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Modeling and Computing Two-Settlement Oligopolistic Equilibrium in a Congested Electricity Network

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A model of two-settlement electricity markets is introduced, which accounts for flow congestion, demand uncertainty, system contingencies, and market power. We formulate the subgame perfect Nash equilibrium for this model as an equilibrium problem with equilibrium constraints (EPEC), in which each firm solves a mathematical program with equilibrium constraints (MPEC). The model assumes linear demand functions, quadratic generation cost functions, and a lossless DC network, resulting in equilibrium constraints as a parametric linear complementarity problem (LCP). We introduce an iterative procedure for solving this EPEC through repeated application of an MPEC algorithm. This MPEC algorithm is based on solving quadratic programming subproblems and on parametric LCP pivoting. Numerical examples demonstrate the effectiveness of the MPEC and EPEC algorithms and the tractability of the model for realistic-size power systems.

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1. Introduction

Electricity restructuring aims at creating new competitive environments that provide long-term consumer benefits. A major obstacle to this goal is market power, both vertical and horizontal. Vertical market power in electricity markets has been substantially mitigated through the unbundling of the generation, transmission and distribution sectors, and through "open access" to transmission grids. However, horizontal and locational market power remains an important issue to policymakers due to the nonstorability of electricity, the lack of demand elasticity, high market concentration, and limited transmission capacities.

Among the many proposed and implemented economic means of mitigating horizontal market power is a two-settlement approach, where forward contracts and spot transactions are settled at different prices. Both theoretical analysis and empirical evidences in Allaz (1992), Allaz and Vila (1993), von der Fehr and Harbord (1992), Green (1999), Newbery (1998), and Powell (1993) have suggested that forward contracting decreases sellers' incentives for manipulating spot market prices because, under two settlements, the volume of trading that can be affected by spot prices is reduced. Allaz (1992) assumes a two-period market and demonstrates that if all producers have access to a forward market, it leads to a prisoners' dilemma type of game among them. Allaz and Vila (1993) show that, as the number of forward trading periods increases, producers lose their ability to raise energy prices above their marginal cost. Kamat and Oren (2004) analyze two-settlement markets over two- and three-node networks and extend the results in Allaz (1992) and Allaz and Vila (1993) to a system with uncertain transmission capacities in the spot market.

Recent work in Yao et al. (2004, 2005) further extends the above results to more realistic multinode and multizone networks. Yao et al. (2004) consider flow constraints, system contingencies, and demand uncertainties in the spot market. Their numerical tests show that, like in the simple cases, generation firms have incentives to engage in forward contracting, which increases social surplus and reduces spot prices. Yao et al. (2005) consider two alternative mechanisms for capping prices. They observe that a forward cap, which can be induced by free entry of new generation capacity, increases firms' incentives for forward contracting, whereas a regulatory cap in the spot market reduces such incentives.

This paper continues the study of two-settlement electricity systems. Our objective is twofold. First, we introduce a new model of Cournot equilibrium in two-settlement markets that overcomes some shortcomings of the formulations in Yao et al. (2004, 2005). As before, the model is formulated as an equilibrium problem with equilibrium constraints (EPEC), where each generation firm solves a mathematical problem with equilibrium constraints

(MPEC) (see Luo et al. 1996) parameterized by the other firms' forward commitments. The model assumes linear demand functions, quadratic generation cost functions, and a lossless DC network, resulting in the preceding equilibrium constraints in the form of a parametric linear complementarity problem (LCP) (see Cottle et al. 1992). This EPEC model presents a computational challenge when applied to realistic-size systems. Therefore, our second goal is to study the computational aspect of this EPEC model and, by exploiting the problem structure, present in detail the solution approach for the EPEC and MPECs arising in our formulation.

Solving an EPEC problem amounts to solving simultaneously a set of MPEC problems, each parameterized by the other MPECs' decision variables (see Pang and Fukushima 2005 for more discussions on related topics). One solution approach is to derive the optimality conditions for the regularization scheme of the MPECs (see Fletcher and Leyffer 2004, Scheel and Scholtes 2000, Scholtes 2001), and then either solve the nonlinear complementarity conditions of the EPEC as a whole (Hu 2002, Su 2005) or iteratively solve the nonlinear complementarity conditions of individual MPECs (Hu 2002, Su 2005).

The second approach we will follow in this research is to iteratively solve MPECs using MPEC-based algorithms. There has been a growing literature on MPEC algorithms. The monograph by Luo et al. (1996) presents a comprehensive study of MPEC problems and provides first- and second-order optimality conditions; it also describes some iterative algorithms, such as the penalty interior point algorithm (PIPA) and the piecewise sequential quadratic programming (PSQP) algorithm (see also Jiang and Ralph 1999). More recent advances in MPEC algorithms can be found, for example, in Chen and Fukushima (2004), Facchinei et al. (1999), Fukushima et al. (1998), Fukushima and Tseng (2002), Fukushima and Lin (2004), Hu and Ralph (2004), and Ralph and Wright (2004). Fukushima et al. (1998) present a sequential quadratic programming approach through a reformulation of the complementarity condition as a system of semismooth equations by means of Fischer-Burmcister functionals. This algorithm shares several common features with the PIPA in terms of computational steps and convergence properties; however, it differs from the PIPA in the way of updating the penalty parameters and determining the step sizes. Chen and Fukushima (2004) consider MPECs whose lower constraints are a parametric P-matrix LCP. They smooth out the complementarity constraints through the use of Fischer-Burmcister functionals, from which the state variables are viewed as implicit functions of the decision variables. The MPECs can thus be solved by a sequence of well-behaved, though nonconvex, nonlinear programs. Fukushima and Tseng (2002) propose an ϵ-active set algorithm for solving MPECs with linear complementarity constraints and establish convergence to B-stationary points under the uniform linear independence constraint qualification on the feasible set. This algorithm generates a sequence of variable value sets such that the objective value is almost decreasing, while maintaining the ϵ-feasibility of the complementarity constraints.

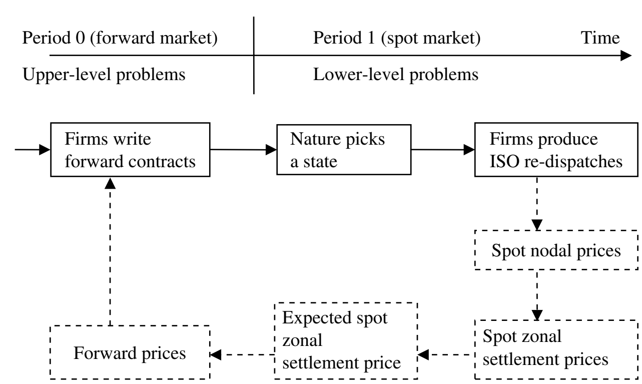
The remainder of this paper is organized as follows. The next section presents the EPEC model of two-settlement markets. In §3, we give a compact representation of the computational problem underlying our model. Section 4 summarizes the proposed MPEC and EPEC algorithms, and §5 reports the computational tests. Finally, we explore some economic implications of our test cases and draw conclusions. More details of the algorithms are given in the online appendix. An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

2. The Model

We view two-settlement markets as a two-period Nash-Cournot game: the forward market (period 0) and the spot market (period 1), and we characterize the equilibrium of this game as a subgame perfect Nash equilibrium (SPNE) (see Fudenberg and Tirole 1991). In period 0, rational firms enter into forward contracts, forming rational expectations regarding the forward commitments of the rivals and the period 1 equilibrium outcomes. Period 1 is a subgame with two stages. In stage 1, nature picks a state defined by a realization of the uncertain demand and system contingencies. In stage 2, the firms whose information sets include the state of nature and all forward commitments compete in a Nash-Cournot manner, while the independent system operator (ISO) transmits electricity and sets congestion prices to maximize social surplus of the entire system. The dynamics of this model are illustrated in Figure 1, where the solid lines represent time progress and the dashed lines denote rational expectations.

From a mathematical perspective, the model is formulated as an EPEC, which comprises a set of MPECs that characterize the decisions of individual firms. In each MPEC, the upper level is the firm's utility-maximization problem in the forward market, and the lower level,

Figure 1. The model dynamics.



shared by all MPECs, consists of the period 1 equilibrium conditions.

The following summarizes the main features of our model that will be elaborated in the rest of this section.

• We consider a lossless DC approximation of an electricity network, where flows on transmission lines are constrained by thermal capacities and random line outages. This simplifies the ISO's decision problem in the spot market. Such approximations are reasonable for an economic-oriented model considering that spot and day-ahead market settlements in major locational-marginal-price (LMP)-based systems like PJM, NYISO, and ISO-NE apply linear programming algorithms to the linearized models of the transmission system.

• The demand side is price taking with elastic demand functions subject to uncertainty (in the form of quantity shifts) at each node. Such an essential assumption in a Cournot type model might be problematic if the spot market represents a real-time balancing market. On the other hand, if the spot market represents a day-ahead market, there is sufficient time for demand response to justify this assumption. Alternative models that do not require the assumption of elastic demand are conjectural variation models that lack theoretical support, or supply function equilibrium (SFE) models that have not yet been sufficiently developed for application in the context of a cuneated network.

• The supply side consists of Cournot producers with multiple generators at various nodes that are subject to random outages, who sell energy to a pool at uniform LMPs set by the ISO.

• Generator outages, transmission line outages, and demand uncertainty are represented in terms of system contingent states that have known probabilities in the forward market and are realized before the spot market commences.[1]

• The forward market is organized at zonal hubs as financial contracts traded at uniform market-clearing prices and settled at spot hub settlement prices based on the nodal LMPs. (Such different granularity in the forward and spot markets is discussed in §2.1.2.)

• In the spot market, producers engage in a Nash-Cournot competition (i.e., setting quantities) while the ISO, who maintains the feasibility of the transmission constraints, behaves a la Bertrand by setting nodal price premiums or, equivalently, congestion charges between nodes.

• The market is efficient; i.e., risk-neutral speculators will arbitrage away any difference between the forward hub prices and expected spot hub settlement prices (see §2.2). To facilitate the computation, we will also assume that all the agents, i.e., the firms and the ISO, are risk neutral, as will be discussed later.

2.1. Period 1: The Spot Market

Electricity restructuring in different markets has been following different blueprints. In the United States, one prevailing design is the so-called centrally dispatched market. This type of market usually consists of a pool run by an ISO that serves as a broker, or auctioneer, for wholesale spot electricity market transactions. The ISO leases the transmission system from transmission owners and controls flows so as to maintain the feasibility of the network. It also sets nodal price premiums and implied congestion charges for bilateral energy transactions.

We consider a centrally dispatched wholesale spot market with demand uncertainty, flow constraints, and system contingencies. The network underlying the spot market consists of a set N of nodes and a set L of transmission lines. There is a set G of competitive firms, each operating the units at a subset of locations $N_g \subseteq N$. We assume that at most one generation firm operates at a node, and if necessary, we can introduce artificial nodes to meet this assumption. We also assume, for convenience, that there is elastic demand at each node so that pure generation nodes are represented by a demand function intersecting the quantity axis at a very small value.

2.1.1. The ISO's Decision Making. In each state $c \in C$, the ISO controls the import/export r_i^c at each node $i \in N$ (using the convention that positive quantities represent imports) and sets the corresponding locational marginal prices. These quantities must satisfy the network feasibility constraints, that is, the resulting power flows should not exceed the thermal limits K_l^c of the transmission lines in both directions. We model the transmission network via a lossless DC (i.e., linear) approximation of Kirchhoff's laws (see Chao and Peck 1996). Specifically, flows on lines can be calculated using power transfer distribution factor (PTDF) $D_{l,i}^c$, which specifies the proportion of flow on a line $l \in L$ resulting from an injection of one-unit electricity at a node $i \in N$ and a corresponding one-unit withdrawal at some fixed reference node (also known as the slack bus). Moreover, because electricity is not economically storable, the load and generation must be balanced at all times so all import and export quantities must add up to zero.

The ISO's objective is to maximize social welfare of the entire system. That is the aggregated area under the nodal inverse demand functions (IDFs) $P_i^c(\cdot)$, which represent the total consumer willingness-to-pay, less the sum of all generation costs $C_i(\cdot)$. Mathematically, the ISO solves the following problem parametric on the firms' production decisions ($\{q_i^c\}_{i\in N}$):

$$\max_{r_i^c:\, i\in N} \sum_{i\in N}\left(\int_0^{r_i^c+q_i^c} P_i^c(\tau_i)\,d\tau_i - C_i(q_i^c)\right)$$

$$\text{subject to}\quad \sum_{i\in N} r_i^c = 0, \tag{1}$$

$$\sum_{i\in N} D_{l,i}^c r_i^c \geqslant -K_l^c, \quad l \in L, \tag{2}$$

$$\sum_{i\in N} D_{l,i}^c r_i^c \leqslant K_l^c, \quad l \in L. \tag{3}$$

In the above formulation, we have excluded the nonnegativity constraints $r_i^c + q_i^c \geqslant 0$, $i \in N$, by implicitly assuming an interior solution with respect to these constraints.

The numerical results in §5 validate this simplification, but that might not be true in general. Let p^c, λ^c_{l-}, and λ^c_{l+} be the Lagrange multipliers corresponding to (1)–(3). The first-order necessary conditions (the Karush-Kuhn-Tucker (KKT) conditions) for the ISO's problem are

$$P_i^c(q_i^c + r_i^c) - p^c - \varphi_i^c = 0, \quad i \in N,$$
$$\varphi_i^c = \sum_{l \in L} (\lambda^c_{l+} D^c_{l,i} - \lambda^c_{l-} D^c_{l,i}), \quad i \in N,$$
$$\sum_{i \in N} r_i^c = 0,$$
$$0 \leqslant \lambda^c_{l-} \perp \sum_{i \in N} D^c_{l,i} r_i^c + K_l^c \geqslant 0, \quad l \in L,$$
$$0 \leqslant \lambda^c_{l+} \perp K_l^c - \sum_{i \in N} D^c_{l,i} r_i^c \geqslant 0, \quad l \in L.$$

The first KKT condition herein implies that

$$q_i^c + r_i^c = (P_i^c)^{-1}(p^c + \varphi_i^c), \quad i \in N,$$

and consequently, due to (1),

$$\sum_{i \in N} q_i^c = \sum_{i \in N} (P_i^c)^{-1}(p^c + \varphi_i^c). \tag{4}$$

This equation represents the aggregate demand function in the network relating the total consumption quantity to the reference node price p^c and the nodal price premiums $\{\varphi_i^c\}_{i \in N}$, which determine the relative nodal prices. The corresponding congestion charges for transmission from node $i \in N$ to node $j \in N$ that will prevent arbitrage between nodal energy transactions and bilateral transactions among nodes must be $\varphi_j^c - \varphi_i^c$.

2.1.2. The Firms' Decision Making. In the spot market, each firm $g \in G$ determines the outputs from its units at N_g. A variety of modeling approaches have been proposed to simulate generation firms' decision making (see, for example, Hobbs 2001, Neuhoff et al. 2005, Smeers and Wei 1997, and Wei and Smeers 1999). One modeling consideration regarding the suppliers' strategic behaviors in these models is whether or not they game the congestion prices set by the ISO. Following Hobbs (2001) and Neuhoff et al. (2005), we classify spot market models into two basic approaches.

The first approach assumes that generation firms anticipate the impact of their production on the congestion prices set by the ISO and take that effect into account in their production decisions. The resulting formulation of the spot market is a multileader, one-follower Stackelberg game (Neuhoff et al. 2005). Each producer g solves the following MPEC, in which the optimality conditions for the ISO's program are the constraints shared by all the firms:

$$\max_{q_i^c : i \in N_g} \sum_{i \in N_g} P_i^c(r_i^c + q_i^c) q_i^c - \sum_{i \in N_g} C_i(q_i^c)$$

subject to $0 \leqslant q_i^c \leqslant \bar{q}_i^c, \quad i \in N_g,$

$$P_i^c(q_i^c + r_i^c) - p^c + \sum_{l \in L} (\lambda^c_{l-} D^c_{l,i} - \lambda^c_{l+} D^c_{l,i}) = 0, \quad i \in N,$$
$$\sum_{i \in N} r_i^c = 0,$$
$$0 \leqslant \lambda^c_{l-} \perp \sum_{i \in N} D^c_{l,i} r_i^c + K_l^c \geqslant 0, \quad l \in L,$$
$$0 \leqslant \lambda^c_{l+} \perp K_l^c - \sum_{i \in N} D^c_{l,i} r_i^c \geqslant 0, \quad l \in L.$$

The equilibrium problem among the above MPECs represents a "generalized Nash game" (see Harker 1991), and it could have zero or multiple equilibria (see Borenstein et al. 2000). On the other hand, even if some pure-strategy equilibrium is found, it can be degenerate; that is, firms will find it optimal to barely congest some transmission lines to avoid congestion charges (see Oren 1997). Moreover, this formulation would lead to a two-settlement model with three decision levels, which makes an equilibrium solution for the two-settlement market computationally intractable.

The second approach assumes that the firms do not fully anticipate the impact of their production decisions on congestion charges (see, for example, Metzler et al. 2003), which can be interpreted as a "bounded rationality" assumption. In this approach, the ISO is a Nash player that moves simultaneously with the generation firms. The firms determine their supply quantities to maximize their profits, but they act as price takers with respect to transmission costs. The market equilibrium is then determined by aggregating the optimality conditions for the firms' and the ISO's problems, which result in a mixed complementarity problem or a variational inequality problem.

There are still two modeling options within this simultaneous-move framework. The first option assumes that the ISO, like the generation firms, is a Cournot player whose strategic variables are the import/export quantities at the nodes (see Neuhoff et al. 2005; and Yao et al. 2004, 2005). Hence, each firm $g \in G$ solves the following profit-maximization problem:

$$\max_{q_i^c : i \in N_g} \sum_{i \in N_g} P_i^c(r_i^c + q_i^c) q_i^c - \sum_{i \in N_g} C_i(q_i^c)$$

subject to $0 \leqslant q_i^c \leqslant \bar{q}_i^c, \quad i \in N_g.$

Note that because this program is parameterized by $\{r_i^c\}_{i \in N}$, it can be decomposed into $|N_g|$ subproblems, each corresponding to the production decision at one node. Therefore, this model will yield a spot market equilibrium that is invariant to the generation resource ownership structure (i.e, it does not matter whether a firm owns one or multiple generators). Moreover, under this formulation, when the network constraints (2)–(3) are nonbinding, the equilibrium solution predicts uniform nodal prices that are systematically higher than the Cournot equilibrium price corresponding to a single market with the aggregated system demand function (Neuhoff et al. 2005). These aspects make the choice of import/export quantities as the ISO's strategic variables (which we have used in our previous work; see Yao et al. 2004, 2005) unsatisfactory.

The second option we employ in this paper is to use the locational price premiums as the ISO's strategic variables. This option can be viewed as a mixed Cournot-Bertrand model, where the ISO behaves a la Bertrand while the generation firms are Cournot players with respect to each other (i.e., set quantities), but treat the ISO as a price setter. Thus, each firm chooses its production quantities to maximize profits with respect to the residual demand defined implicitly by (4). In this formulation, the reference bus price p^c is determined implicitly by the aggregate production decisions of all the generation firms, just as in a regular Cournot game. However, these production decisions and the implied reference node price also depend on the nodal premiums $\{\varphi_i^c\}$ set by the ISO. The resulting problem solved by each generation firm is

$$\max_{q_i^c: i\in N_g,\, p^c} \quad \sum_{i\in N_g}(p^c+\varphi_i^c)q_i^c - \sum_{i\in N_g} C_i(q_i^c)$$

$$\text{subject to}\quad 0\leqslant q_i^c \leqslant \bar{q}_i^c, \quad i\in N_g,$$

$$\sum_{i\in N} q_i^c = \sum_{i\in N}(P_i^c)^{-1}(p^c+\varphi_i^c).$$

This modeling option takes account of the resource ownership structure and, when the network constraints are relaxed, the locational price premiums go to zero so that the model produces the same equilibrium solution as the Cournot equilibrium applied to the aggregate system demand. Unfortunately, this approach has another shortcoming which manifests itself if we reduce the capacity of a radial transmission line to zero or, more realistically, if it is common knowledge that a radial line is constantly congested. In such situations, subnetworks connected by saturated radial lines are effectively decoupled from a competitive interaction point of view. The demand functions on both sides of the saturated line will be shifted by the import/export quantities, but their slope stays the same so generators will behave as local monopolists. For example, in the case of a symmetric two-node one-line network, reducing the line capacity to zero creates two symmetric local monopolies. However, in this situation, our model will produce a symmetric duopoly equilibrium with prices that are systematically lower than the locational monopoly prices. Unfortunately, there is no satisfying solution to this problem because a Nash equilibrium in a congestion-prone network depends on the conjectured common knowledge with regard to the extent of possible competition across transmission lines. Such conjectures affect the perceived elasticity of the residual demand by the competing firms and hence their strategic behaviors. The discontinuities in reaction functions and the resulting equilibrium prices when a single transmission line separating two competitors in a two-node system switches from a congested to an uncongested regime, have been eloquently demonstrated in Borenstein et al. (2000). Such discontinuities become intractable in a meshed system with multiple nodes.

We partially address the above issue in a sequel paper (Yao et al. 2006) through a hybrid approach that requires a prior identification of "systematically congested" links (e.g., path 15 in California or the link between France and the United Kingdom across the English Channel), which effectively decouples the network into strategic subnetworks. In this paper, however, we will assume that the network is fully connected physically and strategically so that competing firms behave as if the demand at all nodes is contestable.

Our two-settlement model permits different granularity in the forward and spot markets. This is achieved by dividing the network into a set Z of zones (or trading hubs), each consisting of a cluster of nodes. By allowing different granularity in the forward and spot markets, we are able to capture the case where the two settlements represent long-term forward contracts typically traded at hubs and nodal day-ahead spot markets, as well as the case where the forward market is in the day ahead and the spot market is at real time, both of which are typically nodal. In particular, our model assumes that the spot market supply and demand at each node are settled at the nodal prices, whereas the forward contracts are traded at zonal hub forward prices and settled at the corresponding spot hub, or zonal settlement, prices $\{u_z^c\}_{z\in Z}$, which are defined as weighted averages of the nodal prices in the respective zones. Thus, the nodal spot prices resulting from the strategic interaction in the spot market will affect the settlement of the forward contracts, which is debited from the firms' spot market profits, through these hub prices. The nodal weights $\{\delta_i\}_{i\in N}$ are assumed to be exogenous parameters based on historical load shares at the nodes. This assumption is consistent with common practice, for example, at the Pennsylvania-Jersey-Maryland (PJM) western hub. In mathematical terms, each firm $g\in G$ solves in the spot market the following profit-maximization problem parametric on the locational price premiums $\{\varphi_i^c\}_{i\in N}$ and on its own forward contracts $\{x_{gz}\}_{z\in Z}$:

$$\max_{q_i^c: i\in N_g,\, p^c} \quad \sum_{i\in N_g}(p^c+\varphi_i^c)q_i^c - \sum_{z\in Z} u_z^c x_{gz} - \sum_{i\in N_g} C_i(q_i^c)$$

$$\text{subject to}\quad u_z^c = \sum_{i: z(i)=z}(p^c+\varphi_i^c)\delta_i,$$

$$q_i^c \geqslant 0, \quad i\in N_g, \tag{5}$$

$$q_i^c \leqslant \bar{q}_i^c, \quad i\in N_g, \tag{6}$$

$$\sum_{i\in N} q_i^c = \sum_{i\in N}(P_i^c)^{-1}(p^c+\varphi_i^c). \tag{7}$$

Let ρ_{i-}^c, ρ_{i+}^c, and β_g^c be the Lagrange multipliers corresponding to (5)–(7). Then, the KKT conditions for firm g's program are (after substituting the first constraint into the objective function)

$$p^c + \varphi_i^c - \beta_g^c - \frac{dC_i(q_i^c)}{dq_i^c} + \rho_{i-}^c - \rho_{i+}^c = 0, \quad i \in N_g,$$

$$\beta_g^c \sum_{i \in N} \frac{d(P_i^c)^{-1}(p^c + \varphi_i^c)}{dp^c} + \sum_{i \in N_g} q_i^c - \sum_{i \in N} \delta_i x_{gz(i)} = 0,$$

$$\sum_{i \in N} q_i^c = \sum_{i \in N} (P_i^c)^{-1}(p^c + \varphi_i^c),$$

$$0 \leqslant \rho_{i-}^c \perp q_i^c \geqslant 0, \quad i \in N_g,$$

$$0 \leqslant \rho_{i+}^c \perp \bar{q}_i^c - q_i^c \geqslant 0, \quad i \in N_g.$$

Here, the first two conditions are the derivatives of the Lagrangian function with respect to q_i^c and p^c, respectively.

2.1.3. Period 1 Equilibrium Conditions. Aggregating the KKT conditions for the firms' and the ISO's programs yields the spot market equilibrium conditions, which, in general, form a mixed nonlinear complementarity problem. It becomes a mixed LCP when both the nodal demand functions and the marginal cost functions are linear, as is assumed in the remainder of this paper.

Let the inverse demand functions and the cost functions be, respectively,

$$P_i^c(q) = a^c - b_i^c q, \quad i \in N,$$

$$C_i(q) = d_i q + \frac{1}{2} s_i q^2, \quad i \in N.$$

Then, the market equilibrium conditions become

$$p^c + \varphi_i^c - \beta_g^c - d_i - s_i q_i^c + \rho_{i-}^c - \rho_{i+}^c = 0, \quad i \in N_g, g \in G, \tag{8}$$

$$p^c = a^c - \frac{\sum_{i \in N} \varphi_i^c / b_i^c}{\sum_{i \in N} 1/b_i^c} - \frac{\sum_{i \in N} q_i^c}{\sum_{i \in N} 1/b_i^c} \tag{9}$$

$$-\beta_g^c \sum_{i \in N} \frac{1}{b_i^c} + \sum_{i \in N_g} q_i^c - \sum_{i \in N} \delta_i x_{gz(i)} = 0, \quad g \in G, \tag{10}$$

$$0 \leqslant \rho_{i-}^c \perp q_i^c \geqslant 0, \quad i \in N, \tag{11}$$

$$0 \leqslant \rho_{i+}^c \perp \bar{q}_i^c - q_i^c \geqslant 0, \quad i \in N, \tag{12}$$

$$\sum_{i \in N} r_i^c = 0, \tag{13}$$

$$a^c - (q_i^c + r_i^c) b_i^c - p^c - \varphi_i^c = 0, \quad i \in N, \tag{14}$$

$$\varphi_i^c = \sum_{l \in L} (\lambda_{l+}^c D_{l,i}^c - \lambda_{l-}^c D_{l,i}^c), \quad i \in N, \tag{15}$$

$$0 \leqslant \lambda_{l-}^c \perp \sum_{i \in N} D_{l,i}^c r_i^c + K_l^c \geqslant 0, \quad l \in L, \tag{16}$$

$$0 \leqslant \lambda_{l+}^c \perp K_l^c - \sum_{i \in N} D_{l,i}^c r_i^c \geqslant 0, \quad l \in L. \tag{17}$$

Here, (8)–(12) are the aggregated KKT conditions for the firms' problems, and (13)–(17) are the KKT conditions for the ISO's problem. Under the assumption of linear demand functions and quadratic convex cost functions, the firms' and the ISO's programs are strictly concave-maximization problems, so (8)–(17) are also sufficient. Note that (9) can be excluded from the preceding market equilibrium conditions because it is implied by (13) and (14).

2.2. Period 0: The Forward Market

The forward market is assumed to be standardized and liquid such that all forward contracts in a zone are settled at equal prices. It is also assumed that there are enough risk-neutral arbitrageurs in the markets, and they will eliminate any arbitrage opportunity arising between the forward prices and the expected spot zonal settlement prices. Consequently, the forward price (h_z) in each zone $z \in Z$ is equal to the expected values of the corresponding spot hub prices over all contingent states ($c \in C$) with respective probabilities ($\Pr(c)$) (we assume for simplicity that the state probabilities and the market risk-neutral probabilities are identical). This is referred to as a "no-arbitrage" or "perfect-arbitrage" condition.

The risk-neutral firms simultaneously determine their forward contract quantities $\{x_{gz}\}_{g \in G, z \in Z}$ so as to maximize the total profit from both the forward contracts and the spot productions, while anticipating the forward commitments of the rivals as well as the equilibrium outcome in period 1. In mathematical terms, each firm $g \in G$ solves the following MPEC program, where $\{(8)\text{–}(17)\}_{c \in C}$ form the inner problem:

$$\max_{\Phi_g} \quad \sum_{z \in Z} h_z x_{g,z} + \sum_{c \in C} \Pr(c) \pi_g^c$$

subject to $\Phi_g = \{\{x_{g,z}\}_{z \in Z}, \{r_i^c, q_i^c, \rho_{i+}^c, \rho_{i-}^c\}_{i \in N, c \in C}, \cdot\{\lambda_{l-}^c, \lambda_{l+}^c\}_{l \in L, c \in C}\},$

$$\pi_g^c = \sum_{i \in N_g} (p^c + \varphi_i^c) q_i^c - \sum_{z \in Z} u_z^c x_{g,z} - \sum_{i \in N_g} C_i(q_i^c), \quad c \in C,$$

$$h_z = \sum_{c \in C} \Pr(c) u_z^c, \quad z \in Z,$$

$$u_z^c = \sum_{i:\, z(i) = z} (p^c + \varphi_i^c) \delta_i, \quad z \in Z, \ c \in C, \quad \text{and}$$

$$(8)\text{–}(17), \quad c \in C.$$

The equilibrium problem among the preceding MPECs is an EPEC. A solution to this EPEC is a set of the variables, including the firms' forward and spot decisions, the ISO's redispatch decisions, and the aforementioned Lagrange multipliers, at which all firms' MPEC problems are simultaneously solved, and no market participant is willing to unilaterally change its decisions in either market.

It is worth noting that, from a philosophical point of view, the above formulation might appear internally inconsistent because firms seem to base their decisions in the forward market on information that is not available to them in the spot market. To resolve this inconsistency, we might assume that forward commitments are based on correct forecast of the expected spot market outcomes rather than on the detailed information we use to replicate that forecast. Furthermore, it is also reasonable to assume that forward contracting decisions and spot market production decisions are made by functionally independent entities within a firm operating on different time horizons and employing different forecasting tools. So while the decisions made in the spot market are informed of the forward contracting positions of the firm, they do not necessarily account for all the global information that led to these contracting decisions.

3. A Compact Representation of the Model

In this section, we compact the notation to streamline the subsequent algorithmic presentation by grouping and relabelling the variables, including the dual variables, as follows:

- x_g $(\in R^{|Z|})$: vector of the forward commitments by firm $g \in G$.
- r^c $(\in R^{|N|})$: vector of the ISO's import/export quantities in state $c \in C$.
- q^c $(\in R^{|N|})$: vector of the firms' generation quantities in state $c \in C$.
- ρ_-^c, ρ_+^c $(\in R^{|N|})$: vectors of the Lagrange multipliers associated with the generation capacity constraints in state $c \in C$.
- λ_-^c, λ_+^c $(\in R^{|L|})$: vectors of the Lagrange multipliers associated with the flow capacity constraints in state $c \in C$.

In addition, the parameters are relabelled as

- Δ $(\in R^{|N|\times|Z|})$: a matrix where the (i, z)th element is -1 if $z(i) = z$, and 0 otherwise.
- $\bar{q}^c$ $(\in R^{|N|})$: vector of the generator capacity bounds in state $c \in C$.
- B_c $(\in R^{|N|\times|N|})$: a diagonal matrix for state $c \in C$, where the (i, i)th element is b_i^c.
- d $(\in R^{|N|})$: vector of the marginal generation costs.
- D_c $(\in R^{|L|\times|N|})$: A PTDF matrix for state $c \in C$, where the (l, i)th element is $D_{l,i}^c$.
- k^c $(\in R^{|L|})$: vector of the flow capacities of the transmission lines in state $c \in C$.
- X_g $(\in R^{|Z|})$: feasible region of x_g for each firm $g \in G$.

3.1. Compact Representation of the Inner Problem $\{(8)\text{–}(17)\}_{c\in C}$

Let $e \in R^{|N|}$ be a vector with all 1s. Then, (13) and (14) become

$$\begin{bmatrix} a^c e \\ 0 \end{bmatrix} - \begin{bmatrix} B_c \\ 0 \end{bmatrix} q^c - \begin{bmatrix} B_c & e \\ e^T & 0 \end{bmatrix} \begin{bmatrix} r^c \\ p^c \end{bmatrix} + \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_- - \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_+ = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Solving r^c and p^c yields

$$\begin{bmatrix} r^c \\ p^c \end{bmatrix} = \begin{bmatrix} Q_c & \dfrac{B_c^{-1} e}{e^T B_c^{-1} e} \\ \dfrac{e^T B_c^{-1}}{e^T B_c^{-1} e} & \dfrac{-1}{e^T B_c^{-1} e} \end{bmatrix} \cdot \left(\begin{bmatrix} a^c e \\ 0 \end{bmatrix} - \begin{bmatrix} B_c \\ 0 \end{bmatrix} q^c + \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_-^c - \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_+^c \right),$$

where

$$Q_c = B_c^{-1} - \frac{B_c^{-1} e e^T B_c^{-1}}{e^T B_c^{-1} e}.$$

Hence,

$$r^c = -Q_c B_c q^c + Q_c (D_c^T \lambda_-^c - D_c^T \lambda_+^c),$$

$$p^c = a^c - \frac{e^T}{e^T B_c^{-1} e} q^c + \frac{e^T B_c^{-1}}{e^T B_c^{-1} e} (D_c^T \lambda_-^c - D_c^T \lambda_+^c).$$

Now, consolidating conditions (8)–(10), we have

$$\rho_-^c = -a^c e + d + H_c q^c + B_c Q_c (D_c^T \lambda_-^c - D_c^T \lambda_+^c) + \rho_+^c + \frac{1}{e^T B_c^{-1} e} \Delta \sum_{g\in G} x_g,$$

where H_c is a matrix such that

$$(h_c)_{ij} = \begin{cases} \dfrac{2 + s_i}{e^T B_c^{-1} e} & \text{if } i = j, \\ \dfrac{2}{e^T B_c^{-1} e} & \text{if } i \neq j, \text{ and the units at nodes } i \text{ and } j \text{ belong to the same firm,} \\ \dfrac{1}{e^T B_c^{-1} e} & \text{otherwise.} \end{cases}$$

Next, let w^c and y^c be two variable vectors, and t^c, A_c, and M_c be constants such that

$$w^c = \begin{bmatrix} \bar{q}^c - q^c \\ \rho_-^c \\ k^c + D_c r^c \\ k^c - D_c r^c \end{bmatrix}, \qquad y^c = \begin{bmatrix} \rho_+^c \\ q^c \\ \lambda_-^c \\ \lambda_+^c \end{bmatrix},$$

$$t^c = \begin{bmatrix} \bar{q}^c \\ -a^c e + d \\ k^c \\ k^c \end{bmatrix}, \qquad A_c = \begin{bmatrix} 0 \\ \dfrac{\Delta}{e^T B_c^{-1} e} \\ 0 \\ 0 \end{bmatrix},$$

$$M_c = \begin{bmatrix} 0 & -I & 0 & 0 \\ I & H_c & B_c Q_c D_c^T & -B_c Q_c D_c^T \\ 0 & -D_c Q_c B_c & D_c Q_c D_c^T & -D_c Q_c D_c^T \\ 0 & D_c Q_c B_c & -D_c Q_c D_c^T & D_c Q_c D_c^T \end{bmatrix}.$$

The preceding applied to (8)–(17) leads to

$$w^c = t^c + A_c \sum_{g\in G} x_g + M_c y^c,$$

$$w^c \geqslant 0, \; y^c \geqslant 0, \; (y^c)^T w^c = 0. \quad (18)$$

Finally, aggregating (18) for all states $c \in C$, we present the inner problem $\{(8)\text{–}(17)\}_{c\in C}$ as

$$w = t + A \sum_{g\in G} x_g + My, \quad w \geqslant 0, \; y \geqslant 0, \; y^T w = 0,$$

where y and w are variables, and t, A, and M are constants as follows:

$$y = [y^c \;\; c \in C], \qquad w = [w^c \;\; c \in C], \qquad t = [t^c \;\; c \in C],$$

$$A = [A_c \;\; c \in C], \qquad M = \begin{bmatrix} M_1 & & & 0 \\ & M_2 & & \\ & & \cdots & \\ 0 & & & M_{|C|} \end{bmatrix}.$$

3.2. Compact Representation of the MPEC Problems

In period 0, each firm $g \in G$ solves the following MPEC problem:

$$\begin{aligned} \mathcal{F}_g(\bar{x}_{-g})\colon \quad & \min_{x_g, y, w} \; f_g(x_g, y, w, \bar{x}_{-g}) \\ & \text{subject to} \;\; x_g \in X_g, \\ & \qquad w = t + A\bar{x}_{-g} + Ax_g + My, \\ & \qquad w \geqslant 0, \; y \geqslant 0, \; y^T w = 0. \end{aligned} \tag{19}$$

In this program, x_g is the decision variable, (y, w) are the state variables, and $\bar{x}_{-g} = \sum_{k \in G\setminus\{g\}} \bar{x}_k$ is a parameter that is the sum of other firms' forward contract quantities.

We denote the EPEC problem in period 0 as $\{\mathcal{F}_g(\cdot)\}_{g\in G}$. An equilibrium of this EPEC problem in period zero is a set $(\{\bar{x}_g\}_{g\in G}, y, w)$ that solves $\mathcal{F}_g(\bar{x}_{-g})$ for all $g \in G$, i.e., $(\bar{x}_g, y, w) \in \mathrm{SOL}(\mathcal{F}_g(\bar{x}_{-g}))$, where $\mathrm{SOL}(\mathcal{F}_g(\bar{x}_{-g}))$ denotes the solution set of $\mathcal{F}_g(\bar{x}_{-g})$.

4. Solution Approach

To solve the EPEC as stated above, we propose an iterative scheme that solves in turn the MPEC problems by holding fixed the decision variables of the other MPEC problems.

4.1. The MPEC Algorithm

The MPEC algorithm is motivated by the following properties of $\mathcal{F}_g(\bar{x}_{-g})$.

(1) $f_g(x_g, y, w, \bar{x}_{-g})$ is quadratic with respect to (x_g, y, w).

(2) M is positive semidefinite. To show this, we first note that H_c is symmetric positive-definite. Second,

$$\begin{aligned} v^T Q_c v &= v^T B_c^{-1} v - \frac{v^T B_c^{-1} e e^T B_c^{-1} v}{e^T B_c^{-1} e} \\ &= \frac{\|B_c^{-1/2} v\|^2 \|B_c^{-1/2} e\|^2 - \|v^T B_c^{-1/2} B_c^{-1/2} e\|^2}{\|B_c^{-1/2} e\|^2} \\ &\geqslant 0, \quad v \in R^{|N|}. \end{aligned}$$

Hence, Q_c is symmetric positive semidefinite. Now, because

$$\frac{M_c + M_c^T}{2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & H_c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ D_c \\ -D_c \end{bmatrix} Q_c \begin{bmatrix} 0 \\ 0 \\ D_c \\ -D_c \end{bmatrix}^T,$$

we conclude that M_c is positive semidefinite.

(3) Given $\bar{x}_{-g}$, the constraint set (19) is an LCP parameterized by x_g. Moreover, for any x_g,

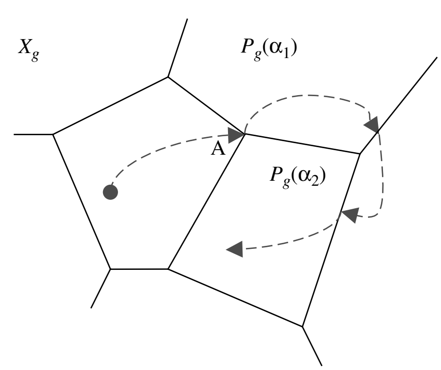
$$w_c = \begin{bmatrix} \bar{q}^c \\ \left(-a^c e + d + \dfrac{\Delta x_{-g} + \Delta x_g}{e^T B_c^{-1} e}\right)^+ \\ k^c \\ k^c \end{bmatrix}, \qquad y_c = \begin{bmatrix} \left(-a^c e + d + \dfrac{\Delta x_{-g} + \Delta x_g}{e^T B_c^{-1} e}\right)^- \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad c \in C,$$

satisfy the linear constraints of this LCP. By Theorem 3.1.2 in Cottle et al. (1992), the LCP problem (19) is always solvable. Recall that such solvability is achieved by assuming that the demand functions are linear and unconstrained and that the cost functions are quadratic.

In addition, we assume that for each state in period 1, the active constraints at the optimal solutions to the period 1 problems are linearly independent. By Theorem 3.1.7 in Cottle et al. (1992), (19) is always uniquely solved for all $x_g \in X_g$, i.e., its solution (y, w) is an implicit function of x_g, and $\mathcal{F}_g(\bar{x}_{-g})$ can be reduced to an optimization problem with respect only to x_g.

We developed an algorithm for solving $\mathcal{F}_g(\bar{x}_{-g})$ via a "divide-and-conquer" approach (see Figure 2). The proposed MPEC algorithm is a variant of the PSQP algorithm in Jiang and Ralph (1999) and Luo et al. (1996), but it specializes the PSQP algorithm by taking advantage of the preceding properties of $\mathcal{F}_g(\bar{x}_{-g})$. Specifically, the partition of X_g is determined by the feasible complementary bases (see Cottle et al. 1992, Definition 1.3.2) of the LCP problem (19). In each polyhedron, we derive the explicit affine functions for the state variables in terms of x_g, and solve a quadratic program involving only x_g for determining a stationary point within this polyhedron. An x_g is a stationary point of $\mathcal{F}_g(\bar{x}_{-g})$ if and only if it is a stationary point

Figure 2. A typical partition of X_g.



with respect to all polyhedra containing itself. Through parametric LCP pivoting, the proposed MPEC algorithm searches in the space of feasible x_g for a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$ along adjacent polyhedra. Details of the algorithm are described in online Appendix 1.

4.2. The EPEC Scheme

To find a B-stationary equilibrium of $\{\mathcal{F}_g(\cdot)\}_{g\in G}$, we start with an arbitrary set $\{\bar{x}_g^0 \in X_g\}_{g\in G}$. At each outer iteration k, we compute $\bar{x}_g^k$ from $\mathcal{F}_g(\bar{x}_{-g}^k)$ for each $g \in G$ while taking $\bar{x}_{-g}^k$ as given. The algorithm terminates when the improvement of the design variables in two consecutive iterations is reduced to a predetermined limit, or when the number of iterations reaches a predetermined upper bound. Online Appendix 2 discusses this scheme. Unlike the approach described in Hobbs et al. (2000), the current scheme carries (y, w), which always solves (19), among the MPEC problems. This offers the flexibility of terminating the MPEC algorithm before it reaches a B-stationary point.

5. Computational Results

We implemented in MATLAB the MPEC and EPEC algorithms that utilize the optimization toolbox for solving quadratic programs. In the implementation, we treat any number below 10^{-16} as zero to account for round-off errors. Tests of the algorithms are performed on both randomly generated problems and representative test cases specific to the context of electricity markets.

5.1. Tests of the MPEC Algorithm

The main computational effort involved in the EPEC scheme is to solve the MPECs. While our MPEC algorithm is guaranteed to terminate in finite steps, its actual performance is not known. Indeed, linear and also quadratic programs with linear complementarity constraints are shown to be NP-hard in Luo et al. (1996). In this section, we test the algorithm on a randomly generated set of generic MPEC problems with quadratic objective functions. Specifically, these MPEC problems are of the form

$$\min_{x,\,y} \quad \frac{1}{2}[x \quad y]P\begin{bmatrix} x \\ y \end{bmatrix} + c^T\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{aligned} \text{subject to}\quad & Ax + a \leqslant 0,\\ & w = Nx + My + q,\\ & w \geqslant 0, \quad y \geqslant 0, \quad w^T y = 0, \end{aligned}$$

where P, A, M (a positive semidefinite matrix), N, c, a, and q are constant matrices and vectors with suitable dimensions. We use the "QPECgen" package by Jiang and Ralph (1999) to generate these MPEC programs.

In the tests, we launch the MPEC algorithm from random starting points. Table 1 summarizes the test results. The first three columns list the dimensions of the decision and state variables, and columns 5 to 7 report the minimum, maximum, and average numbers of iterations, respectively. We observe that:

- The average number of iterations increases moderately as the dimension of the MPEC problems grows (except for the case of $n = 150$ and $m = 100$), but there does not exist such a trend for the minimum and maximum numbers of iterations.
- The algorithm is able to effectively solve MPEC problems with relatively large dimensions. Note that all instances in Table 1 have greater dimensions than those reported in Jiang and Ralph (1999).

Table 1. Test results of the MPEC algorithm.

				Iterations		
Dim(x)	Dim(y)	Dim(w)	Total dimension	Min	Max	Average
25	50	50	125	3	34	16
50	50	50	150	7	35	18
50	100	100	250	2	49	22
100	100	100	300	10	43	23
150	100	100	350	2	30	14
100	200	200	500	3	38	23
200	200	200	600	2	88	29
200	500	500	1,200	2	76	43

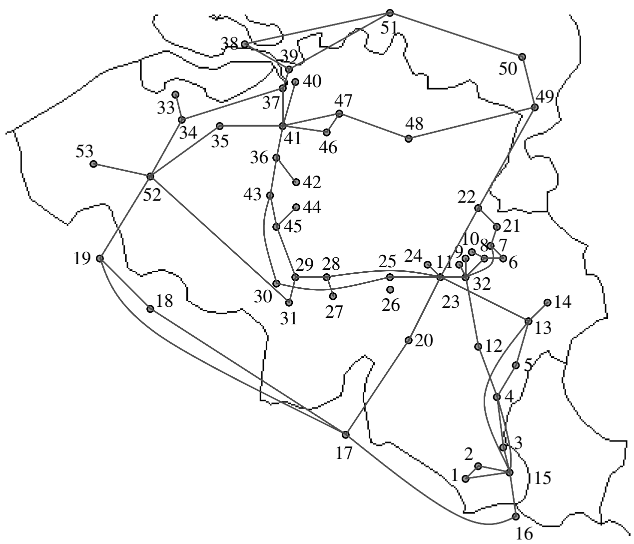
5.2. Tests of the EPEC Scheme

We now test the MPEC/EPEC algorithms on an EPEC problem derived from the stylized Belgian electricity system which was also used in our previous work (Yao et al. 2005). This system is originally composed of 92 380 kv and 220 kv transmission lines including some lines in neighboring countries for capturing the effect of loop flow. Parallel lines between the same pairs of nodes have been collapsed into single lines with equivalent electric characteristics. In total, the stylized network comprises 71 transmission lines and 53 nodes (see Figure 3). Generation units in this system are located, respectively, at the nodes $\{7, 9, 10, 11, 14, 22, 24, 31, 33, 35, 37, 40, 41, 42, 44, 47, 48, 52, 53\}$. The ownership structure, zonal aggregation in the forward market, and contingency states are fictitious and so are the nodal demand functions, although they are calibrated to actual demand information.

Table 2 lists the nodal information for this test problem, including the IDF slopes, the marginal generation costs (marginal costs are constant in this example), and the capacity bounds of the generation units. Table 3 summarizes the impedance of the transmission lines and the corresponding thermal limits. Only lines 22-49, 29-45, 30-43, and 31-52 are assumed prone to congestion in this example. The method for calculating the state-dependent PTDF matrices from the network data can be found in standard electrical engineering textbooks (e.g. Hambley 2004) and will be omitted here due to space limitation.

We assume six independent contingency states in the spot market.[2] The first three states correspond to demand

Figure 3. Belgian high-voltage network.



uncertainty, while all generation units and all transmission lines are rated at their full capacities. States 4, 5, and 6 have the same demand levels as state 2, but they represent the system contingencies resulting from transmission or generator outages. State 4 denotes a transmission breakdown on lines 31-52. States 5 and 6 capture the unavailability of two generation units at nodes 10 and 41, respectively. The price intercepts of the hypothetical IDFs and the probabilities of the six states are given in Table 4.

The stylized system has the dimension $2|C| \times (|N| + |L|) = 684$ of y (and w), and the total number of possible partitions is 2^{684}. In this implementation, we terminate the EPEC algorithm at an outer iteration k if the relative improvement of the MPECs' decision variables (forward commitments) is no greater than 10^{-8}, i.e., $\|\{\bar{x}_g^k - \bar{x}_g^{k-1}\}_{g\in G}\| \leqslant 10^{-8}\|\{\bar{x}_g^{k-1}\}_{g\in G}\|$.

We ran the tests with different numbers of zones and firms, and for each test we start with randomly generated decision variables of the MPECs. In the implementation, the MPEC algorithm was limited to execute a single inner iteration. We also tried some other rules for terminating the MPEC algorithm; however, they do not provide comparable results.[3] The test results are summarized in Table 5. Columns 4 to 9 list the minimum, maximum, and average numbers of outer iterations and quadratic programs, respectively. In addition, Tables 6 and 7 report the outer iterations of the firms' total forward commitments for the cases of two zones and two or three firms. We find that:

- For all test problems, the EPEC scheme converges rapidly.
- There exists no clear relationship between the problem dimension and the number of iterations. However, the total number of quadratic programs grows as the number of firms increases.
- In the tests, the EPEC scheme quickly reaches the proximity of the B-stationary equilibrium, after which it only improves the significant decimal digits (see, for example, Tables 6 and 7).

6. Economic Interpretation of the Results

The EPEC algorithm is not guaranteed to locate a (global) Nash equilibrium; however, as we will demonstrate in this section, it produced results that are consistent with economic intuition.

In particular, we considered two hypothetical generator ownership structures with two zones in the stylized Belgian network: nodes 1 through 32 belong to zone 1, and the remaining nodes to zone 2. The first structure has two firms,

Table 2. Nodal data.

Node	IDF slope $/MW^2	Marg. cost $/MW	Capac. MW	Node	IDF slope $/MW^2	Marg. cost $/MW	Capac. MW	Node	IDF slope $/MW^2	Marg. cost $/MW	Capac. MW
1	1	—	0	19	0.68	—	0	37	1	10	1,399
2	0.82	—	0	20	1.05	—	0	38	0.85	—	0
3	1.13	—	0	21	1	—	0	39	1	—	0
4	1	—	0	22	1.1	19	602	40	1.15	10	1,378*
5	0.93	—	0	23	1	—	0	41	1	21	522
6	0.85	—	0	24	0.75	10	2,985	42	0.79	18	385
7	1	45	70	25	1	—	0	43	0.68	—	0
8	1	—	0	26	0.8	—	0	44	1.03	20	538
9	0.88	18	460	27	1.13	—	0	45	1	—	0
10	0.9	18	121*	28	1	—	0	46	1	—	0
11	1	20	124	29	0.93	—	0	47	1	—	0
12	0.73	—	0	30	0.85	—	0	48	0.73	22	258
13	1	—	0	31	1	18	712	49	1.2	—	0
14	0.85	13	1,164	32	1	—	0	50	1.5	—	0
15	1	—	0	33	0.88	20	496	51	1	—	0
16	1.3	—	0	34	0.5	—	0	52	1	20	879
17	1	—	0	35	1	25	1,053	53	0.7	58	95
18	0.79	—	0	36	0.73	—	0				

*These numbers are zeros in states 5 and 6, respectively.

Table 3. Belgian transmission network data.

Line	Imped. (Ohm)	Capac. (MW)	Line	Imped. (Ohm)	Capac. (MW)	Line	Imped. (Ohm)	Capac. (MW)
1-2	23,716	345	16-17	2,633	5,154	34-37	7,048	1,350
1-15	6,269	345	17-18	4,236	1,715	34-52	12,234	1,350
2-15	8,534	345	17-19	1,939	5,140	35-41	14,204	1,350
3-4	5,339	240	17-20	8,071	1,179	35-52	9,026	1,420
3-15	11,686	240	18-19	1,465	13,710	36-41	15,777	2,770
4-5	6,994	510	19-52	11,321	1,179	36-42	11,186	2,840
4-12	5,887	405	20-23	13,165	1,316	36-43	15,408	2,770
4-15	3,644	240	21-22	47,621	1,420	37-39	66,471	1,420
5-13	6,462	510	22-23	11,391	1,350	37-41	21,295	1,350
6-7	23,987	300	22-49	9,138	1,350	38-39	10,931	1,650
6-8	9,138	400	23-24	41,559	5,540	38-51	17,168	946
7-21	14,885	541	23-25	16,982	1,420	39-51	8,596	1,650
7-32	5,963	410	23-28	8,610	1,350	40-41	11,113	2,770
8-9	45,360	400	23-32	33,255	1,350	41-46	11,509	2,840
8-10	26,541	800	25-26	134,987	1,420	41-47	13,797	1,420
8-32	11,467	400	25-30	11,991	1,420	43-45	34,468	1,350
9-11	20,157	410	27-28	64,753	1,420	44-45	47,128	1,420
9-32	10,012	375	28-29	38,569	1,350	46-47	34,441	1,420
11-32	18,398	375	29-31	284,443	1,350	47-48	14,942	1,420
12-32	4,567	405	29-45	14,534	1,350	48-49	6,998	1,420
13-14	121,410	2,700	30-31	269,973	1,420	49-50	5,943	3,784
13-15	5,094	790	30-43	10,268	1,420	50-51	2,746	5,676
13-23	5,481	2,770	31-52*	1,453	400	52-53	1,279	2,840
15-16	8,839	400	33-34	40,429	1,420			

*This line breaks down in state 4.

where the units at the node set $\{9, 11, 22, 31, 35, 37, 41, 47, 52, 53\}$ belong to the first firm and the remaining units to the second firm. The second structure is composed of three firms, operating the units at $\{7, 11, 33, 37, 41, 52, 53\}$, $\{10, 14, 24, 40, 44, 48\}$, and $\{9, 22, 31, 35, 42, 47\}$, respectively.

We observe that, under both resource structures, firms have strategic incentives for forward contracting as reported in Tables 6 and 7. However, unlike in the single-node case where all firms have strategic incentives for short forward positions (i.e., to sell forward) as shown in Allaz (1992), here some firms might take long forward positions (i.e., buy forward). In the duopoly case, for example, firm 1 buys 552 MWh in the forward market, but firm 2 is short 1,747 MWh, so the supply side as a whole sells forward, which is qualitatively consistent with the result in Allaz (1992). A generation firm could take long forward positions when an arbitrage opportunity is raised from a discrepancy between its expected generator-location-weighted average nodal price in a zone and the zonal settlement price. In such a case, a firm's speculative incentive for long positions may overwhelm the strategic "Allaz-Vila"-type incentive for selling forward. Such an action, however, will typically induce rival firms to increase forward sales due to the increased treat of low spot prices, as observed in our duopoly example.

In Figure 4, we plot the expected spot nodal prices under two settlements and contrast them with the corresponding

Table 4. States of the Belgian spot market.

State	IDF intercept ($/MW^2)	Probability	Type and description
1	1,000	0.20	Demand uncertainty: Demands are on the peak.
2	500	0.50	Demand uncertainty: Demands are at shoulder.
3	250	0.20	Demand uncertainty: Demands are off-peak.
4	500	0.03	Contingency of line breakdown: Line 31-52 goes down.
5	500	0.03	Contingency of generation outage: Plant at node 10 goes down.
6	500	0.04	Contingency of generation outage: Plant at node 41 goes down.

Table 5. Test results of the EPEC algorithm.

			Outer iterations			Quadratic programs		
$\lvert Z\rvert$	$\lvert G\rvert$	Dim(y)	Min	Max	Average	Min	Max	Average
2	2	684	4	8	6	15	29	24
2	3	684	8	11	9	47	65	55
2	4	684	7	12	10	55	94	77
3	2	684	2	4	3	7	15	11
3	3	684	4	6	5	23	34	29
3	4	684	4	10	8	32	79	62
4	2	684	3	9	7	11	36	27
4	3	684	7	21	13	41	126	80
4	4	684	9	25	15	70	197	122

Table 6. Iterations of the firms' total forward commitments (two firms).

Outer iteration	Firm 1	Firm 2
0	0.000000	0.000000
1	−513.063752	575.219726
2	−331.223467	1,546.721883
3	−545.254227	1,747.692181
4	−552.287608	1,747.692181
5	−552.287608	1,747.692181

nodal prices in the equilibrium of a single-settlement market which is obtained by constraining all firms' forward positions to zero. We first note that, whether or not there exists a forward market, the three-firm structure yields lower spot equilibrium prices than the Duopoly structure, as one would expect. Moreover, under both the two- and three-firm structures, a two-settlement equilibrium results in lower spot equilibrium prices at most nodes than a single settlement. However, nodes 29 and 31 do not follow this trend. Consequently, two settlements increase social welfare and consumer surplus. These results suggest that the welfare-enhancing effect described in Allaz (1992) and Allaz and Vila (1993) generalizes to the case with flow congestion, system contingency, and demand uncertainty, although that effect is quantitatively different due to generator capacities and transmission limits.

7. Concluding Remarks

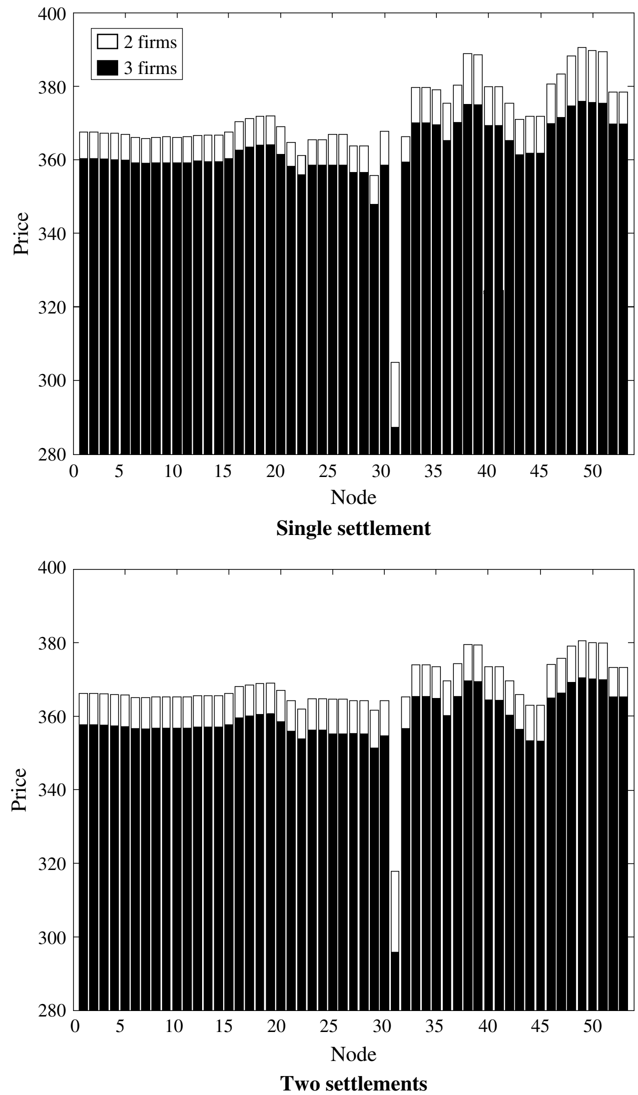
We study the Nash-Cournot equilibrium in two-settlement electricity markets. We develop an EPEC model of this equilibrium, in which each firm solves an MPEC problem parameterized by the design variables of the other MPECs.

We propose an MPEC algorithm by taking advantage of the special properties of the problems at hand. This algorithm partitions the feasible region of the decision variables into a set of polyhedra, and projects the state variables into the space of the decision variables. The algorithm solves a quadratic program for a stationary point in each polyhedron and pivots through adjacent polyhedra while maintaining

Table 7. Iterations of the firms' total forward commitments (three firms).

Outer iteration	Firm 1	Firm 2	Firm 3
0	0.000000	0.000000	0.000000
1	6,739.889190	−16.249658	−288.471837
2	6,739.889190	246.601419	−103.536223
3	6,851.687937	556.357457	71.319790
4	7,001.487699	849.405693	154.719273
5	7,154.268773	1,001.093059	149.846951
6	7,237.416442	1,006.167745	149.619740
7	7,239.775870	1,006.342137	149.611431
8	7,239.859233	1,006.348165	149.611140
9	7,239.862110	1,006.348374	149.611129
10	7,239.862110	1,006.348382	149.611130
11	7,239.862110	1,006.348382	149.611130

Figure 4. Expected spot nodal prices.



feasibility of the linear complementarity constraints. We establish the finite global convergence of this MPEC algorithm. An EPEC scheme is constructed by deploying the MPEC algorithm iteratively. Numerical tests on randomly generated quadratic MPECs and on the EPEC derived from a stylized Belgian electricity network demonstrate the effectiveness of the algorithms.

One limitation of our model is the assumption of risk neutrality on the part of the generating firms. Unfortunately, introducing risk aversion will make the objective functions of the MPECs nonquadratic, which significantly increases the computational complexity of the model.

On the other hand, we like to point out that although the MPEC and EPEC algorithms are presented here in the context of two-settlement electricity markets, they can be applied to other quadratic EPEC problems, provided that the linear complementarity constraints yield unique values of the state variables.

8. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

Endnotes

1. This is a limited representation of uncertainty that can capture major contingencies and, to some extent, demand variations within the forward contract period such as seasonal variations. Because we assume that uncertainties are realized before the spot market decisions occur, we do not capture uncertainties that are not common knowledge in the spot market and are revealed implicitly through the spot prices. SFE models are better suited to capture uncertainties that are revealed after spot market decision making, but using SFE models with transmission constraints is at this point not well understood and is computationally prohibitive.
2. In this example, we simplify the problem by assuming that forward contracts are for fixed quantities, and we represent systematic demand variation within the contract period as demand uncertainty that is realized prior to the spot market decisions. In reality, systematic demand changes can be handled through forward contracts with variable quantities that attempt to follow systematic variations in load. To handle contracts that specify different forward quantities and prices for peak and off-peak periods, for example, we could break up the problem into two separate problems for peak and off-peak demand periods.
3. The idea of not solving subproblems to completion is quite common in mathematical programming. Examples of such approaches include nonlinear programming procedures with inexact line search and sequential quadratic programming, which can be interpreted as a penalized Lagrangian method where a single Newton step is performed each time and the estimates of the Lagrange multipliers are updated.

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