Running a More Complete Market With the SLP-IV-ACOPF

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***Abstract*—The ACOPF is at the core of competitive electricity market design. It is the only ac-based algorithm that simultaneously co-optimizes real and reactive power dispatch for steady-state operations on ac power systems. In practice, independent system operators (ISOs) oversimplify the physical problem and settle the markets on locational marginal price (LMPs) based on real power but do not price reactive power dispatch or voltage control. This work proposes a market dispatch and pricing procedure based on the ACOPF to provide a more complete pricing mechanism. We formulate the dual problem of the SLP-IV-ACOPF as shown in [1], which is a successive linear program shown to solve the ACOPF to an acceptable quality of convergence to a best-known solution with linear scaling of computational time in proportion to network size. Therefore, the dual problem is also a linear program; as a result, the marginal value pricing of the optimal solution to the dual problem supports the market dispatch due to strong duality, i.e., this solution technique results in a revenue adequate and more complete market design. Furthermore, we show how to distribute the complete real-time market settlements. The analysis includes a direct comparison to DCOPF-based approaches similar to those applied in current ISO markets.**

***Index Terms*—Power generation dispatch, power system economics, reactive power, mathematical programming.**

NOMENCLATURE

Sets:

$\mathcal{N}$	Buses (nodes) $\{1,\ldots,N\}$; $n,m \in \mathcal{N}$.
$\mathcal{K}$	Lines $\{1,\ldots,K\}$; $k \in \mathcal{K}$.
$\mathcal{A}(n)$	Buses adjacent to bus n; $m \in \mathcal{A}(n)$.
$\mathcal{G}$	Buses with generators; $\{1,\ldots,G\}$.
$\mathcal{S}$	Line segments forming a polygonal outer approximation to the voltage feasible region; $s \in S$.
$\mathcal{F}$	Set of flows $\{1,\ldots,2K\}$; $k(n,m) \in \mathcal{F}$.
$\mathcal{L}$	Piecewise linear segments to approximate real power generated; $l \in \mathcal{L}$.
$\mathcal{H}$	Set of iterations $\{1,\ldots,h^*\}$ to solve the problem. $h \in \mathcal{H}$.

Indices:

h^*	Iteration when the SLP-IV-ACOPF meets its convergence criteria (detailed in [1]).
$k(n,m)$	Flow on transmission element k at bus n connecting to m.
$k(n)$	Flows on line k at the n end; $k(\cdot) \in \mathcal{F}$.

Variables:

For each bus n:

p_n^g	Total linearized real power generation.
$p_{n,l}^g$	Linear segment l of generation.
q_n^g	Reactive power generation.
v_n^{sq}	Linearization of $(v_n)^2$.
$v_n^r,\ v_n^j$	Real and imaginary voltage parts.
$i_n^r,\ i_n^j$	Real and imaginary current injection parts.
$p_n^{\mathrm{viol},-}$, $p_n^{\mathrm{viol},+}$	Violation of min and max real power.
$q_n^{\mathrm{viol},-}$, $q_n^{\mathrm{viol},+}$	Violation of min and max reactive power.
$v_n^{\mathrm{viol},-}$, $v_n^{\mathrm{viol},+}$	Violation of min and max voltage.

For each line k:

$i_{k(n,m)}^r$, $i_{k(n,m)}^j$	Real and imaginary parts of current.
$p_{k(n,m)}$	Real power flowing on line k at bus n.

Dual Variables: Marginal Values of the Following Constraints:

For each bus n:

$\rho_n^{\mathrm{stepsum}}$	Sum of real power linearized segments.
$\rho_{n,l}^{\mathrm{steplim}}$	Upper bound on $p_{n,l}^g$.
ρ_n^{low}, ρ_n^{high}	Min and max p_n^g.
γ_n^{low}, γ_n^{high}	Min and max q_n^g.
ν_n^{low}, ν_n^{high}	Min and max voltage limits.
$\iota_n^{r,eq}$, $\iota_n^{j,eq}$	Real and imaginary current bus defn.
λ_n^P, λ_n^Q	Real and reactive power nodal balance.
ν_n^{mag}	Voltage squared defn.
$\nu_{\mathrm{ns}}^{\mathrm{poly}}$	Polygonal constraint on voltage.
$\nu_n^{\mathrm{iter(h)}}$	Iterative voltage cuts.
$\nu_n^{r,\mathrm{LB}}$, $\nu_n^{r,\mathrm{UB}}$	Lower and upper bounds (LB & UB) on v_n^r.
$\nu_n^{j,\mathrm{LB}}$, $\nu_n^{j,\mathrm{UB}}$	LB & UB on v_n^j.
$\rho_{k(n)}^{\max}$	Real power line limit.

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$\nu_n^{r,\text{low}}, \nu_n^{r,\text{high}}$	Real voltage box constraint LB & UB.
$\nu_n^{j,\text{low}}, \nu_n^{j,\text{high}}$	Imaginary voltage box constraint LB & UB.

For each line k:

$\iota_{k(n,m)}^{r,eq}, \iota_{k(n,m)}^{j,eq}$	Real and imaginary current line defn.

Parameters:

For each bus n:

$\hat{v}_n^{r(h)}, \hat{v}_n^{j(h)}$	Real and imaginary voltage parts at h.
$\hat{i}_n^{r(h)}, \hat{i}_n^{j(h)}$	Real and imaginary current parts at h.
$\hat{v}_n^{r(*)}, \hat{v}_n^{j(*)}$	Voltage real and imaginary parts at h^*.
$\hat{i}_n^{r(*)}, \hat{i}_n^{j(*)}$	Current real and imaginary parts at h^*.
$G_n^{\text{sh}}, B_n^{\text{sh}}$	Shunt conductance and susceptance.
P_n^d, Q_n^d	Real and reactive power demand.
$P_n^{\min}, P_n^{\max}$	Min and max p_n^g.
$Q_n^{\min}, Q_n^{\max}$	Min and max q_n^g.
$V_n^{\min}, V_n^{\max}$	Min and max voltage magnitudes.
$V_n^{(h)}$	Step-size bound on the voltage in iter h.
$C_n^{g,1}$	Linear cost coefficient for generation.
$C_{n,l}^g$	Linear segment l of the quadratic cost.
C_n^q	Linear cost coefficient for reactive power.
$P_{n,l}^g$	Maximum length of piecewise segment l.
$P_n^\epsilon, \overline{P}_n^\epsilon$	Dispatch run (DR) and pricing run (PR) penalty costs of real power violations.
$Q_n^\epsilon, \overline{Q}_n^\epsilon$	DR & PR penalty costs of q_n^g violations.
$V_n^\epsilon, \overline{V}_n^\epsilon$	DR & PR penalty costs of voltage violations.

For each line k:

$G_{k(n)}, B_{k(n)}$	Series conductance and susceptance.
$\hat{i}_{k(n,m)}^{r(h)}, \hat{i}_{k(n,m)}^{j(h)}$	Current real and imaginary parts at h.
$\hat{i}_{k(n,m)}^{r(*)}, \hat{i}_{k(n,m)}^{j(*)}$	Current real and imaginary parts at h^*.
$P_k^{\max}$	Maximum real power magnitude.
$P_k^\epsilon, \overline{P}_k^\epsilon$	DR & PR penalty costs of p_k violations.

I. Introduction

INDEPENDENT System Operators (ISOs) and Regional Transmission Organizations (RTOs) provide unbiased access to transmission, maintain grid reliability, and maximize social welfare. One way they fulfill their responsibilities is running energy markets, including day-ahead, fifteen-minute-ahead, and five-minute-ahead markets. The optimal power flow (OPF) problem is the base of all these markets. The marginal values of constraints in the OPF provide market prices, and the dual objective of the OPF determines market settlements. This paper focuses on the market prices of the basic alternating current (AC) OPF in the five-minute-ahead (real-time) power market. We detail the market prices and settlements of the SLP-IV-ACOPF as a possible way to run the real-time market.

A. Running the Power Market

To run the real-time power market, ISOs typically iterate between an AC power flow (ACPF) solver and a modified DCOPF. ISOs use a modified DCOPF rather than the ACOPF to clear markets because the DCOPF is fast, linear, and has well-defined marginal values; the ACOPF is nonlinear and nonconvex, which means its marginal values may not support the hyperplane of the solution. The unmodified DCOPF approximation assumes that resistances and voltage angles are relatively small, ignores reactive power, and restricts voltage magnitudes to a constant. To better suit the DC approximation to the physical system, ISOs first run the DCOPF and input the power generation solution to an ACPF solver; ISOs use the ACPF solution to estimate real power losses, set voltage magnitudes, and add in nomogram constraints to approximate voltage constraints and reactive power needs and then resolve the modified DCOPF. Nomogram constraints typically limit total power flow across interfaces and may be added iteratively when the DC solution is not AC feasible. Locational marginal prices (LMPs) are the dual values on the nodal power balance constraint. Since the modified DCOPF is only based on real power, reactive power is not explicitly priced or compensated within the market, although reactive power is required for normal operation of the power system and influences voltages and system stability. Reactive power can be provided by transmission, generation, or load, and is typically compensated outside of the power generation market.

B. Reactive Power Compensation

ISOs compensate reactive power production in several ways. Most ISOs pay a fixed amount per unit of reactive capacity over the year [2]–[5]. All but MISO use one set price for all areas; MISO pays different rates for each zone. Generators are required to be able generate reactive power over a specified power factor range at maximum real power output. The ISO sends a dispatch signal to the generator for reactive power or specifies a voltage level to be maintained at the bus. CAISO, ISONE, and PJM pay units a lost opportunity cost for backing down real power if the units are forced out of the power factor range [5]. ISONE and PJM pay startup and no load costs if a generator is committed out of merit order for reactive support purposes [5]. Reactive power and voltage support cost between 0.5–1% of the total power procurement cost.. PJM's 2015 annual reactive requirement was \$280M [2]; total billing for services was \$50B in 2014 [3]. ISONE paid \$5M in 2009 and 2010 and \$5.9M for voltage in 2011; its total energy market settlement was \$5.9B in 2009, \$7.3B in 2010, and \$6.7B in 2011 [4], [6].

C. Issues With the Market

While the ISO OPF solution process produces a good approximation of the flows and prices of the power system, it has some drawbacks. Different modeling choices to represent the same physical constraint lead to different LMP profiles. The same voltage or reactive power limit can be represented by several different nomogram constraints, which can result in inconsistent nodal prices [7]. The selection of the reference bus for losses also changes the LMP profiles for the same real power dispatch [8]. It is also unclear on how the voltage limits impact the pricing and whether the ISO's reactive power requirement is appropriate compensation for the actual reactive power provided. Some ISOs find that the current iterative process may not satisfy all power system requirements. MISO has reported that nomograms do not work well for local voltage constraints. In CAISO, the main cause of voltage instability is when the power system cannot meet reactive power demand, and this issue is becoming worse with real power transfer increasing [9].

While transmission is compensated for static reactive power, dynamic power (from generators) is often not compensated, even if it is often more valuable [5]. Many capacity contracts for reactive power ignore how often a unit is in service and the location of the reactive power provider. However, reactive power provides very localized services due to high reactive losses in transit over lines with high loadings. Reactive power loss is proportional to the square of the current times reactance; since reactance is typically much greater than resistance for transmission lines, reactive power losses are much larger than real power losses. A concern for market-based reactive power pricing is that it may be easier to exercise market power with reactive generation; however, the wide range of possible participants (generators, transmission, and load) in reactive markets may lessen market power. Additionally, the current system does not incentivize reactive power production or installing equipment for reactive power [5]; moving to a more comprehensive compensation of reactive power may incentivize investment. While it may be possible that market pricing alone will still not be sufficient to incentive reactive investment [26], market pricing of reactive power will at least reduce the current out-of-market procedures needed today.

D. Related Work

Zhong and Bhattacharya [10] suggest making reactive power payments based on three parts: availability, operation/loss cost, and opportunity payments. In these different settlements, the real power solution is solved beforehand and used as input for the other mechanisms. Kumar anf Kumar examine three different ways to price reactive power and examine the corresponding real and reactive nodal prices [11]. Baughman and Siddiqi [12] show that using power factor penalties does not accurately capture the value of reactive power and that real-time reactive power prices are greatly impacted by voltage constraints.

Xie *et al.* [13] find LMPs and nodal prices for reactive power (LMRPs) with the interior point method. Baughman *et al.*[14], [15] and Liu *et al.* [16] show how to find AC LMPs using the Lagrangian; Baughman also finds reactive prices and dispatch quantities. Momoh *et al.* [17] shows how to find LMPs and LMRPs and breaks them down into congestion and loss components using an iterative process. Conejo *et al.* [18] show the sensitivity of LMPs to parameters of the network. O'Neill *et al.* [19] examine running a power market based on the dual of the power-voltage formulation of the ACOPF. While these papers derive prices for real and reactive power, they do not guarantee zero duality gap or discuss how to settle voltage limit prices.

E. Contributions of the Paper

Past research on the ACOPF has largely focused on the primal formulations of the OPF, which are designed to either be faster to solve, more optimal, or more feasible [20]–[22]. This research focuses on showing how to run a real-time market using the dual formulation of the SLP-IV-ACOPF algorithm. This formulation yields explicit reactive power and voltage prices in addition to more accurate and reference-independent real power prices and congestion rent. Although we do not discuss these issues in this paper due to length, the SLP-IV-ACOPF can also be embedded into the unit commitment problem [23].

Knowing the true real-time prices, especially over time, is beneficial for better understanding the system operation and value of different players in the market. By understanding the way voltage limits and reactive power demands affect the system, better decisions can be made for investment in reactive power equipment. By including reactive power into a market context, we can reduce uplift and adjust constraints that may be set much more tightly than necessary. In addition to its solution time, another reason the ACOPF has not been used to run power markets is because there may be a duality gap due to the nonconvexity of the ACOPF. This would mean that the amount paid into the market would not necessarily equal the amount paid out to the market; the prices derived would not necessarily support the market. Since the SLP-IV-ACOPF is linear, there is no duality gap and the prices support the market.

F. Organization of the Paper

Section II gives the formulation of the SLP-IV-ACOPF and its dual. Section IV describes the DC formulation with piecewise linear losses to which we compare the SLP-IV-ACOPF. Section III defines the market prices and market settlements, from the dual of the SLP-IV-ACOPF. Section V shows how the LMPs converge. Section VI displays a side-by-side comparison of market prices and settlements for running AC and DC markets. Section VII concludes the paper.

II. SLP-IV-ACOPF Market Solution Procedure

First, we find the optimal real and reactive power dispatch by running the SLP-IV-ACOPF until the algorithm terminates. If the algorithm terminates with an acceptable outcome, we do a pricing run where we modify the penalty values. The market prices and settlements are taken from this pricing run.

A. Dispatch Run

We use the methodology in [1] to solve for the optimal dispatch with one change: replacing the limits on maximum line current [1, Eqs. (27), (32), (33), and (37)] with constraints on maximum line power transfer, shown in (1)

$$\begin{aligned} &\hat{v}_n^r i_{k(n,m)}^r + v_n^r \hat{i}_{k(n,m)}^r + \hat{v}_n^j i_{k(n,m)}^j + v_n^j \hat{i}_{k(n,m)}^j \\ &- p_{k(n,m)}^{\text{viol}} \le P_k^{\max} + \hat{v}_n^r \hat{i}_{k(n,m)}^r + \hat{v}_n^j \hat{i}_{k(n,m)}^j \qquad \rho_{k(n)}^{\max}. \quad (1) \end{aligned}$$

In the dispatch run, the objective is to maximize market surplus with fixed demand; this is equivalent to minimizing the total bid cost plus the penalty for violating constraints (2). The limits on real and reactive generation and voltage are considered as 'soft' limits to assist in solving the problem. In the dispatch run, penalties are set as described in [1]

$$\begin{aligned} \min \ & \sum_{n\in\mathcal{G}} \left[\sum_{l\in\mathcal{L}} C_{n,l}^{g,2} p_{n,l}^g + C_n^{g,1} p_n^g + C_n^q q_n^g \right] \\ &+ \sum_{k\in K} P_k^{\epsilon} p_{k(n,m)}^{\text{viol}} + \sum_{n\in\mathcal{N}} \Big[P_n^{\epsilon} \left(p_n^{\text{viol},-} + p_n^{\text{viol},+} \right) \\ &+ Q_n^{\epsilon} \left(q_n^{\text{viol},-} + q_n^{\text{viol},+} \right) + V_n^{\epsilon} \left(v_n^{\text{viol},-} + v_n^{\text{viol},+} \right) \Big]. \quad (2) \end{aligned}$$

As in [1], there are four potential outcomes of the SLP solution when the algorithm terminates: (1) ACOPF KKT optimal solution, (2) ACOPF feasible but not optimal, (3) SLP feasible but ACOPF infeasible, and (4) Infeasible. We accept solutions with outcome (1) or (2); the pricing run is a formality similar to executing one more iteration. With outcome (3), the operator would decide whether the 'infeasibility' was within an acceptable range; for example, it may be acceptable to slightly exceed one line limit. In outcome (3), the pricing run may have a significant impact on the prices, and the penalty costs need to be set carefully. If outcome (3) is not acceptable or outcome (4) results, we would rerun the problem with a different starting points until outcome (1) or (2) or a satisfactory outcome (3) is achieved, or investigate what additional resources are needed for feasibility.

B. Pricing Run

Similar to the ISOs, we do an extra pricing run after the problem converges to a solution. The purpose of this pricing run is to modify the penalty values to reflect opportunity costs. The different ISOs have different penalty values for the pricing run than the dispatch run for real power mismatch and transmission limit. For the purposes of this paper, we set each penalty in the pricing run to 20% of its value in the dispatch run (e.g., $\overline{P}_n^{\epsilon} = 0.2P_n^{\epsilon}$), which mimics the ISO process [24].

In the dispatch run, we limit the change in voltage between iterations (26)–(29) to reduce the error between the estimated and actual power and to help the problem converge. These constraints are only to aid convergence and are not physical limits imposed on the system. Ideally, we would remove these constraints as they impact prices. However, without these constraints, the real and reactive power generated is different than what would result from the voltages and currents in the system. Therefore, we cannot remove these constraints during the pricing run. The objective (3) is subject to the constraints listed in (4) through (31), with the corresponding dual variable for each constraint given to the right of the constraint

$$\begin{aligned} C^{*}\min \; & \sum_{n\in\mathcal{G}}\left[\sum_{l\in\mathcal{L}} C_{n,l}^{g,2} p_{n,l}^{g} + C_n^{g,1} p_n^g + C_n^q q_n^g\right] \\ & + \Bigg[\sum_{k\in K}\overline{P}_k^{\epsilon} p_{k(n,m)}^{\text{viol}} + \sum_{n\in\mathcal{N}}\Big[\overline{P}_n^{\epsilon}\left(p_n^{\text{viol},-}+p_n^{\text{viol},+}\right) \\ & + \overline{Q}_n^{\epsilon}\left(q_n^{\text{viol},-}+q_n^{\text{viol},+}\right) + \overline{V}_n^{\epsilon}\left(v_n^{\text{viol},-}+v_n^{\text{viol},+}\right)\Big]\Bigg] \end{aligned} \quad (3)$$

subject to

$$- p_n^g + \sum\nolimits_{l\in\mathcal{L}} p_{n,l}^g = -P_n^{\min} \qquad \rho_n^{\text{stepsum}} \quad (4)$$

$$- p_{n,l}^g \geq -\left(P_n^{\max} - P_n^{\min}\right)/|\mathcal{L}| \qquad \rho_{n,l}^{\text{steplim}} \quad (5)$$

$$p_n^g + p_n^{\text{viol},-} \geq P_n^{\min} \qquad \rho_n^{\text{low}} \quad (6)$$

$$- p_n^g + p_n^{\text{viol},+} \geq -P_n^{\max} \qquad \rho_n^{\text{high}} \quad (7)$$

$$q_n^g + q_n^{\text{viol},-} \geq Q_n^{\min} \qquad \gamma_n^{\text{low}} \quad (8)$$

$$- q_n^g + q_n^{\text{viol},+} \geq -Q_n^{\max} \qquad \gamma_n^{\text{high}} \quad (9)$$

$$v_n^{\text{sq}} + v_n^{\text{viol},-} \geq \left(V_n^{\min}\right)^2 \qquad \nu_n^{\text{low}} \quad (10)$$

$$v_n^{\text{sq}} - v_n^{\text{viol},+} \leq \left(V_n^{\max}\right)^2 \qquad \nu_n^{\text{high}} \quad (11)$$

$$\begin{aligned} & i_{k(n,m)}^r - G_{k(n)} v_n^r + G_{k(m)} v_m^r \\ & + B_{k(n)} v_n^j - B_{k(m)} v_m^j = 0 \end{aligned} \qquad \iota_{k(n,m)}^{r,eq} \quad (12)$$

$$\begin{aligned} & i_{k(n,m)}^j - B_{k(n)} v_n^r + B_{k(m)} v_m^r \\ & - G_{k(n)} v_n^j + G_{k(m)} v_m^j = 0 \end{aligned} \qquad \iota_{k(n,m)}^{j,eq} \quad (13)$$

$$i_n^r - B_n^{\text{sh}} v_n^j + G_n^{\text{sh}} v_n^r - \sum\nolimits_{k(\cdot,n)} i_{k(\cdot,n)}^r = 0 \qquad \iota_n^{r,eq} \quad (14)$$

$$i_n^j + B_n^{\text{sh}} v_n^r + G_n^{\text{sh}} v_n^j - \sum\nolimits_{k(\cdot,n)} i_{k(\cdot,n)}^j = 0 \qquad \iota_n^{j,eq} \quad (15)$$

$$\begin{aligned} & p_n^g - \hat{v}_n^{r(*)} i_n^r - \hat{v}_n^{j(*)} i_n^j - v_n^r \hat{i}_n^{r(*)} - v_n^j \hat{i}_n^{j(*)} \\ & = P_n^d - \hat{v}_n^{r(*)} \hat{i}_n^{r(*)} - \hat{v}_n^{j(*)} \hat{i}_n^{j(*)} \end{aligned} \qquad \lambda_n^P \quad (16)$$

$$\begin{aligned} & q_n^g + \hat{v}_n^{r(*)} i_n^j - \hat{v}_n^{j(*)} i_n^r + v_n^r \hat{i}_n^{j(*)} - v_n^j \hat{i}_n^{r(*)} \\ & = Q_n^d + \hat{v}_n^{r(*)} \hat{i}_n^{j(*)} - \hat{v}_n^{j(*)} \hat{i}_n^{r(*)} \end{aligned} \qquad \lambda_n^Q \quad (17)$$

$$v_n^{\text{sq}} - 2\hat{v}_n^{r(*)} v_n^r - 2\hat{v}_n^{j(*)} v_n^j = -\left(\hat{v}_n^{r(*)}\right)^2 - \left(\hat{v}_n^{j(*)}\right)^2 \qquad \nu_n^{\text{mag}} \quad (18)$$

$$\begin{aligned} & - v_n^r \cos(2\pi s/S) - v_n^j \sin(2\pi s/S) + v_n^{\text{viol},+} \\ & \geq -\left(V_n^{\max}\right)^2 \end{aligned} \qquad \nu_{\text{ns}}^{\text{poly}} \quad (19)$$

$$- \hat{v}_n^{r(1)} v_n^r - \hat{v}_n^{j(1)} v_n^j + v_n^{\text{viol},+} \geq -\left(V_n^{\max}\right)^2 \qquad \nu_n^{\text{iter}}(1) \quad (20)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$- \hat{v}_n^{r(*)} v_n^r - \hat{v}_n^{j(*)} v_n^j + v_n^{\text{viol},+} \geq -\left(V_n^{\max}\right)^2 \qquad \nu_n^{\text{iter}}(*) \quad (21)$$

$$v_n^r \geq -V_n^{\max} \qquad \nu_n^{r,\text{LB}} \quad (22)$$

$$- v_n^r \geq -V_n^{\max} \qquad \nu_n^{r,\text{UB}} \quad (23)$$

$$v_n^j \geq -V_n^{\max} \qquad \nu_n^{j,\text{LB}} \quad (24)$$

$$- v_n^j \geq -V_n^{\max} \qquad \nu_n^{j,\text{UB}} \quad (25)$$

$$- v_n^r \geq -\hat{v}_n^{r(*)} - V_n^{(h)} \qquad \nu_n^{\text{r,high}} \quad (26)$$

$$v_n^r \geq \hat{v}_n^{r(*)} - V_n^{(h)} \qquad \nu_n^{\text{r, low}} \quad (27)$$

$$- v_n^j \geq -\hat{v}_n^{j(*)} - V_n^{(h)} \qquad \nu_n^{\text{j, high}} \quad (28)$$

$$v_n^j \geq \hat{v}_n^{j(*)} - V_n^{(h)} \qquad \nu_n^{\text{j,low}} \quad (29)$$

$$\begin{aligned} & \hat{v}_n^{r(*)} i_{k(n,m)}^r + v_n^r \hat{i}_{k(n,m)}^{r(*)} + \hat{v}_n^{j(*)} i_{k(n,m)}^j + v_n^j \hat{i}_{k(n,m)}^{j(*)} \\ & - p_{k(n)}^{\text{viol}} \geq P_k^{\max} + \hat{v}_n^{r(*)} \hat{i}_{k(n,m)}^{r(*)} + \hat{v}_n^{j(*)} \hat{i}_{k(n,m)}^{j(*)} \qquad \rho_{k(n)}^{\max} \end{aligned} \quad (30)$$

$$\begin{aligned} & p_{n,l}^g,\; p_n^{\text{viol},+}, p_n^{\text{viol},-}, q_n^{\text{viol},+}, q_n^{\text{viol},-}, v_n^{\text{viol},+}, \\ & v_n^{\text{viol},-} \geq 0. \end{aligned} \quad (31)$$

C. Lagrangian Dual Formulation of the Pricing Run

The dual objective of the SLP-IV-ACOPF is given in (32). The constraints are given in (33) through (48) with the corresponding primal variable to the right of each constraint. The dual objective is composed of the load payment, generation rent for real and reactive power, voltage payment, congestion rent, and shunt compensation. The LMP and LMRP are part of several different dual constraints

$$\begin{aligned}
\max \ & \sum_{n\in N} P_n^d \lambda_n^P + Q_n^d \lambda_n^Q \\
& - \sum\nolimits_{n\in\mathcal{G}} \Bigg[\Bigg[\sum\nolimits_{l\in\mathcal{L}} \left(P_n^{\max} - P_n^{\min}\right) / |\mathcal{L}|\, \rho_{n,l}^{\text{steplim}} \Bigg] - \Bigg(P_n^{\max} \rho_n^{\text{high}} \\
& - P_n^{\min} \rho_n^{\text{low}} + P_n^{\min} \rho_n^{\text{stepsum}} + Q_n^{\max} \gamma_n^{\text{high}} - Q_n^{\min} \gamma_n^{\text{low}} \Bigg) \Bigg] \\
& - \sum_{n\in\mathcal{N}} \Bigg[\left(V_n^{\max}\right)^2 \nu_n^{\text{high}} - \left(V_n^{\min}\right)^2 \nu_n^{\text{low}} \\
& + \left(\left(\hat{v}_n^{r(*)}\right)^2 + \left(\hat{v}_n^{j(*)}\right)^2 \right) \nu_n^{\text{mag}} + \left(V_n^{\max}\right)^2 \left(\nu_n^{\text{iter(*)}} + \sum_s \nu_{\text{ns}}^{\text{poly}} \right) \\
& + V_n^{\max} \left(\nu_n^{r,\text{UB}} + \nu_n^{r,\text{LB}} + \nu_n^{j,\text{UB}} + \nu_n^{j,\text{LB}} \right) \\
& + \nu_n^{r,\text{high}} \left(\hat{v}_n^{r(*)} + V_n^{(h)} \right) + \nu_n^{r,\text{low}} \left(-\hat{v}_n^{r(*)} + V_n^{(h)} \right) \\
& + \nu_n^{j,\text{high}} \left(\hat{v}_n^{j(*)} + V_n^{(h)} \right) + \nu_n^{j,\text{low}} \left(-\hat{v}_n^{j(*)} + V_n^{(h)} \right) \\
& + \left(\hat{v}_n^{r(*)} \hat{i}_n^{r(*)} + \hat{v}_n^{j(*)} \hat{i}_n^{j(*)} \right) \lambda_n^P + \left(-\hat{v}_n^{r(*)} \hat{i}_n^{j(*)} + \hat{v}_n^{j(*)} \hat{i}_n^{r(*)} \right) \lambda_n^Q \Bigg] \\
& - \sum_{k\in K} \left[\left(P_k^{\max} + \hat{v}_n^{r(*)} \hat{i}_{k(n)}^{r(*)} + \hat{v}_n^{j(*)} \hat{i}_{k(n)}^{j(*)} \right) \rho_{k(n)}^{\max} \right] \qquad (32)
\end{aligned}$$

subject to

$$\lambda_n^P + \rho_n^{\text{low}} - \rho_n^{\text{high}} - \rho_n^{\text{stepsum}} = C_n^{g,1} \qquad p_n^g \qquad (33)$$

$$-\rho_{n,l}^{\text{steplim}} + \rho_n^{\text{stepsum}} \le C_{n,l}^{g,2} \qquad p_{n,l}^g \qquad (34)$$

$$\gamma_n^{\text{high}} - \gamma_n^{\text{low}} - \lambda_n^Q = C_n^q \qquad q_n^g. \qquad (35)$$

Equations (33) through (34) give the dual constraints corresponding to real and reactive power

$$\rho_n^{\text{low}} \le \overline{P}_n^{\epsilon} \qquad p_n^{\text{viol},-} \qquad (36)$$

$$\rho_n^{\text{high}} \le \overline{P}_n^{\epsilon} \qquad p_n^{\text{viol},+} \qquad (37)$$

$$\gamma_n^{\text{low}} \le \overline{Q}_n^{\epsilon} \qquad q_n^{\text{viol},-} \qquad (38)$$

$$\gamma_n^{\text{high}} \le \overline{Q}_n^{\epsilon} \qquad q_n^{\text{viol},+} \qquad (39)$$

$$-\nu_n^{\text{low}} \le V_n^e \qquad v_n^{\text{viol},-} \qquad (40)$$

$$-\nu_n^{\text{high}} - \nu_{\text{ns}}^{\text{poly}} - \nu_n^{\text{iter(*)}} \le V_n^e \qquad v_n^{\text{viol},+}. \qquad (41)$$

Equations (36) through (41) give the dual constraints corresponding to the penalty variables on real power, reactive power, and voltage

$$\begin{aligned}
& -\sum_{m\in\mathcal{A}(n)} \Big[G_{k(n)} \iota_{k(n,m)}^{r,\text{eq}} + B_{k(n)} \iota_{k(n,m)}^{j,\text{eq}} + G_{k(m)} \iota_{k(m,n)}^{r,\text{eq}} \\
& + B_{k(m)} \iota_{k(m,n)}^{j,\text{eq}} \Big] + G_n^s \iota_n^{r,\text{eq}} + B_n^s \iota_n^{j,\text{eq}} - \hat{i}_n^{r(*)} \lambda_n^P + \hat{i}_n^{j(*)} \lambda_n^Q \\
& - \sum_{m\in\mathcal{A}(n)} \hat{i}_{k(n,m)}^{r(*)} \rho_{k(n)}^{\max} \\
& - \nu_n^{r,\text{UB}} + \nu_n^{r,\text{LB}} - \nu_n^{r,\text{high}(*)} + \nu_n^{r,\text{low}(*)} - 2\hat{v}_n^{r(*)} \nu_n^{\text{mag}} \\
& - \sum_{s\in\mathcal{S}} \cos(2\pi s/S) \nu_{\text{ns}}^{\text{poly}} - \hat{v}_n^{r(*)} \nu_n^{\text{iter(*)}} = 0 \qquad v_n^r \qquad (42)
\end{aligned}$$

$$\begin{aligned}
& \sum_{m\in\mathcal{A}(n)} \Big[B_{k(n)} \iota_{k(n,m)}^{r,\text{eq}} - G_{k(n)} \iota_{k(n,m)}^{j,\text{eq}} - B_{k(m)} \iota_{k(m,n)}^{r,\text{eq}} \\
& + G_{k(m)} \iota_{k(m,n)}^{j,\text{eq}} \Big] - B_n^s \iota_n^{r,\text{eq}} + G_n^s \iota_n^{j,\text{eq}} - \hat{i}_n^{j(*)} \lambda_n^P - \hat{i}_n^{r(*)} \lambda_n^Q \\
& - \sum_{m\in\mathcal{A}(n)} \hat{i}_{k(n,m)}^{j(*)} \rho_{k(n)}^{\max} \\
& - \nu_n^{j,\text{UB}} + \nu_n^{j,\text{LB}} - \nu_n^{r,\text{high}(*)} + \nu_n^{r,\text{low}(*)} - 2\hat{v}_n^{j(*)} \nu_n^{\text{mag}} \\
& - \sum_{s\in\mathcal{S}} \sin(2\pi s/S) \nu_{\text{ns}}^{\text{poly}} - \hat{v}_n^{j(*)} \nu_n^{\text{iter(*)}} = 0 \qquad v_n^j \qquad (43)
\end{aligned}$$

$$\nu_n^{\text{mag}} - \nu_n^{\text{high}} + \nu_n^{\text{low}} = 0 \qquad v_n^{\text{sq}}. \qquad (44)$$

Equations (42) through (44) give the dual constraints corresponding to the voltage components

$$\iota_{k(n,m)}^{r,\text{eq}} - \iota_n^{r,\text{eq}} + \iota_m^{r,\text{eq}} - \rho_{k(n)}^{\max} \hat{v}_n^{r(*)} = 0 \qquad i_{k(n,m)}^r \qquad (45)$$

$$\iota_{k(n,m)}^{j,\text{eq}} - \iota_n^{j,\text{eq}} + \iota_m^{j,\text{eq}} - \rho_{k(n)}^{\max} \hat{v}_n^{j(*)} = 0 \qquad i_{k(n,m)}^j \qquad (46)$$

$$\iota_n^{r,\text{eq}} - \hat{v}_n^{r(*)} \lambda_n^P - \hat{v}_n^{j(*)} \lambda_n^Q = 0 \qquad i_n^r \qquad (47)$$

$$\iota_n^{j,\text{eq}} - \hat{v}_n^{j(*)} \lambda_n^P + \hat{v}_n^{r(*)} \lambda_n^Q = 0 \qquad i_n^j. \qquad (48)$$

Equations (45) through (48) give the dual constraints corresponding to the nodal and line current components

$$\begin{aligned}
& \rho_{n,l}^{\text{steplim}},\ \rho_n^{\text{low}},\ \rho_n^{\text{high}},\ \gamma_n^{\text{low}},\ \gamma_n^{\text{high}},\ \nu_n^{\text{low}},\ \nu_n^{\text{high}},\ \nu_{\text{ns}}^{\text{poly}}, \\
& \nu_n^{\text{iter(*)}},\ \nu_n^{r,\text{LB}},\ \nu_n^{r,\text{UB}},\ \nu_n^{j,\text{LB}},\ \nu_n^{j,\text{UB}}, \\
& \nu_n^{r,\text{high}(*)},\ \nu_n^{r,\text{low}(*)},\ \nu_n^{j,\text{high}(*)},\ \nu_n^{j,\text{low}(*)},\ \rho_k^{\max} \ge 0. \qquad (49)
\end{aligned}$$

Equation (49) gives the nonnegative requirements for dual variables.

III. Market Terms For the SLP-IV-ACOPF

This Section describes market prices and market settlements for a real-time market running the SLP-IV-ACOPF.

A. Market Prices

This Section details the nodal prices for real and reactive power as well as the flowgate prices on transmission lines.

1) Locational Marginal Price (LMP): The LMP, λ_n^P, is the price for real power at location n. It is a function of generation limits and generator costs (50). The LMP for the SLP-IV-ACOPF cannot directly be broken down into reference, congestion, and loss terms due to voltage being a variable rather than a parameter. Papers that have broken the LMPs down into these components either fix voltage [25] or do an empirical sensitivity analysis [17]

$$\lambda_n^P = \rho_n^{\text{high}} - \rho_n^{\text{low}} + \rho_n^{\text{stepsum}} - C_n^{g,1}. \qquad (50)$$

2) Locational Marginal Reactive Price (LMRP): The LMRP, λ_n^Q, is the price for reactive power at location n. It is influenced by the generator's reactive limits and reactive cost. Even if reactive power is priced at zero, providing reactive power may have a non-zero price if a generator is at its upper or lower reactive limit (51). C_n^q may be also set as the cost of the reactive equipment and its commodity costs, similar to how real power is priced. We could also extend this framework to price reactive power as a linearized quadratic cost

$$\lambda_n^Q = \gamma_n^{\text{high}} - \gamma_n^{\text{low}} - C_n^q. \tag{51}$$

3) Flowgate Prices: The flowgate price $\rho_{k(n)}^{\max}$ is the value of one more unit of real power capacity on the line. It is a function of dual values on current definition constraints and the voltage evaluation point

$$\rho_{k(n)}^{\max} = \frac{\iota_{k(n,m)}^{r,\text{eq}} + \iota_n^{r,\text{eq}} - \iota_m^{r,\text{eq}}}{\hat{v}_n^r} = \frac{\iota_{k(n,m)}^{j,\text{eq}} + \iota_n^{j,\text{eq}} - \iota_m^{j,\text{eq}}}{\hat{v}_n^j}. \tag{52}$$

4) Voltage Prices: While voltage is considered a public good, constraints on its limits mean there are marginal values of the voltage constraints. Due to the representation of voltage in rectangular coordinates and the linearization, there are many of these such constraints and therefore corresponding voltage prices. The prices ν_n^{mag}, ν_n^{low}, ν_n^{high}, $\nu_{\text{ns}}^{\text{poly}}$, and $\nu_n^{\text{iter(h)}}$ pertain to the voltage magnitude. The prices $\nu_n^{r,\text{LB}}$, $\nu_n^{r,\text{UB}}$, $\nu_n^{j,\text{LB}}$, and $\nu_n^{j,\text{UB}}$ pertain to the real and imaginary components of voltage; these prices can only be non-zero if the voltage component is at the maximum voltage. The prices $\nu_n^{r,\text{high}}$, $\nu_n^{r,\text{low}}$, $\nu_n^{j,\text{high}}$, $\nu_n^{j,\text{low}}$ are for voltage staying within its box convergence constraints. All the prices except those on the box convergence constraints are system requirements; the box constraints are not system requirements and are used only for convergence. Generators are incentivized to follow the market solution for all prices except for those on the box convergence constraints. For example, if $\nu_n^{r,\text{UB}} > 0$, then the generator has the incentive to push its real voltage component as high as possible; since $\nu_n^{r,\text{UB}} > 0$, by complementary slackness, that means that $v_n^r = V_n^{\max}$, so the generator will be served by setting $v_n^r = V_n^{\max}$ to obtain the highest payment possible.

However, the box convergence prices $\nu_n^{r,\text{high}}$, $\nu_n^{r,\text{low}}$, $\nu_n^{j,\text{high}}$, $\nu_n^{j,\text{low}}$ should stay internal to the ISO and not be viewable to generators. For example, if $\nu_n^{r,\text{high}} > 0$ and the generator is strictly within the voltage limits, the generator has an incentive to increase voltage although the ISO's desired behavior is for the generator to stay at its present voltage point. If the generator does not see box convergence prices, then it only sees a voltage prices of zero, so its incentive would be to stay at its present voltage setting. ISOs do not publish prices for the nomogram constraints, and not publishing the box convergence prices is analogous to this behavior.

B. Market Settlements

This Section shows the equivalence between the minimization of generation cost and the total market settlement. The dual objective is comprised of the load payment, generator profit, congestion rent, voltage payment, and shunt compensation. The term $(\hat{v}_n^{r(*)}\hat{i}_n^{r(*)} + \hat{v}_n^{j(*)}\hat{i}_n^{j(*)})\lambda_n^P + (-\hat{v}_n^{r(*)}\hat{i}_n^{j(*)} + \hat{v}_n^{j(*)}\hat{i}_n^{r(*)})\lambda_n^Q$ is broken into congestion rent and shunt compensation later in this section.

1) Load Payment: As in the current market system, load pays its demand for real power times the LMP. Here, it would also pay its demand for reactive power times the LMRP. For bus n, the real power load payment LP_n^p is given in (53), the reactive power load payment LP_n^q is given in (54), and the total load payment LP_n^{tot} is the sum of the real and reactive payments (55)

$$LP_n^p = P_n^d \lambda_n^P \tag{53}$$

$$LP_n^q = Q_n^d \lambda_n^Q \tag{54}$$

$$LP_n^{\text{tot}} = P_n^d \lambda_n^P + Q_n^d \lambda_n^Q. \tag{55}$$

2) Generator Rent: The generator rent in the SLP-IV-ACOPF is the same as in the DCOPF case with an additional term for reactive power. At bus n, the real power component of the generation rent, GR_n^p is given in (56) and the reactive power generation rent, GR_n^q is given in (57); the total rent GR_n^{tot} paid to the generator at bus n is the sum of these two terms (58)

$$GR_n^p = P_n^{\max}\rho_n^{\text{high}} - P_n^{\min}\rho_n^{\text{low}} + P_n^{\min}\rho_n^{\text{stepsum}} - \sum_{l\in\mathcal{L}}\left(\left(P_n^{\max} - P_n^{\min}\right)\rho_{n,l}^{\text{steplim}}\right) / |\mathcal{L}| \tag{56}$$

$$GR_n^q = Q_n^{\max}\gamma_n^{\text{high}} - Q_n^{\min}\gamma_n^{\text{low}} \tag{57}$$

$$GR_n^{\text{tot}} = GR_n^p + GR_n^q. \tag{58}$$

As in present markets, a generator gets paid $\lambda_n^P P_n^g$; with this formulation, the generator additionally receives a market payment of $\lambda_n^q Q_n^g$. The generator payment equals the generator cost plus generator rent. Similar to bid cost recovery, the ISO may need to total the reactive payments from the market and ensure that they cover the generator's reactive costs.

3) Congestion Rent: The value of power on the line k paid to the line operator (congestion rent for line k), CR_k^{tot} (61) is composed of real power congestion rent, CR_k^p, (59) and reactive power congestion rent, CR_k^q (60)

$$CR_k^p = \hat{p}_{k(n,m)}\rho_{k(n)} + \hat{p}_{k(m,n)}\rho_{k(m)} + P_k^{\max}\left(\rho_{k(n)}^{\max} + \rho_{k(m)}^{\max}\right) + \hat{p}_{k(n,m)}\lambda_n^P + \hat{p}_{k(m,n)}\lambda_m^P \tag{59}$$

$$CR_k^q = \hat{q}_{k(n,m)}\lambda_n^Q + \hat{q}_{k(m,n)}\lambda_m^Q \tag{60}$$

$$CR_k^{\text{tot}} = CR_k^p + CR_k^q. \tag{61}$$

Since the differences between iterations decrease with each iteration, the difference in the power on lines between the last iteration $\hat{p}_{k(n,m)}$ and current iteration $p_{k(n,m)}$ should be small. As will be seen in Section VI, in some cases, the lines pay the ISO due to line losses. If there are no line limits, we can write the congestion rent as in (62)

$$CR_k^{\text{nolim}} = \hat{p}_{k(n,m)}\left(\lambda_n^P - \lambda_m^P\right) + \left(\hat{p}_{k(n)} + \hat{p}_{k(m)}\right)\lambda_m^P + \hat{q}_{k(n,m)}\left(\lambda_n^Q - \lambda_m^Q\right) + \left(\hat{q}_{k(n)} + \hat{q}_{k(m)}\right)\lambda_m^Q. \tag{62}$$

If we assume that $\hat{p}_{k(n)} < 0$ and $|\hat{p}_{k(m)}| < \hat{p}_{k(n)}$ and the same for reactive power, then $CR_k^{\text{nolim}} = \hat{p}_{k(n,m)}\left(\lambda_m^P - \lambda_n^P\right)$

$-\lambda_m^P p_k^{\text{loss}} - \lambda_m^Q q_k^{\text{loss}}$ and we could have that $CR_k^{\text{nolim}} < 0$. While operators may earn money on the transfer between the two buses, they may have to pay the ISO for line losses.

4) Shunt Settlement: Owners of shunts (transmission equipment) receive shunt compensation at bus n, SC_n, shown in (63)

$$SC_n = G_n^{\text{sh}} \hat{v}_n^2 \lambda_n^P - B_n^{\text{sh}} \hat{v}_n^2 \lambda_n^Q. \tag{63}$$

If there are multiple shunt compensation devices at the bus, then the settlement would be allocated based on the proportion of admittance that each owner's devices contributed. If $SC_n < 0$, the owner(s) will pay the ISO; if $SC_n > 0$, the owner will get paid by the ISO. If $\lambda_n^P > 0$, the conductive shunt owner gets paid as the shunt is improving the power factor; if $\lambda_n^P < 0$, the owner gets penalized as it would help increase power transfer and we would rather reduce it. For susceptive shunt equipment, if $\lambda_n^Q < 0$, then the network desires less reactive power; since susceptive shunt compensation tends to improve real power transfer and raise the power factor [27], the susceptive shunt owner gets paid more when $\lambda_n^Q < 0$. If $\lambda_n^Q > 0$, then the network desires more reactive power; since susceptive shunt compensation tends to reduce reactive power, the susceptive equipment owner is penalized.

5) Voltage Support: Voltage is a similar type of service to reserves, both ancillary services and reliability unit commitments (RUCs), which are considered public goods. Voltage is a considered a public good since it benefits the whole system and depends on system conditions across the entire network but cannot be directly attributable to marginal changes in power. In the current day-ahead market, there are constraints that require a certain amount of reserves to be procured. Reserves are considered a public good since they support the system at a zonal and overall level. The payments for these reserves are provided by the load, where the charge to each load is proportional to that load divided by the overall system load. Therefore, the voltage support settlement that results from the constraints on voltage should be considered as a public good and be paid for and distributed as such. While voltage cannot be as easily tied to controllable quantities, we can measure how changing voltage limits would change the objective by examining the marginal values of those constraints. Additionally, the total voltage support payment $\sum_{n \in \mathcal{N}} VV_n$ is part of the dual objective. The total voltage support can be broken down to each node n. This value is shown in (64) and is a function of the limits on maximum and minimum voltage

$$\begin{aligned} VV_n = {} & V_n^{\max} \nu_n^{\text{high}} - V_n^{\min} \nu_n^{\text{low}} + \left(\left(\hat{v}_n^{r(*)} \right)^2 + \left(\hat{v}_n^{j(*)} \right)^2 \right) \nu_n^{\text{mag}} \\ & + (V_n^{\max})^2 \left(\sum_{h=1}^{h^*} \nu_n^{\text{iter(h)}} + \sum_s \nu_{\text{ns}}^{\text{poly}} \right) \\ & + V_n^{\max} \left(\nu_n^{r,\text{UB}} + \nu_n^{r,\text{LB}} + \nu_n^{j,\text{UB}} + \nu_n^{j,\text{LB}} \right) \\ & + \nu_n^{r,\text{high}} \left(\hat{v}_n^{r(*)} + V_n^{(h)} \right) + \nu_n^{r,\text{low}} \left(-\hat{v}_n^{r(*)} + V_n^{(h)} \right) \\ & + \nu_n^{j,\text{high}} \left(\hat{v}_n^{j(*)} + V_n^{(h)} \right) + \nu_n^{j,\text{low}} \left(-\hat{v}_n^{j(*)} + V_n^{(h)} \right). \end{aligned} \tag{64}$$

The market would distribute the total voltage support $\sum_{n \in \mathcal{N}} VV_n$ rather than settle with each bus its VV_n. The revenue to the ISO from the voltage support settlement can be distributed in a number of different ways. It can be considered as extra revenue for the ISO to pay for reactive/voltage support, whether with fixed contracts or paying for opportunity cost. Also, knowing the voltage value over a year can help determine how to price long-term contracts. ISOs may also decide to allocate voltage support in the shorter-term, paying more to generators that provide voltage support to buses where the support is more crucial, who provide more reactive power, and those closer to the source. This term for voltage support is analogous to the value of the interface limits times their duals or the cost of real power losses in the present energy market.

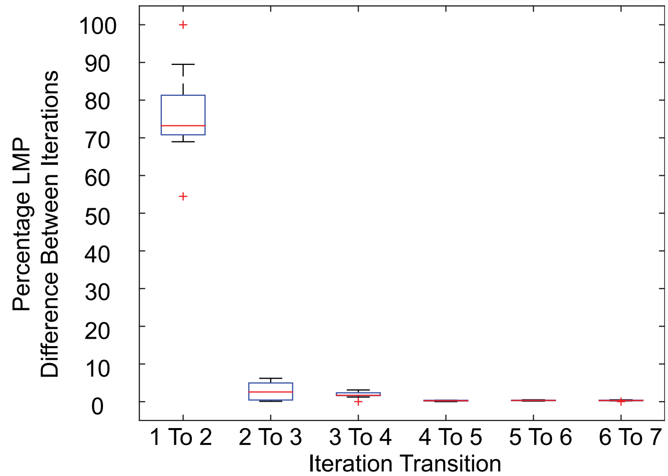

Fig. 1. LMP differences between iterations for IEEE case 14 with a flat start.

IV. DC With Losses Formulation

We compare the SLP-IV-ACOPF to a DC formulation that includes losses, modifying the formulation in [28] to minimize a piecewise linear cost. The formulation in [28] estimates the losses with a convex piecewise linear function. For this implementation of [28], we chose breakpoints at the angle differences of 0°, 0.25°, 0.5°, 0.75°, 1°, 1.5°, 2°, 3°, 4°, 5°, 7°, 10°, 15°, 20°, 30°, and 60°. The market settlements follow from the traditional DC settlements with one extra term: a loss payment, which is the dual value of the constraint on the angle difference breakpoint (equations (9) and (10) in [28]) times the limit on the angle difference breakpoint. The specific details follow in the online supplement [29].

V. LMP Convergence

This Section reports the LMP deviations between the iterations. The following box plots in Figs. 1 and 2 aggregate the data for all the buses and show the mean absolute percentage difference, minimum and maximum, and error bars for the LMP differences from iteration to iteration. The outliers are represented as red pluses. Fig. 1 shows the LMP convergence for the IEEE 14 bus problem (no line limits) when the SLP-IV-ACOPF uses a flat start. Some of the LMPs in the first iteration are negative, and they are very different from the final results. From the first to the second iteration, the LMPs change on average

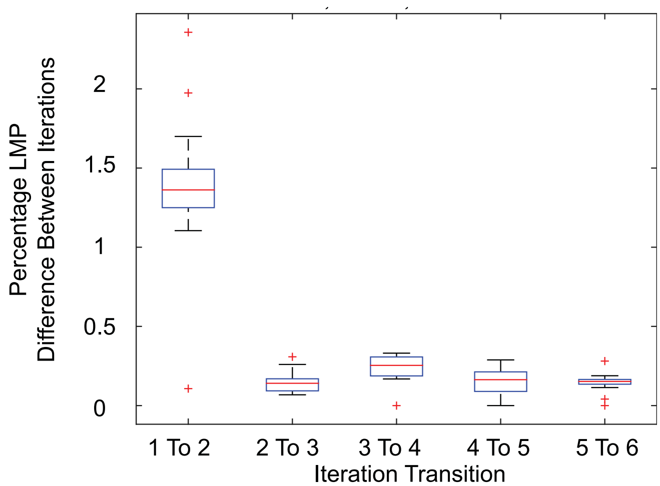

Fig. 2. LMP differences between iterations for IEEE case 14 with a DC start.

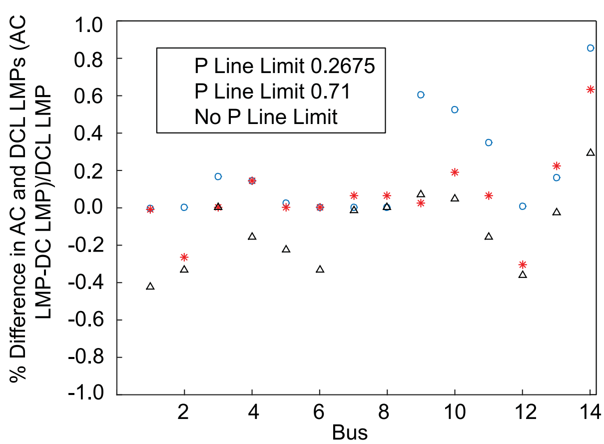

Fig. 3. Differences in AC and DCL LMPs for IEEE case 14 with different line limits.

by over 70%. After the second iteration, the largest change the LMPs is 6%, from the second to the third iteration. After the 4th iteration, the largest LMP difference is under 1%. As seen in Fig. 2, if a DC start is used instead of a flat start the LMP convergence improves greatly. The difference from the first to second iteration does not exceed 2.5% for any bus; after the first iteration, the difference between iterations is less than 0.5%.

VI. Economic Analysis

This Section discusses the differences in the market prices and payments when the OPF is solved with the DCOPF with losses (DCL) versus the SLP-IV-ACOPF. It shows the magnitudes of the new payments under the AC system: reactive power, voltage support, and reactive congestion rent.

A. Nodal Power Prices for Case 14

The LMP differences between the AC and DCL cases are shown in Fig. 3. The *y* axis shows (AC LMP - DCL LMP)/(DCL LMP.) With no line limits, the DCL case overestimates half of the LMPs, underestimates the LMP for bus 14, and accurately

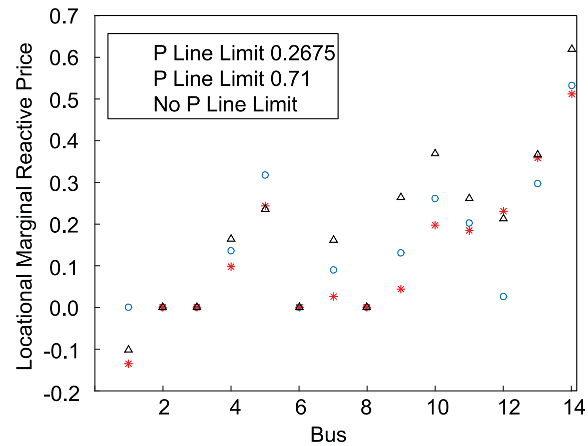

Fig. 4. LMRP for case 14.

estimates the other LMPs. With a 0.71 line limit, the DCL over and under estimates by the same amount. With the 0.2675 line limit case, 5 LMPs are underestimated and the rest are fairly close. In all line limit cases, the greatest error in the DCL LMP is less than 1%. The LMRPs are shown in Fig. 4. The range of the LMRPs tends to become smaller as the line limits are tightened. In these cases, where there is plentiful reactive power, the LMRP is small.

B. Market Settlements for Case 14

Table I shows the difference in aggregate payments under a DCL and an AC system for IEEE Case 14 with three types of line limits: no line limits, a 0.71 and a 0.2675 p.u. limit on real power across lines. The tighter the line limits, the higher the congestion rent and the lower the reactive power and voltage support. This impact is likely because limiting real power across lines also limits the difference in voltages between two ends of a bus. From this case, it also appears that the AC reactive power and voltage support plus congestion rent is close to the DCL congestion rent plus loss cost for the two cases with line limits. This may mean that when ISOs use nomogram constraints, much of the voltage support and reactive power value is going to congestion rent. Since the ISO pays congestion rent, a reactive requirement, and in some cases an opportunity cost for providing reactive power, it may be paying for reactive power/voltage support multiple times. It appears that reactive power and voltage support are being incorporated into the congestion rent; that is, we may be overpaying in FTRs when part of these funds could be used to pay for reactive power and voltage support.

C. Market Settlements for Case 2383wp

The market settlements for the Polish case 2383wp are shown in Table II for both the case without line limits and with letting the limit on power across the line be the same as the limit given for apparent power on the line. In this case, the DCL load payment with line limits is higher than the AC load payment with line limits. The DCL case yields more congestion rent than the AC case; in the DCL case, the net congestion rent is paid

TABLE I
AC VERSUS DCL MARKET PAYMENTS FOR IEEE 14-BUS PROBLEM

	No Line Limits			0.71 PLine Limit			0.2675 PLine Limit		
	AC	DCL	% Diff	AC	DCL	% Diff	AC	DCL	% Diff
Objective Function	8092.2	8085.1	0.1%	8488.2	8478.6	0.1%	9323.6	9324.1	0.0%
Real Power Load Payment, LP^p	10391.9	10398.4	0.1%	10640.0	10634.2	0.05%	11046.4	11020.3	0.2%
Reactive Power Load Payment, LP^q	12.3	0	100%	8.5	0	100%	7.2	0	100%
Real Power Generation Rent, GR^p	1904.5	1940.8	1.9%	1101.2	1105.0	0.35%	881.70	875.0	0.8%
Reactive Power Generation Rent, GR^q	0	0	0%	0	0	0%	0	0	0%
Voltage Support, VV	814.9	0	100%	320.2	0	100%	188.4	0	100%
Real Power Congestion Rent, CR^p	-395.0	0	100%	747.4	902.9	20.80%	667.2	743.9	11.5%
Reactive Power Congestion Rent, CR^q	-6.9	0	100%	-5.9	0	100%	-6.3	0	100%
Shunt Compensation, SC	-5.4	0	0%	-2.6	0	0%	-0.9	0	0%
DC Loss Payment, LoP	0	372.5	100%	0	147.6	100%	0	77.3	100%
Objective excluding LP^p and GR^p:									
-LP^q+GR^q+VV+CR^p+CR^q+SC+LoP	395.3	372.5	5.8 %	1050.6	1050.5	0.0%	841.1	821.2	2.4%

TABLE II
DIFFERENCES IN AC AND DCL MARKETS FOR CASE 2383WP

	No Line Limits			Line Limits		
	AC	DCL	% Diff	AC	DCL	% Diff
Objective Function	18589.386	18609.823	4.87%	18602.482	18830.058	3.42%
Real Power Load Payment, LP^p	36259.842	36285.08	2.75%	36334.991	41006.009	5.92%
Reactive Power Load Payment, LP^q	58.561	0	100%	62.411	0	100.00%
Real Power Generation Rent, GR^p	16812.5	16818.388	4.54%	16694.553	17689.74	1.66%
Reactive Power Generation Rent, GR^q	16.795	0	100%	15.33	0	100%
Voltage Support, VV	1795.439	0	100%	1758.131	0	100%
Real Power Congestion Rent, CR^p	-856.950	0	100%	-625.8894	3476.74	467.30%
Reactive Power Congestion Rent, CR^q	-41.767	0	100%	-47.2054	0	100%
Shunt Compensation, SC	0	0	0%	0	0	0%
DC Loss Payment	0	856.869	100%	0	829.708	100%
Total of Non-Real Power Load Payments		856.9			4306.211	

from the ISO; in the AC case, the net congestion rent is paid to the ISO.

VII. CONCLUSION AND FUTURE WORK

The power limits in this formulation are represented as box constraints, with limits on real and reactive power independently. These limits could be represented as D-curves, and generators would also receive the value of lost opportunity cost. This formulation does not discuss reserves and their pricing. It could be extended into a reliability unit commitment model that would yield reserve prices and ramping products. Since reactive power requirements can be solved through investment in reactive equipment instead of market pricing [30], it would also be interesting to compare these approaches.

This work defines a more complete real-time market by pricing reactive power and voltage. It shows that the current representation of the market may be paying transmission for both congestion rent and reactive support when part of these funds should be going to reactive support. It produces LMPs based on an OPF formulation that includes voltage and reactive power; these LMPs converge as the primal dispatch converges, and one more iteration after convergence will have a minimal impact on prices. This approach gives the explicit cost of reactive power and voltage support, which can assist in deciding where to cite more equipment and how to price long-term contracts. This representation shows how reactive power and voltage impact prices. The 14-bus example illustrates that tightening line limits as surrogates for voltage and reactive power limits likely means that voltage support is being paid twice; once via congestion rent and again as uplift or in the reactive capacity contract. With this formulation, we can explicitly separate the reactive power compensation from the congestion rent. Additionally, since we model losses, reactive power, and voltage directly, the LMPs are independent of the reference bus and nomogram constraint selections.

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