Analyzing Valid Inequalities of the Generation Unit Commitment Problem

Kory W. Hedman, *Student Member, IEEE*, Richard P. O'Neill, and Shmuel S. Oren, *Fellow, IEEE*

***Abstract*--The use of Mixed Integer Programming (MIP) within the electric industry is increasing. Many US ISOs are testing and planning to use MIP in the near future or they are already using MIP. There are various MIP formulations published for generation unit commitment with little consensus as to which formulation is preferred. In particular, various valid inequalities are used to model the minimum up and down time constraints for generation unit commitment. In this paper, we first discuss valid inequalities and facet defining valid inequalities. We then present and compare these previously published valid inequalities and we demonstrate why certain valid inequalities dominate other valid inequalities. We also present previously published facet defining valid inequalities.**

***Index Terms*--Mixed integer programming, generation unit commitment, power generation dispatch, power system economics, valid inequalities, facets, convex hull**

NOMENCLATURE

Indices
g: generator
t: period

Variables
$u_{g,t}$: binary, unit commitment variable for generator g, period t (0 down, 1 operational)
$v_{g,t}$: startup variable for generator g, period t (1 for startup, 0 otherwise)
$w_{g,t}$: shutdown variable for generator g, period t (1 for shutdown, 0 otherwise)

Parameters
UT_g: minimum up time for generator g
DT_g: minimum down time for generator g
T: number of periods

I. INTRODUCTION

Generation unit commitment is a well-known, difficult problem to solve within the electric industry. There are various proposed ways to solve generation unit commitment problems [1], [2], and [3]. Within this paper, we focus on the use of Mixed Integer Programming (MIP) to solve the generation unit commitment problem. This research focuses on how to better formulate the problem.

The use of MIP within the electric industry is growing. Recently, PJM switched from a Lagrangian Relaxation (LR) approach to a MIP approach for their generation unit commitment software [4] and for their real-time market look-ahead [5]. These changes are estimated to save PJM over 150 million dollars per year [4], [5], and [6]. CAISO is planning on switching from LR to MIP as well; they estimate the savings to be in the millions [7]. Furthermore, most US ISOs are testing and planning to switch to MIP in the near future [8].

Published in 2005, [9] discusses the tradeoffs between LR and MIP for PJM and a recent presentation, [10], discusses the implementation of MIP in PJM. There also have been improvements in MIP software. In particular, CPLEX has shown improvements; [8] provides an overview of how CPLEX has progressed over the years for basic generation unit commitment models.

With this increased use of MIP within the electric industry, it is important to have a sound understanding of mixed integer programming. LR is commonly taught within power engineering classrooms today with little mention of MIP even as many ISOs are in the process of changing from LR to MIP [8].

With this increased emphasis on MIP for generation unit commitment, we analyze various published valid inequalities used within generation unit commitment models. In particular, the minimum up and down time constraints are one key complicating source within the generation unit commitment problem. For this reason, we investigate the use of valid inequalities and facets for the minimum up and down time constraints as these inequalities help improve the computational time and the lower bound associated with the generation unit commitment problem. In depth analysis of the inequalities used in generation unit commitment formulations has not been an emphasis in the past. With the advances in MIP and with ISOs switching to MIP, it is now more advantageous to understand which valid inequalities are preferred. We do not present our own, novel valid inequalities for the generation unit commitment problem; rather, our purpose of this research is to present, analyze, and discuss these previously published valid inequalities.

The paper is organized as follows. Section II discusses valid inequalities, facets, the convex hull, and a brief overview of the generation unit commitment problem; we also discuss and present facet defining valid inequalities for the generation unit commitment problem. In section III, we present previously published valid inequalities used to model the minimum up and down time constraints and we analyze these various valid inequalities in order to determine which valid inequalities are preferred. Section IV provides a discussion on possible future work and section V concludes this paper.

II. VALID INEQUALITIES WITHIN GENERATION UNIT COMMITMENT FORMULATIONS

Generation unit commitment is a well-known, difficult problem to solve within the electric industry. For this reason, it is important to examine ways to reduce the computational complexity of the problem. Within this section, we provide a

978-1-4244-3811-2/09/$25.00 ©2009 IEEE

brief overview of valid inequalities and generation unit commitment; we also compare the different valid inequalities that have been proposed for the minimum up and down time constraints.

A. *Valid Inequalities, Facets, and the Convex Hull*

Mixed integer programs can be enhanced by applying cuts, i.e. constraints, to the problem. It is a common misconception that adding additional constraints only complicates an optimization problem. The purpose of applying valid inequalities, or cuts, is to "cut off" integer infeasible solutions that are feasible solutions within the LP relaxed problem of the MIP without cutting off feasible integer solutions. The LP relaxed problem is the problem where the integrality constraints of the MIP problem are relaxed. Applying these cuts can provide a tighter, better lower bound (i.e. for minimization problems this approach can increase the LP relaxed optimal solution), reduce the number of branch and bound nodes required to be searched, and it can improve the computational time.

Facets are a special type of valid inequality and they can be used to produce the convex hull of the MIP. The convex hull is the minimal convex set that contains all feasible points to the MIP. Facets are the valid inequalities that define this minimal convex set.

By relaxing the integrality constraints within a MIP, the problem can be solved by linear programming (LP). However, the optimal solution to this relaxed problem is generally not integer feasible, i.e. it is not a feasible solution to the actual MIP problem. The important property with convex hulls is that every extreme point solution of the convex hull is a feasible solution to the MIP. Consequently, the convex hull of a MIP problem can be solved by LP and the resulting LP optimal solution will be the optimal solution to the MIP problem.

Valid inequalities can be compared numerically by comparing the solutions to the LP relaxation, the number of branch and bound nodes, and the CPU time. Another way to test the strength of a valid inequality is to determine its dimensional face. Let X define the feasible region of the MIP problem. A valid inequality is said to be an s-dimensional face for X if there exist $s+1$ affinely independent points in X that satisfy the valid inequality when the valid inequality is set as an equality constraint rather than an inequality constraint. If $X \in \Re^n$, an $n\text{-}1$ dimensional face is called a facet. Valid inequalities that are higher dimensional faces are preferred. Valid inequalities can also be compared based on whether one inequality dominates another. We discuss the strengths of these valid inequalities in section III by seeing which inequalities dominate other inequalities, see [11] for further discussion on faces, dimensions of faces, valid inequalities, and the convex hull.

B. *Generation Unit Commitment Overview*

Generators have startup costs, minimum operating levels, minimum up and down times, etc and these characteristics require binary variables so that they can be properly modeled. There is a binary, unit commitment variable, $u_{g,t}$, that takes on a value of one (zero) when the unit is on (off). Every unit is assumed to have a startup cost; therefore, there is a startup binary variable, $v_{g,t}$. When the unit is turned on during period t, this variable takes on a value of one; otherwise, the startup variable's value is zero.

Even if shutdown costs are not included in the optimization problem, it may be advantageous to include shutdown variables. The shutdown binary variable, w, can actually be treated as a continuous variable since, by the constraints that are set within the model, its final value will always be binary once the unit commitment, u variables are determined. The same is the case for the startup, v variable as you can relax the integrality constraint since the variable will take on a binary value. Thus, the u variables are modeled as binary variables while the v and w variables are continuous variables with a lower bound of zero and an upper bound of one. Adding additional binary variables usually adds to the complexity of the problem. Since the shutdown variable can be modeled as continuous and it is helpful within some of the inequalities, we include it. By incorporating the shutdown variable, we have the relationship defined in (1). The use of this constraint can be found in papers that were published over forty years ago, [12]

$$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1}, \ \forall g,t. \tag{1}$$

C. *Minimum Up/Down Polytopes and Facet Defining Valid Inequalities*

Two papers investigated and proved that they have determined facet defining valid inequalities for particular projections of the generation unit commitment problem. [13] defines the *alternating up/down* inequalities, which are facets for the polytope[1] of the minimum up and down time constraints. This polytope is a projection of the u variables from the generation unit commitment problem. [13] did not consider startup or shutdown costs so they did not incorporate the startup and shutdown variables. To define the *alternating up* inequalities, first choose a set of $2k+1$ indices over the interval [1,..,T] such that (2) and (3) hold. Call this set $\overline{\Omega}$. For the *alternating down* inequalities, the set of indices must satisfy (2) and (4). Let this set be listed as $\underline{\Omega}$. Based on these sets of indices, the *alternating up* inequalities are defined by (5) and the *alternating down* inequalities are defined by (6)

$$\phi(1) < \psi(1) < \phi(2) < \psi(2) < ... < \phi(k) < \psi(k) < \phi(k+1) \tag{2}$$

$$\phi(k+1) - \phi(1) \le UT_g \tag{3}$$

$$\phi(k+1) - \phi(1) \le DT_g \tag{4}$$

$$-\sum_{j=1}^{k+1} u_{g,\phi(j)} + \sum_{j=1}^{k} u_{g,\psi(j)} \le 0, \ \forall g, \overline{\Omega}_g \tag{5}$$

$$\sum_{j=1}^{k+1} u_{g,\phi(j)} - \sum_{j=1}^{k} u_{g,\psi(j)} \le 1, \ \forall g, \underline{\Omega}_g. \tag{6}$$

[14] examined the polytope of the minimum up and down time constraints assuming there are startup and shutdown costs. They define the *turn on/off* inequalities, which are facets

[1] A bounded polyhedron is called a polytope.

for the projection of the u and v variables from the generation unit commitment problem. [14] also proved that these inequalities dominate, i.e. they are stronger valid inequalities, the *alternating up/down* facet defining inequalities from [13], for the generation unit commitment problem with startup and shutdown costs. The *turn on/off* inequalities, along with other trivial, facet defining inequalities produce the convex hull of this projection of variables u and v. The set of facets in [13] produce the convex hull of a projection of the generation unit commitment problem; they do not produce the convex hull of the entire generation unit commitment problem.

The *turn on/off* inequalities are defined by (7) and (8). By using (1), (7) and (8) can be re-written into (7a) and (8a). This can be useful in case someone has only startup (shutdown) costs and prefers to only use startup (shutdown) variables. Equations (7) and (8) are not repeated for all periods but for the periods $t \in \{UT,..,T\}$ and $t \in \{DT,..,T\}$ because [14] proved that the remaining inequalities are dominated by (7) and (8)[2]

$$\sum_{q=t-UT_g+1}^{t} v_{g,q} \le u_{g,t}, \ \forall g, t \in \{UT_g,..,T\} \tag{7}$$

$$\sum_{q=t-DT_g+1}^{t} w_{g,q} \le 1 - u_{g,t}, \ \forall g, t \in \{DT_g,..,T\} \tag{8}$$

$$\sum_{q=t-UT_g+1}^{t} w_{g,q} \le u_{g,t-UT_g}, \ \forall g, t \in \{UT_g,..,T\} \tag{7a}$$

$$\sum_{q=t-DT_g+1}^{t} v_{g,q} \le 1 - u_{g,t-DT_g}, \ \forall g, t \in \{DT_g,..,T\}. \tag{8a}$$

III. Analyzing Valid Inequalities Used for Min Up and Down Time Constraints

It is common to model the startup and shutdown variable as

$$v_{g,t} \ge u_{g,t} - u_{g,t-1}, \ \forall g, t \tag{9}$$

$$w_{g,t} \ge u_{g,t-1} - u_{g,t}, \ \forall g, t. \tag{10}$$

[14] showed that (9) is a facet for the projection of variables u and v from the generation unit commitment problem. By introducing w and (1) into the optimization formulation, it can be easily shown that (9) and (10) are dominated by (1). Equation (1) can be written as a greater than or equal to constraint and a less than or equal to constraint. The greater than or equal to constraint can then be re-written as (1a) and the less than or equal to constraint can be re-written as (1b)

$$v_{g,t} \ge u_{g,t} - u_{g,t-1} + w_{g,t}, \ \forall g, t \tag{1a}$$

$$w_{g,t} \ge u_{g,t-1} - u_{g,t} + v_{g,t}, \ \forall g, t. \tag{1b}$$

[2] [14] defines $t \in \{1,..,T\}$ and defines (7) and (8) over $t \in \{UT_g + 1,..,T\}$ and $t \in \{DT_g + 1,..,T\}$. We define (7) and (8) over different sets, $t \in \{UT_g,..,T\}$ and $t \in \{DT_g,..,T\}$, since we model all generators with an initial state, *t=0*.

Since both w and v are always non-negative, it is evident that (1a) and (1b) dominate (9) and (10) respectively, i.e. (1) dominates both (9) and (10). Since (1a) is the same as (9) with the exception of the addition of w, a non-negative variable, on the right hand side, the value of v from (1a) will always satisfy (9) but not vice versa, thereby demonstrating why (1a) dominates (9). Similarly, it is easy to see that (1b) dominates (10). When the integrality constraints are relaxed, any solution that satisfies (1) will be satisfied by (9) and (10) but there may be solutions that are satisfied by (9) and (10) that violate (1). Since these inequalities are all valid, (1a) and (1b) are stronger valid inequalities than (9) and (10), i.e. (1) dominates (9) and (10) as the relaxed feasible region with (1) is tighter than with (9) and (10).

Since a generator cannot be turned on and turned off during the same period, another inequality that is commonly used, see [15] and [16], is

$$v_{g,t} + w_{g,t} \le 1, \ \forall g, t. \tag{11}$$

However, it is easy to see that this inequality, (11), is dominated by other valid inequalities. The startup variable can never take on a value of one unless the unit commitment variable has a value of one during the same period. Likewise, the startup variable does not have a value of one unless the previous period the unit commitment variable has a value of zero. These two simple logic statements produce (12) and (13); by similar logic, it is easy to produce (14) and (15) for the shutdown variable

$$v_{g,t} \le u_{g,t}, \ \forall g, t \tag{12}$$

$$v_{g,t} \le 1 - u_{g,t-1}, \ \forall g, t \tag{13}$$

$$w_{g,t} \le u_{g,t-1}, \ \forall g, t \tag{14}$$

$$w_{g,t} \le 1 - u_{g,t}, \ \forall g, t. \tag{15}$$

Adding (12) and (15) together or adding (13) and (14) together will produce (11). This clearly demonstrates that (11) is dominated by these stronger valid inequalities as whatever is satisfied by (12)-(15) will be satisfied by (11) but not vice versa. However, (7) clearly dominates (12), (8a) dominates (13), (7a) dominates (14), and (8) dominates (15). Thus, by no surprise, the facets defined in [14] dominate (11).

[17] and [18] provide the valid inequalities (18) and (19) as ways to enforce the minimum up and down time constraints. The equations state that once the generator is turned on (off) in period t, the following u variables corresponding to the minimum up (down) time constraint must be equal to one (zero) as well

$$\overline{\tau_{g,t}} \in \{t+1,.., \min(t + UT_g - 1, T)\}, \ \forall g, t \tag{16}$$

$$\underline{\tau_{g,t}} \in \{t+1,.., \min(t + DT_g - 1, T)\}, \ \forall g, t \tag{17}$$

$$u_{g,t} - u_{g,t-1} \le u_{g,\overline{\tau_{g,t}}}, \ \forall g, \overline{\tau_{g,t}}, t \tag{18}$$

$$u_{g,t-1} - u_{g,t} \le 1 - u_{g,\underline{\tau_{g,t}}}, \ \forall g, \underline{\tau_{g,t}}, t. \tag{19}$$

[17] does not include startup or shutdown costs so there is no need to include the startup and shutdown variables. For generation unit commitment problems that do include these variables, it is preferred to modify (18) and (19) as shown below by (18a) and (19a). First, it is essential that (18a) and (19a) are still valid inequalities to the generation unit commitment problem. This is easily verified since v and w will take on the exact same value as the left hand sides of (18a) and (19a) respectively when the unit commitment variables are integers. The reason (18a) and (19a) are preferred is due to the fact that they cut off integer infeasible solutions that are feasible for the relaxed problem, i.e. the unit commitment problem with all integrality constraints relaxed. This can be seen by (1a) and (1b); v and w can both have non-integer values when the unit commitment variables are not binary. By (1a) and (1b), it is clear that, for such a situation, v and w are then larger than the left hand sides of (18) and (19) respectively. Since (18a) and (19a) are still valid inequalities, it is therefore preferable to use them over (18) and (19) for problems with startup and shutdown variables. Note that we have updated $\overline{\tau}$ and $\underline{\tau}$ to include t within (16a) and (17a) as this is permissible since the left hand side of (18a) and (19a) now include v and w

$$\overline{\tau_{g,t}} \in \{t,..,\min(t+UT_g-1,T)\},\ \forall g,t \tag{16a}$$
$$\underline{\tau_{g,t}} \in \{t,..,\min(t+DT_g-1,T)\},\ \forall g,t \tag{17a}$$
$$v_{g,t} \le u_{g,\overline{\tau_{g,t}}},\ \forall g,\overline{\tau_{g,t}},t \tag{18a}$$
$$w_{g,t} \le 1-u_{g,\underline{\tau_{g,t}}},\ \forall g,\underline{\tau_{g,t}},t\,. \tag{19a}$$

Similar to (18a) and (19a), one can determine the valid inequalities that are listed by (22) and (23). Equation (18a) states that the unit commitment variable must take on a value of one for the periods defined by $\overline{\tau}$ when v takes on a value of one for period t. This enforces the minimum up time constraint. Using similar logic, we have (22), which states that the unit commitment variable must take on a value of one for the periods defined by $\overline{\Gamma}$ if the w has a value of one for period t. Likewise, we can see how (23) is another way to enforce the minimum down time constraints. When v has a value of one for period t, then the unit must be off for at least the previous DT periods

$$\overline{\Gamma_{g,t}} \in \{\max(t-UT_g,1),..,t-1\},\ \forall g,t \tag{20}$$
$$\underline{\Gamma_{g,t}} \in \{\max(t-DT_g,1),..,t-1\},\ \forall g,t \tag{21}$$
$$w_{g,t} \le u_{g,\overline{\Gamma_{g,t}}},\ \forall g,\overline{\Gamma_{g,t}},t \tag{22}$$
$$v_{g,t} \le 1-u_{g,\underline{\Gamma_{g,t}}},\ \forall g,\underline{\Gamma_{g,t}},t\,. \tag{23}$$

These equations, (16a)-(23), can be rewritten as (24)-(31). The left hand side of (26) can be summed from $t\text{-}UT\text{+}1$ to t, i.e. for all values of $\overline{\tau}$, and this sum would still be $\le u$. This is still a valid inequality because there can never be more than one v variable that has a value of one over any UT consecutive periods. In addition, any time that the u variable has a value of one, there must be exactly one v variable that has a value of one over the last UT periods. By summing up only the left hand side of (26) over $\overline{\tau}$, we reproduce (7). Likewise, one can easily create (8) from (27), (7a) from (30), and (8a) from (31). This demonstrates where the facet defining valid inequalities come from and it also demonstrates that (16)-(31) are dominated by them

$$\overline{\tau_{g,t}} \in \{\max(t-UT_g+1,1),..,t\},\ \forall g,t \tag{24}$$
$$\underline{\tau_{g,t}} \in \{\max(t-DT_g+1,1),..,t\},\ \forall g,t \tag{25}$$
$$v_{g,\overline{\tau_{g,t}}} \le u_{g,t},\ \forall g,\overline{\tau_{g,t}},t \tag{26}$$
$$w_{g,\underline{\tau_{g,t}}} \le 1-u_{g,t},\ \forall g,\underline{\tau_{g,t}},t\,. \tag{27}$$
$$\overline{\Gamma_{g,t}} \in \{\max(t-UT_g+1,1),..,t\},\ \forall g,t \tag{28}$$
$$\underline{\Gamma_{g,t}} \in \{\max(t-DT_g+1,1),..,t\},\ \forall g,t \tag{29}$$
$$w_{g,\overline{\Gamma_{g,t}}} \le u_{g,t-UT_g},\ \forall g,\overline{\Gamma_{g,t}},t \tag{30}$$
$$v_{g,\underline{\Gamma_{g,t}}} \le 1-u_{g,t-DT_g},\ \forall g,\underline{\Gamma_{g,t}},t\,. \tag{31}$$

There are many different ways to write the minimum up and down time constraints. [15] and [16] use (32) and (33) to enforce the minimum up and down time constraints

$$v_{g,t} + \sum_{q=t+1}^{\min(t+UT_g-1,T)} w_{g,q} \le 1,\ \forall g,t \tag{32}$$
$$w_{g,t} + \sum_{q=t+1}^{\min(t+DT_g-1,T)} v_{g,q} \le 1,\ \forall g,t\,. \tag{33}$$

The first noticeable fact is that (32) and (33) are dominated by (32a) and (33a). This is verified by the fact that v and w cannot have a value of one at the same time, as stated by the valid inequality (11). Therefore, there is no reason to have the index of the sum, q, start at $t\text{+}1$ rather than having it start at t. As a result, (32a) and (33a) cut off integer infeasible solutions, which are feasible for the relaxed problem, that are not cut off by (32) and (33); therefore, (32a) and (33a) are preferred over (32) and (33)

$$v_{g,t} + \sum_{q=t}^{\min(t+UT_g-1,T)} w_{g,q} \le 1,\ \forall g,t \tag{32a}$$
$$w_{g,t} + \sum_{q=t}^{\min(t+DT_g-1,T)} v_{g,q} \le 1,\ \forall g,t\,. \tag{33a}$$

Seeing that (32a) and (33a) dominate (32) and (33) respectively is straightforward. However, it is possible to further improve these valid inequalities. If v takes on a value of one for period t, it cannot take on a value of one again until on or after period $t\text{+}DT\text{+}UT$. The exact same is true for w. Both v and w must be zero for at least $DT\text{+}UT\text{-}1$ periods immediately following a period where it has a value of one.

Therefore, the index for (32) and (33) should not begin at *t+1* but rather the index can begin at *t-DT+1* for (32) and *t-UT+1* for (33). This gives us the valid inequalities defined by (32b) and (33b) below

$$v_{g,t} + \sum_{q=\max(t-DT_g+1,1)}^{\min(t+UT_g-1,T)} w_{g,q} \le 1,\ \forall g,t \tag{32b}$$

$$w_{g,t} + \sum_{q=\max(t-UT_g+1,1)}^{\min(t+DT_g-1,T)} v_{g,q} \le 1,\ \forall g,t\,. \tag{33b}$$

A simple proof to see that (32b) and (33b) are still valid inequalities is to use (1) to replace *w* in (32b) and replace *v* in (33b). Expanding these equations, you can obtain (32b.1) and (33b.1). It is then easy to see that they each become the sum of two of the facet defining valid inequalities listed by (7)-(8a) once the index for the parts in parentheses are adjusted. Thus, these are valid inequalities and they are obviously dominated by the facet defining valid inequalities presented in [14]. We have shown how the published valid inequalities listed by (32) and (33) can be easily enhanced and we have demonstrated that they are indeed dominated by the facets defined within [14]

$$\sum_{q=t-DT_g+1}^{t} v_{g,q} + \left(\sum_{q=t}^{t+UT_g-1} v_{g,q}\right) \le 1 - u_{t-DT_g} + \left(u_{t+UT_g-1}\right),\ \forall g,t \tag{32b.1}$$

$$\sum_{q=t-UT_g+1}^{t} w_{g,q} + \left(\sum_{q=t}^{t+DT_g-1} w_{g,q}\right) \le u_{t-UT_g} + \left(1 - u_{t+DT_g-1}\right),\ \forall g,t\,. \tag{33b.1}$$

There are other ways of modeling the minimum up and down time constraints, see [19] and [20]; these methods use big M values, i.e. they use a large multiplier. Big M values are known to complicate MIPs and it is often preferred to avoid using big M values if possible. Considering that we have these facet defining valid inequalities, these inequalities are preferred over these methods that use big M values.

We are now able to formulate the generation unit commitment problem with the preferred valid inequalities. Equation (1) is used to define the relationship between the startup, the shutdown, and the unit commitment variables. It is preferred to use (1) over (9) and (10) for the reasons previously discussed. The facet defining valid inequalities (7) and (8) are the preferred valid inequalities to model the minimum up and down time constraints as they have been shown to dominate the other valid inequalities. The only other constraints to be included are the lower and upper bound constraints for the variables. These constraints are listed below

$$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1},\ \forall g,t \tag{1}$$

$$\sum_{q=t-UT_g+1}^{t} v_{g,q} \le u_{g,t},\ \forall g,t \in \{UT_g,..,T\} \tag{7}$$

$$\sum_{q=t-DT_g+1}^{t} w_{g,q} \le 1 - u_{g,t},\ \forall g,t \in \{DT_g,..,T\} \tag{8}$$

$$0 \le v_{g,t} \le 1,\ \forall g,t \tag{34}$$

$$0 \le w_{g,t} \le 1,\ \forall g,t \tag{35}$$

$$u_{g,t} \in \{0,1\},\ \forall g,t\,. \tag{36}$$

IV. Future Work

Generation unit commitment is a challenging MIP problem by itself. The motivation of this research is to assist our research on another topic, optimal transmission switching [21]-[24] and, in particular, [25]. Optimal transmission switching introduces binary variables to represent the state of the transmission elements. Our current research, [25], focuses on determining the benefits when co-optimizing both generator commitment schedules and the topology of the network. Combining these two difficult MIP problems together is challenging, thereby requiring us to examine the valid inequalities that are used within the generation unit commitment formulation in order to improve the computational time of the problem. We are currently testing the various valid inequalities discussed within this paper on the RTS 96 model, [26] and [27].

V. Conclusions

Many ISOs have switched or will be switching over to MIP for generation unit commitment models in the near future. This increases the need to properly understand and analyze the techniques used in MIP. Valid inequalities are commonly used in MIP to improve solution times and lower bounds. These improvements are especially crucial due to the difficulty of generation unit commitment problems.

Within this paper, we provide an overview of valid inequalities and generation unit commitment; we also compare different valid inequalities that have been published and used for the minimum up and down time constraints. We have shown how to analyze these valid inequalities and we have shown how some of these published valid inequalities can be easily enhanced.

References

[1] G. B. Sheblé and G. N. Fahd, "Unit commitment literature synopsis," *IEEE Trans. Power Syst.,* vol. 9, no. 1, pp. 128-135, Feb. 1994.

[2] N. P. Padhy, "Unit commitment – a bibliographical survey," *IEEE Trans. Power Syst*., vol. 19, no. 2, pp. 1196-1205, May 2004.

[3] H. Y. Yamin, "Review on methods of generation scheduling in electric power systems," *Electric Power Systems Research,"* vol. 69, no. 2-3, pp. 227-248, May 2004.

[4] PJM, "News Release: August 14th, 2003: Outage Anniversary," Available: http://www.pjm.com/contributions/news-releases/2004/20040811-aug-14-outage-anniversary.pdf, pp. 3, visited October 2007.

[5] A. Ott, VP, PJM, Norristown, PA, private communication, July 2008.

[6] PJM, "PJM's new generation-scheduling software to save customers estimated $56 million – unit commitment system uses first of its kind technique," PJM News Release, June 24, 2004.

[7] M. Rothleder, "Mixed-integer programming solution at the CAISO," HEPG Forty-Ninth Plenary Session, Dec. 2007.

Available: www.hks.harvard.edu/hepg/rlib_rp_competitive_models.html

[8] R. P. O'Neill, "It's getting better all the time (with mixed integer programming)," HEPG Forty-Ninth Plenary Session, Dec. 2007. Available: www.hks.harvard.edu/hepg/rlib_rp_competitive_models.html

[9] D. Streiffert, R. Philbrick, and A. Ott, "A mixed integer programming solution for market clearing and reliability analysis," *IEEE PES General Meeting, 2005,* vol. 3, pp. 2724-2731, June 2005.

[10] A. Ott, "MIP-based unit commitment implementations in the PJM markets," HEPG Forty-Ninth Plenary Session, Dec. 2007. Available: www.hks.harvard.edu/hepg/rlib_rp_competitive_models.html

[11] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*. New York: Wiley-Interscience, 1999.

[12] L. L. Garver, "Power generation scheduling by integer programming – development of theory," *AIEE Trans*., vol. 81, no. 3, pp. 730-734, Apr. 1962.

[13] J. Lee, J. Leung, and F. Margot, "Min-up/min-down polytopes," *Discrete Optimization,* vol. 1, no. 1, pp: 77-85, June 2004.

[14] D. Rajan and S. Takriti, "Minimum up/down polytopes of the unit commitment problem with start-up costs," IBM Research Report, June 2005.

[15] T. Li and M. Shahidehpour, "Price-based unit commitment: a case of Lagrangian relaxation versus mixed integer programming," *IEEE Trans. Power Syst.,* vol. 20, no. 4, pp: 2015-2025, Nov. 2005.

[16] G. W. Chang, Y. D. Tsai, C. Y. Lai, and J. S. Chung, "A practical mixed integer linear programming based approach for unit commitment," *IEEE PES General Meeting 2004,* vol. 1, pp. 221-225, June 2004.

[17] S. Takriti, B. Krasenbrink, and L. S. Y. Wu, "Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem," Operations Research, vol. 48, no. 2, pp. 268-280, March 2000.

[18] S. Takriti and J. R. Birge, "Using integer programming to refine Lagrangian-based unit commitment solutions," *IEEE Trans. Power Syst*., vol. 15, no. 1, pp. 151-156, Feb. 2000.

[19] J. M. Arroyo and A. J. Conejo, "Optimal response of a thermal unit to an electricity spot market," *IEEE Trans. Power Syst.*, vol. 15, no. 3, pp. 1098-1104, Aug. 2000.

[20] R. D. Naidoo, "A mixed integer programming formulation of generator startup costs," *IPEC 2007,* pp. 173-176, Dec. 2007.

[21] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," *IEEE Trans Power Syst.*, vol. 23, no. 3, pp. 1346-1355, Aug. 2008.

[22] K. W. Hedman, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching – sensitivity analysis and extensions," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1469-1479, Aug. 2008.

[23] K. W. Hedman, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching with contingency analysis," *IEEE Trans. Power Syst.*, submitted for publication.

[24] K. W. Hedman, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching applied to ISO networks," working paper.

[25] K. W. Hedman, R. P. O'Neill, and S. S. Oren, "Optimal transmission switching and generation unit commitment," working paper.

[26] University of Washington, Dept of Electrical Engineering, "Power System Test Case Archive," July 2007. Available: https://www.ee.washington.edu/research/pstca/index.html

[27] C. Grigg, P. Wong, *et al.*, "The IEEE reliability test system-1996. A report prepared by the reliability test system task force of the application of probability methods subcommittee," *IEEE Trans. Power Syst.*, vol. 14, Issue 3, pp. 1010-1020, Aug. 1999.

BIOGRAPHY

Kory W. Hedman (S'05) received the B.S. degree in electrical engineering and the B.S. degree in economics from the University of Washington, Seattle, in 2004 and the M.S. degree in economics and the M.S. degree in electrical engineering (thesis option) from Iowa State University, Ames, in 2006 and 2007, respectively. He is pursuing the Ph.D. degree at the University of California at Berkeley in the Department of Industrial Engineering and Operations Research. He is currently researching optimal transmission switching.

Richard P. O'Neill received the B.S. degree in chemical engineering and the Ph.D. degree in operations research from the University of Maryland at College Park. Currently, he is the Chief Economic Advisor in the Federal Energy Regulatory Commission (FERC), Washington, DC. He was previously on the faculty of the Department of Computer Science, Louisiana State University, and the Business School at the University of Maryland at College Park.

Shmuel S. Oren (M'72-SM'99-F'02) received the B.Sc. and M.Sc. degrees in mechanical engineering and in materials engineering from the Technion Haifa, Israel, and the MS. and Ph.D. degrees in engineering economic systems from Stanford University, Stanford, CA, in 1972. He is a Professor of IEOR at the University of California at Berkeley. He is the Berkeley site director of the Power System Engineering Research Center (PSERC). He has published numerous articles on aspects of electricity market design and has been a consultant to various private and government organizations including the Brazilian regulatory commission, the Alberta Energy Utility Board, the Public Utilities Commission, the Polish system operator, and the Public Utilities Commission of Texas, where he is currently a Senior Advisor to the Market Oversight Division. Dr. Oren is a Fellow of INFORMS.