-based surplus.

1) Unit Surplus Comparisons: Table II provides summary statistics of the offer-based surpluses of five select units across the 11 LR solutions, showing the extent to which the surplus of individual units can vary between the different solutions and deviate from the MIP optimum. For computation of surplus per MWh generated, this value is set to zero for a unit which receives no dispatch. Furthermore, the columns labeled "cv" report the coefficient of variation, which is the ratio of the standard deviation and mean, and provides a unit-free metric of surplus variability. Units 1 and 5, for instance, are not committed at all under the MIP optimum but are dispatched in a number of the LR solutions, always running at a net loss. Unit 1, when committed by the LR algorithm, runs at a substantial loss as it is given little dispatch with which to recover the large startup cost it incurs. Although some units are made worse off by the LR commitments, others such as units 3 and 4 are generally run at a net gain under the LR solutions whereas they are run at a loss in the MIP optimum.

Table II demonstrates a further complication in markets with nonconvex costs—that linear prices can be confiscatory—indeed, all of our LR and the MIP-optimal commitment leave

[3]Indeed, the 17-unit model studied by Johnson *et al.* could not be tractably solved by MIP solvers a decade ago.

[4]If a generator's revenues resulting from its commitment and dispatch is given by R and its actual cost of meeting the schedule is c, then the generator's actual profits will be given by $\Pi = R - c$. The generator, in making an offer to generate to the ISO, will state its costs are given by $\hat{c}$, which may be different from c; thus, its offer-based surplus will be $\hat{S} = R - \hat{c}$.

[5]Although it is possible to include ramping constraints in a multiperiod OPF, most ISOs solve each period individually and do not include these constraints.

TABLE II
OFFER-BASED SURPLUS OF SELECTED UNITS UNDER LR SOLUTIONS AND MIP OPTIMUM WITH LINEAR ENERGY PAYMENTS ONLY

Unit	MIP Offer-Based Surplus ($/MWh)	MIP Offer-Based Surplus ($)	LR Offer-Based Surplus ($/MWh)				LR Offer-Based Surplus ($)			
			Mean	Max	Min	cv	Mean	Max	Min	cv
1	0.00	0.00	-685.40	0.00	-952.56	-0.64	-7,458.07	0.00	-13,715.84	-0.73
2	-7.97	-6,886.92	-0.40	0.48	-0.69	-0.87	-348.94	414.92	-595.33	-0.83
3	-16.58	-2,387.39	3.44	4.68	3.98	0.73	478.53	673.22	-764.94	0.83
4	-1.73	-18,498.82	0.02	0.47	-0.13	9.89	152.48	4,186.96	-1,182.71	10.20
5	0.00	0.00	-42.43	-20.33	-50.03	-0.19	-12,564.29	-2,642.30	-19,510.97	-0.39
All	30.06	9,047,685.89	30.90	31.18	30.65	0.01	9,298,276.20	9,385,277.46	9,224,431.24	0.01

TABLE III
TOTAL SETTLEMENTS OF SELECTED UNITS UNDER LR SOLUTIONS AND MIP OPTIMUM WITH LINEAR ENERGY AND MAKE-WHOLE PAYMENTS

Unit	MIP Settlements ($)	LR Settlements ($)			
		Mean	Max	Min	cv
1	0.00	7,967.03	14,485.50	0.00	0.80
2	59,887.67	60,006.78	60,380.20	59,965.29	0.00
3	11,800.00	12,259.62	15,400.00	11,832.53	0.09
4	638,994.42	552,554.96	559,340.71	535,352.53	0.01
5	0.00	36,270.18	51,557.40	17,585.80	0.27
All	17,283,791.33	17,643,817.52	17,719,380.91	17,567,988.90	0.00

some generators with net losses. Absent a nonlinear pricing scheme, the potential for confiscation could lead generators to withhold themselves from the market or to distort the cost or constraint parameters in their offers to ensure themselves sufficiently high energy rents, with the potential to lead to an inefficient commitment. Most ISOs overcome this confiscation problem by paying uniform hourly energy and ancillary service prices with supplemental make-whole payments, which guarantee that a unit will recover any portion of its offer-based costs not covered by inframarginal energy and ancillary service rents over the planning horizon.[6] One shortcoming of this make-whole payment scheme is that the linear prices are not market-clearing, which is to say that if price-taking generators could adjust their generation and reserved capacity to maximize profits, there would generally be load imbalances. Recently, O'Neill *et al.* [7] propose a set of nonlinear prices which yield a Walrasian equilibrium and ensure nonconfiscation in a general competitive market with indivisible units or other nonconvexities, overcoming these issues. Despite the attractive properties of the prices proposed by O'Neill *et al.*, ISOs do not implement their pricing scheme.

Table III shows the total settlements paid to the five units in Table II, assuming that the market makes energy payments only (i.e., no payments for ancillary services) and includes a make-whole provision. Comparing the range of settlements paid to individual units and the range of total settlements for all the units between the different solutions, we see that payments to individual units can vary between different LR solutions, even though total payments to all generators tend to be relatively close. The high settlement cost of unit 1 in some of the LR solutions, for example, reflects the fact that unit 1 is started up in these solutions and must be given a supplemental make-whole payment, but because it displaces another unit, the impact on total settlement costs is small.

[6]Using our notation from before, if a generator's revenues resulting from its commitment and dispatch is given by R and its offer-based costs of generating according its commitment and dispatch is given by $\hat{c}$, then its make-whole payment will be $\max\{0, \hat{c} - R\}$, which ensures that the generator's net offer-based surplus is $\widehat{NS} = R - \hat{c} + \max\{0, \hat{c} - R\} \geq 0$.

An added benefit of including a make-whole provision is that it can help to "smooth out" the payoff differences between the LR solutions and MIP optimum and reduce the volatility of the LR payoffs by truncating the distribution at zero. Moreover, when compared to the MIP optimum, LR solutions generally commit the correct baseload and mid-merit units and differ mainly in the dispatch of marginal units. Because a marginal unit often receives little inframarginal energy rents and requires make-whole payments, the net offer-based surplus of such a unit is zero regardless of whether it is committed in an LR solution. As such, if the stated cost in the offers of these marginal units reflect their actual costs, then these marginal units which must be made whole will break even, regardless of whether or not they are committed. Table IV presents summary statistics comparing the absolute value of differences in net offer-based surplus to individual generators between each LR solution and the MIP optimum, with and without make-whole payments. The results show that when measured on a per-MWh-generated basis, the make-whole payments reduce the difference in and variability of payoffs, both by truncating the distribution and smoothing out net payoffs to near-marginal units. When looking at total surplus, the average in the deviations is reduced; however, the maxima are not affected since some units receive much higher payoffs under the LR commitment than under the MIP-optimum, which would not be affected by a make-whole provision. Importantly, comparing across the 11 LR solutions, we see that with a make-whole provision, the payoff differences are on average the same (with the one exception of solution 4), despite nontrivial differences in the optimality gap of the LR solutions, indicating that with regard to the equity of generators' offer-based surplus, there are only modest gains from closing the optimality gap of a near-optimal LR commitment.

2) Energy Pricing: Because an LR solution generally yields a suboptimal commitment, it is possible that the energy prices found in the least-cost dispatch will be "incorrect" in the sense that they do not reflect the correct marginal cost of dispatch from a least-cost commitment. Such commitment and the resulting pricing errors have the potential to affect generator incentives, send incorrect price signals to market participants, and leave

TABLE IV
ABSOLUTE VALUE OF UNIT OFFER-BASED SURPLUS DEVIATIONS BETWEEN LR SOLUTIONS AND MIP OPTIMUM

	($/MWh)						($)					
	Without Make-Whole Payments			With Make-Whole Payments			Without Make-Whole Payments			With Make-Whole Payments		
Solution	Mean	Max	cv	Mean	Max	cv	Mean	Max	cv	Mean	Max	cv
1	61.19	942.24	3.66	0.33	4.68	1.76	1,605.50	26,950.13	2.54	972.35	26,950.13	3.59
2	71.92	945.92	3.38	0.33	4.80	1.79	1,716.95	26,950.13	2.42	972.63	26,950.13	3.59
3	73.66	939.30	3.31	0.33	4.80	1.79	1,686.54	26,950.13	2.43	972.63	26,950.13	3.59
4	57.46	943.55	3.33	0.39	4.80	1.61	1,806.36	33,641.06	2.63	1,225.27	33,641.06	3.56
5	7.28	952.56	8.20	0.33	4.80	1.79	1,404.04	26,950.13	2.82	972.63	26,950.13	3.59
6	71.90	945.92	3.38	0.29	4.54	1.80	1,604.52	23,556.04	2.41	845.49	23,556.04	3.60
7	73.64	939.30	3.31	0.29	4.54	1.80	1,574.11	23,556.04	2.42	845.49	23,556.04	3.60
8	5.57	353.56	5.00	0.29	4.54	1.80	1,261.20	23,556.04	2.87	845.49	23,556.04	3.60
9	62.95	931.94	3.56	0.29	4.54	1.80	1,462.95	23,556.04	2.49	845.49	23,556.04	3.60
10	61.22	942.24	3.66	0.29	4.54	1.80	1,493.36	23,556.04	2.48	845.49	23,556.04	3.60
11	5.60	353.56	4.97	0.33	4.80	1.79	1,373.63	26,950.13	2.87	972.63	26,950.13	3.59

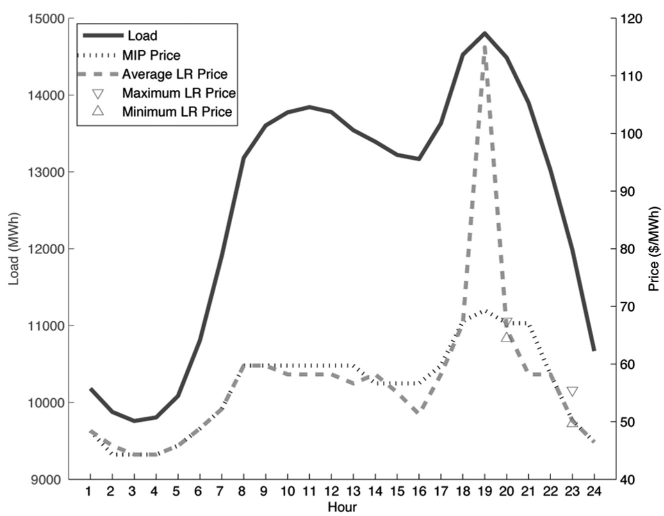


Fig. 1. LR and MIP prices.

uncommitted generators who would be "in the money" out of profitable dispatch. Moreover, these pricing errors can further confound the deviations in generator payoffs.

Fig. 1 plots the load profile for our test problem, the MIP-optimal energy prices, and energy prices from the LR commitments. In 22 of the hours, the prices in the LR solutions are identical to one another and the range of the prices are plotted for the other 2, which show little variability. Furthermore, the LR prices are the same as the MIP-optimal prices in ten of the hours, indicating that our LR algorithm is capable of dispatching the "correct" marginal unit in some hours. There is, however, a large deviation in hour 19 with the LR commitments yielding energy prices which are more than $45 higher than the corresponding MIP-optimal price. Importantly, this mispricing appears in all our LR solutions—even the ones with smaller optimality gaps—reflecting the fact that such errors can occur in general, regardless of the size of the duality gap. One of the issues of such mispricings is that it further confounds the issue of payoff deviations resulting from LR solutions. Unit 3 in Table II, for instance, receives its MIP-optimal commitment and dispatch in most of the LR solutions, yet because much of its generation is dispatched in hour 19, it runs at a net offer-based gain in the LR commitments whereas it should run at a net loss in the MIP optimum from linear energy-only payments.

This result highlights the fact that an individual generator's payoff is dependent not only on it receiving its correct commitment but also on the commitments of other units. Moreover, because the LR dual solution is generally primal-infeasible, the marginal unit in some hours may be determined by the feasibility heuristic, meaning the final set of energy prices can be sensitive to the specifics of the heuristics utilized in restoring feasibility of the dual solution. In our simple model with fixed loads, these mispricings represent wealth transfers between suppliers and consumers, meaning there are no welfare losses from the prices themselves.[7] In a more general setting with price-elastic demand, transaction, or virtual bids, such mispricings could cause efficiency losses both from prices being too low, which could result in inefficient trade, or prices being too high, which could result in efficient trades being priced out of the market.

B. MIP Implementation

Although recent computational and algorithmic advances make direct solution of the unit commitment by B&B tractable, ISOs cannot currently solve their commitment problems to complete optimality within the allotted timeframe. Most ISOs, upon receiving generation offers and other market data day-ahead, must return commitments and a schedule to generators within a few hours. The formation of these schedules oftentimes requires solving multiple unit commitment, OPF, and other optimization problems. As such, ISOs which have implemented or are proposing to use MIP in their unit commitment set limits on the solution time and rely on the best integer-feasible solution found at the end of that time. Although ISOs boast their ability to find feasible solutions with minuscule optimality gaps, if an ISO is left to rely on an intermediate integer-feasible but suboptimal solution, the same issues of generator payoffs, energy pricing, and inequity of the resulting dispatch arise as with suboptimal LR commitments.

Table V summarizes the progression of the MIP optimizer in CPLEX 9.120 solving our simple unit commitment with the default settings. The problem was formulated using AMPL 10.100 using default presolver settings. We note that unlike commercial MIP-based unit commitment software packages, we did not fine-tune the formulation of the problem, the settings in CPLEX,

[7]Although the payments are wealth transfers which do not incur consumer welfare losses, the suboptimal commitment and dispatch of the system obviously results in productive efficiency losses.

TABLE V
PROGRESSION OF INTEGER-FEASIBLE SOLUTIONS FOUND IN B&B TREE OF SIMPLE UNIT COMMITMENT PROBLEM WITH MAKE-WHOLE PAYMENTS

Solution	Total Cost ($)	MIP Gap (%)	Absolute Value of Unit Offer-Based Surplus Deviations ($/MWh)		
			Mean	Max	cv
1	8,074,400.39	0.0049275	0.00	0.02	1.56
2	8,074,045.70	0.0005345	0.00	0.00	n/a
3	8,074,020.25	0.0002192	0.00	0.00	n/a
4	8,074,014.91	0.0001531	0.00	0.00	n/a
5	8,074,003.06	0.0000063	0.00	0.00	n/a

TABLE VI
PROFITABILITY OF SELECT UNITS UNDER OPTIMAL AND INTERMEDIATE B&B SOLUTIONS OF SIMPLE UNIT COMMITMENT WITH LINEAR ENERGY PAYMENTS ONLY

Unit	MIP Offer-Based Surplus ($/MWh)	Suboptimal MIP Solution Offer-Based Surplus ($/MWh)			
		Mean	Max	Min	cv
6	0.00	-102.61	-100.70	-105.49	-0.03
7	-90.70	-72.56	0.00	-90.70	-0.56
8	-90.70	-18.14	0.00	-90.70	-2.24
All	30.06	30.06	30.08	30.06	0.00

TABLE VII
ENERGY PRICES OF INTERMEDIATE SOLUTIONS FOUND IN BRANCH AND BOUND TREE

Hour	Energy Price of Solution ($/MWh)					
	1	2	3	4	5	Optimum
2	45.84	44.30	44.30	44.30	44.30	44.30
11	114.95	64.49	59.72	59.72	59.72	59.72
23	48.96	50.24	48.96	50.24	50.24	50.24

or introduce problem-specific cutting plane algorithms to improve the solutions or solution times of the problem.[8] CPLEX finds five intermediate suboptimal integer-feasible solutions, all of which have smaller optimality gaps than our LR commitments. Moreover, should the ISO use one of the intermediate solutions but include a make-whole provision, the net offer-based surplus to each unit is identical to that under the MIP optimum in all but the first solution, with the largest deviation being $0.02/MWh in the first solution.

Table VI shows that should the ISO not include a make-whole provision, then the surplus of individual units can differ between the suboptimal solutions and the MIP optimum, indicating intermediate solutions from a B&B algorithm will suffer the same issues as LR commitments, regardless of the size of the optimality gap. Units 7 and 8, for instance, are identical in terms of the stated cost and operating constraint parameters in their offers, and they receive the exact same commitment and dispatch in the MIP optimum but are given different commitments and dispatches in every intermediate MIP solution.

Table VII shows that as with suboptimal LR commitments, the intermediate MIP solutions can also yield incorrect energy prices due to an incorrect dispatch. With our simple problem, there are only differences in three of the hours and only in the first three solutions—energy prices in all other hours equal the MIP-optimal prices in all the intermediate solutions, and the last two intermediate solutions had the same energy prices as the MIP optimum. Nonetheless, the first solution, the optimality gap of which is a fraction of a percent, yields an energy price in hour 11 which is nearly twice the MIP-optimal price.

Although our results show that when the market includes make-whole payments the intermediate solutions to our unit commitment problem have virtually eliminated the generator payoff issues prevalent with LR commitment, the formulation used is a simplification of any actual unit commitment solved by an ISO and excludes many important system details. Table VIII summarizes the progression of integer-feasible solutions found by CPLEX in solving ISONE's complete unit commitment problem, which includes virtual, transaction, and demand bids, time-dependent startup costs, stepped generation costs, multiple types of ancillary service requirements, and a dc-load flow model. Due to the inclusion of the demand bids, the problem is formulated to maximize total social surplus from energy traded. The problem is once again formulated using AMPL 10.100 and solved by CPLEX 9.120 using default settings except that the integrality and optimality tolerances in CPLEX were set to zero to ensure the final solution given is the MIP-optimum. CPLEX iterates through 38 suboptimal integer-feasible solutions before finding the MIP optimum.

As the table clearly highlights, complex unit commitment formulations, which are more reminiscent of the actual problems which must be solved by ISOs, still present market design issues, even when solved using a B&B algorithm. Solutions which are a minuscule fraction of a percent away from optimal nonetheless result in different payoffs to individual generators. Moreover, the progression of solutions shows that the surplus deviations are not a decreasing function of the size of the optimality gap. Solution 17, for instance, gives generator surpluses which are on average within $0.04/MWh of the MIP optimum, yet the next four solutions, all of which have smaller optimality gaps, result in larger average and maximum surplus deviations.

Just as with the suboptimal LR commitments, intermediate solutions found by CPLEX in solving the MIP to optimality can lead to energy pricing errors, with some extreme cases of the intermediate prices being more than 10% from the MIP-optimal prices. Although our unit commitment model included network flow constraints, none were binding in the optimum or in any of the intermediate solutions, and as such, the locational marginal prices were identical across the network. Table IX summarizes the range of energy price deviations between the 38 intermediate solutions and the MIP optimum, showing again that solutions which are a millionth of a percent away from optimal can nonetheless have substantive price differences. Moreover, the size of the deviations is not monotone in the size of the optimality gap, as shown by comparing solution 31 to all the intermediate solutions found after it.

[8]We did set the integrality and optimality gap tolerances to zero in order to ensure the final solution given by CPLEX is indeed the MIP-optimum.

TABLE VIII
PROGRESSION OF INTEGER-FEASIBLE SOLUTIONS FOUND IN B&B TREE OF COMPLETE UNIT COMMITMENT PROBLEM WITH MAKE-WHOLE PAYMENTS

Solution	Total Surplus	Optimality Gap (%)	Absolute Value of Unit Offer-Based Surplus Deviations ($/MWh)		
			Mean	Max	cv
Optimum	10,873,267.23				
1	10,743,126.26	0.011968893	0.44	3.07	1.39
2	10,780,344.54	0.008545977	0.49	2.97	1.38
3	10,791,726.53	0.00749919	0.71	3.64	1.37
4	10,836,621.47	0.003370262	0.62	2.54	1.37
5	10,837,058.63	0.003330057	0.59	2.43	1.37
6	10,837,740.91	0.003267309	0.76	3.01	1.37
7	10,837,952.97	0.003247806	0.76	3.01	1.37
8	10,839,184.76	0.00313452	0.73	2.91	1.37
9	10,839,379.80	0.003116582	0.66	3.04	1.37
10	10,839,458.47	0.003109347	0.71	3.04	1.37
11	10,839,613.51	0.003095088	0.69	2.94	1.37
12	10,839,764.98	0.003081158	0.67	2.88	1.37
13	10,842,307.12	0.00284736	0.25	1.04	1.38
14	10,859,035.14	0.001308906	0.19	0.82	1.38
15	10,859,165.69	0.0012969	0.20	0.85	1.38
16	10,859,165.78	0.001296892	0.19	0.82	1.38
17	10,859,280.44	0.001286347	0.04	0.32	1.46
18	10,863,244.27	0.000921798	0.19	0.84	1.38
19	10,866,290.84	0.000641609	0.17	4.13	1.90
20	10,866,499.64	0.000622406	0.20	5.61	2.06
21	10,866,761.13	0.000598357	0.19	4.30	1.81
22	10,870,211.13	0.000281065	0.04	4.41	6.90
23	10,871,577.89	0.000155366	0.23	1.67	1.41
24	10,871,675.08	0.000146428	0.23	1.81	1.41
25	10,871,987.07	0.000117735	0.25	1.37	1.39
26	10,872,163.04	0.000101551	0.27	1.44	1.39
27	10,872,312.67	0.00008779	0.22	4.35	1.71
28	10,872,789.77	0.000043911	0.24	3.92	1.60
29	10,872,868.80	0.000036643	0.23	2.01	1.42
30	10,872,961.30	0.000028136	0.01	0.79	4.34
31	10,872,961.38	0.000028129	0.01	0.73	7.45
32	10,872,995.32	0.000025007	0.03	0.20	1.39
33	10,872,995.41	0.000024999	0.02	0.14	1.41
34	10,873,156.39	0.000010194	0.26	1.08	1.38
35	10,873,156.48	0.000010186	0.25	1.03	1.38
36	10,873,193.80	0.000006753	0.24	0.99	1.38
37	10,873,256.39	0.000000997	0.02	0.11	1.40
38	10,873,267.14	0.000000008	0.01	0.07	1.41

TABLE IX
PERCENT DIFFERENCES IN ENERGY PRICE DEVIATIONS BETWEEN INTERMEDIATE INTEGER-FEASIBLE SOLUTIONS AND MIP OPTIMUM

Solution	Mean	Max	Min	Solution	Mean	Max	Min
1	2.27%	7.13%	-2.36%	20	0.89%	5.03%	-2.08%
2	2.51%	9.87%	-4.40%	21	0.89%	5.03%	-2.08%
3	3.65%	10.22%	-2.08%	22	0.14%	5.03%	-3.03%
4	3.17%	10.22%	-3.44%	23	1.18%	5.03%	-1.83%
5	3.05%	10.22%	-3.44%	24	1.21%	5.03%	-1.06%
6	3.90%	10.40%	-5.08%	25	1.33%	5.03%	-1.83%
7	3.90%	10.40%	-5.08%	26	1.41%	5.03%	-0.47%
8	3.81%	10.22%	-2.08%	27	1.10%	4.60%	-1.83%
9	3.31%	10.22%	-5.08%	28	1.20%	4.60%	-0.47%
10	3.60%	10.22%	-5.08%	29	1.20%	4.60%	-0.47%
11	3.48%	10.22%	-5.08%	30	0.04%	1.43%	-0.47%
12	3.38%	10.22%	-5.08%	31	-0.02%	0.00%	-0.47%
13	1.29%	4.60%	-4.40%	32	0.17%	3.13%	-0.47%
14	1.00%	4.60%	-4.40%	33	0.11%	3.13%	-0.47%
15	1.05%	4.60%	-4.40%	34	1.35%	4.60%	0.00%
16	1.00%	4.60%	-4.40%	35	1.29%	4.60%	0.00%
17	-0.19%	5.03%	-7.19%	36	1.23%	4.60%	-1.06%
18	1.03%	5.03%	-7.19%	37	-0.13%	0.00%	-3.03%
19	0.77%	5.03%	-4.40%	38	0.06%	1.43%	0.00%

III. CONCLUSION

We have revisited some of the issues raised by Johnson *et al.* [1] surrounding the design and implementation of a centralized unit commitment. We demonstrated that different near-, but sub-optimal commitments from an LR or B&B algorithm can result in different payoffs to individual generators, calling into question the incentive properties of a market with centralized unit commitment. We further showed that the suboptimal commitments will generally yield incorrect energy prices, which will present additional efficiency implications in a market with price-responsive demand. With an LR algorithm, these mispricings can be further seen as nontransparent or "black-box" pricing insomuch as these energy prices will be determined by the marginal unit in each hour which is often found by the feasibility heuristic, which may be viewed as opaque, arbitrary, or convoluted.

We showed that make-whole payments can reduce the problem of generator surplus differences. Make-whole payments help to smooth out the net offer-based surplus earned by near-marginal units, and if the stated costs of these units reflect actual costs, these units would be indifferent between receiving their optimal commitments with make-whole payments or not being committed. While the use of a B&B algorithm which could solve the problem to complete optimality can, in theory, overcome these issues entirely, this is as of yet intractable.[9] If the ISO must, instead, rely on an intermediate near-optimal solution, we demonstrated the same issues of surplus and price differences will arise, even with solutions which are a fraction of a percent away from optimal.

As such, one should expect the issues raised by Johnson *et al.* [1] to remain in centrally committed markets, regardless of the solution technique used. Moreover, our results showed that payoff and price deviations are not monotone in the size of the optimality gap—which is contrary to common belief. Guan *et al.* [8] state, for example, that the 1%–2% duality gaps achieved with LR were sufficient for monopoly utilities but that the development of competitive markets in which generators compete to provide their products has increased the need for more accurate unit commitment solutions. Our results show, however, that this market design issue will loom, regardless of how accurate the unit commitment solution is, unless an optimum can be found. As such, even though the B&B algorithm can typically achieve smaller optimality gaps than LR, this does not guarantee that the equity issues will be "smaller" under MIP than LR. Overall, our results demonstrate that centralized markets in which the ISO makes binding commitment decisions will suffer from the issues raised by Johnson *et al.* [1], which cannot be fully addressed with current computational limits.

APPENDIX
PROBLEM FORMULATION

The simple unit commitment formulation used in our comparison of MIP and LR solutions is presented. We first define the following notation:

Problem Parameters

- I: generator index set
- T: number of planning periods
- SU_i: startup cost of unit i's offer
- N_i: no-load cost of unit i's offer
- MC_i: marginal generating cost of unit i's offer
- K_i^-: minimum generating capacity of unit i's offer
- K_i^+: maximum generating capacity of unit i's offer
- R_i: maximum ramp rate of unit i's offer
- SP_i: maximum spinning capacity of unit i's offer
- n_i: minimum up-time of unit i's offer
- f_i: minimum down-time of unit i's offer
- D_t: load forecast in period t
- ρ: percentage of load which must be available in additional spinning reserves

Decision Variables

- $q_{i,t}$: generation provided by unit i in period t
- $r_{i,t}$: spinning reserve provided by unit i in period t
- $u_{i,t}$: binary variable indicating if unit i is up in period t
- $s_{i,t}$: binary variable indicating if unit i is started in period t
- $h_{i,t}$: binary variable indicating if unit i is stopped in period t

The problem is formulated as minimizing total commitment costs:

$$\min_{q,r,u,s,h} \sum_{i,t} (MC_i q_{i,t} + N_i u_{i,t} + SU_i s_{i,t})$$

subject to load-balance:

$$\sum_i q_{i,t} = l_t, \; \forall\, t$$

spinning-reserve requirement:

$$\sum_i (q_{i,t} + r_{i,t}) \geq (1+\rho) l_t, \; \forall\, t$$

unit minimum-load requirement:

$$K_i^- u_{i,t} \leq q_{i,t}, \; \forall\, i,t$$

unit maximum-load requirement:

$$q_{i,t} + r_{i,t} \leq K_i^+ u_{i,t}, \; \forall\, i,t$$

unit spinning capacity:

$$0 \leq r_{i,t} \leq SP_i u_{i,t}, \; \forall\, i,t$$

unit ramping limit:

$$-R_i \leq q_{i,t} - q_{i,t-1} \leq R_i, \; \forall\, i,t$$

unit minimum up-time:

$$\sum_{\tau=t-n_i+1}^{t} s_{i,\tau} \leq u_{i,t}, \; \forall\, i,t$$

unit minimum down-time:

$$\sum_{\tau=t-f_i+1}^{t} h_{i,\tau} \leq 1 - u_{i,t}, \; \forall\, i,t$$

[9]Similarly, if an LR algorithm could yield a MIP-optimal commitment, then these issues would be resolved.

startup definition:

$$s_{i,t} \geq u_{i,t} - u_{i,t-1},\ \forall\, i,t$$

shutdown definition:

$$h_{i,t} \geq u_{i,t-1} - u_{i,t},\ \forall\, i,t$$

and variable integrality:

$$u_{i,t}, s_{i,t}, h_{i,t} \in \{0,1\},\ \forall\, i,t$$

constraints.

ACKNOWLEDGMENT

The authors would like to thank E. Litvinov, R. Entriken, seminar participants at FERC, and five anonymous referees for comments and suggestions; E. Litvinov and T. Zheng for invaluable help in providing us with unit commitment data; and A. Sorooshian for implementation and modeling suggestions.

REFERENCES

[1] R. B. Johnson, S. S. Oren, and A. J. Svoboda, "Equity and efficiency of unit commitment in competitive electricity markets," *Util. Pol.*, vol. 6, pp. 9–19, Mar. 1997.

[2] D. M. Newbery, , F. P. Sioshansi and W. Pfaffenberger, Eds., "Electricity liberalization in Britain and the evolution of market design," in *Electricity Market Reform: An International Experience*. New York: Elsevier, 2006.

[3] D. Streiffert, R. Philbrick, and A. Ott, "A mixed integer programming solution for market clearing and reliability analysis," in *Proc. IEEE Power Eng. Soc. General Meeting*, San Francisco, CA, 2005.

[4] T. Li and M. Shahidehpour, "Price-based unit commitment: A case of Lagrangian relaxation versus mixed integer programming," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 2015–2025, Nov. 2005.

[5] B. Lu and M. Shahidehpour, "Short-term scheduling of combined cycle units," *IEEE Trans. Power Syst.*, vol. 19, no. 3, pp. 1616–1625, Aug. 2004.

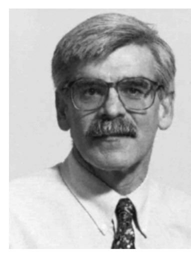
[6] L. A. Wolsey, *Integer Programming*. New York: Wiley-Interscience, 1998.

[7] R. P. O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf, and W. R. Stewart, "Efficient market-clearing prices in markets with nonconvexities," *Eur. J. Oper. Res.*, vol. 164, pp. 269–285, Jul. 1, 2005.

[8] X. Guan, Q. Zhai, and A. Papalexopoulos, "Optimization based methods for unit commitment: Lagrangian relaxation versus general mixed integer programming," in *Proc. IEEE Power Eng. Soc. General Meeting*, 2003.

Ramteen Sioshansi received the B.A. degree in economics and applied mathematics and the M.S. and Ph.D. degree in industrial engineering and operations research from the University of California, Berkeley, and the M.Sc. degree in econometrics and mathematical economics from The London School of Economics and Political Science, London, U.K.

He is a postdoctoral researcher in the Industrial Engineering and Operations Research Department at the University of California, Berkeley. His research focuses on the design of restructured competitive electricity markets.



Richard O'Neill received the Ph.D. degree in operations research from the University of Maryland, College Park.

He is Chief Economic Advisor in the Federal Energy Regulatory Commission, Washington, DC. He previously was on the faculties of the University of Maryland and Louisiana State University.

Shmuel S. Oren (F'02) received the B.Sc. and M.Sc. degrees in mechanical engineering from the Technion, Haifa, Israel, and the M.S. and Ph.D. degrees in engineering economic systems from Stanford University, Stanford, CA.

He is Earl J. Isaac Chair Professor in the Industrial Engineering and Operations Research department at the University of California, Berkeley. He is the Berkeley site director of PSERC—a multi-university Power System Engineering Research Center sponsored by the National Science Foundation and industry members. He has published numerous articles on aspects of electricity market design and has been a consultant to various private and government organizations, including the Brazilian Electricity Regulatory Agency, the Alberta Energy Utility Board, the Polish System Operator, the Public Utility Commission of Texas, and the California Public Utility Commission.

Dr. Oren is a Fellow of INFORMS.