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ORIGINAL PAPER

Constructing transmission line current constraints for the IEEE and polish systems

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Abstract All power system operators ensure their systems adhere to thermal limits on transmission lines in order to avoid line deformation or stability problems. The IEEE test problems do not include data on thermal limits for the 14-, 57-, 118-, and 300-bus systems. The Polish systems contain limits on the apparent power on the line, but these apparent power limits needs to be translated to current limits to solve for the optimal power flow using a current-voltage formulation. The purpose of this paper is to develop new test problems that contain current constraints. It presents a simple method for constructing these current magnitude constraints. This paper finds limits on the maximum allowable current magnitude that result in a feasible solution for the 14-, 30-, 57-, 118-, and 300-bus IEEE test problems. For each test problem, a single limit is applied to all lines that makes the optimal solution without these limits infeasible. For each problem, tight and loose limits on the current magnitude are developed. The resulting problems are solved using the current voltage formulation. Additionally, apparent power limits on the Polish systems are converted to limits on current. Including these constraints in the ACOPF increases power production costs (the objective function) up to 14 %.

Keywords Power system operations · Optimal power flow · Optimization · Test system standards

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List of symbols

Superscripts:

D	Demand
G	Generator
r	Real (x-axis) component of the vector
j	Imaginary (y-axis) components of the vector

Subscripts:

n, m	Buses
$k(n, m)$	Line k connecting buses n and m, in the n to m direction
$k(n)$	Property of line k at bus n

Sets:

$N(n)$	The set of buses that are adjacent to bus n; that is, buses that are directly connected by a line to bus n

Variables:

$i^r_{k(n,m)}$	Real component of current on line $k(n, m)$ at bus n
$i^j_{k(n,m)}$	Imaginary component of current on line $k(n, m)$ at bus n
i^r_n	Real component of current injection at bus n
i^j_n	Imaginary component of current injection at bus n
$p_{k(n,m)}$	Real power across line $k(n, m)$
p^G_n	Real power generated at bus n
$p^G_{n,t}$	Step division of p^G_n
$p^{viol,-}_n$	Amount by which the minimum real power generation at bus n is violated
$p^{viol,+}_n$	Amount by which the maximum real power generation at bus n is violated
$q_{k(n,m)}$	Reactive power across line $k(n, m)$ at bus n
q^G_n	Reactive power generated at bus n
$q^{viol,-}_n$	Amount by which the minimum reactive power generation at bus n is violated
$q^{viol,+}_n$	Amount by which the maximum reactive power generation at bus n is violated
$s_{k(n,m)}$	Apparent power across line $k(n, m)$ at bus n
$v^{viol,-}_n$	Amount by which the minimum voltage requirement at bus n is violated
$v^{viol,+}_n$	Amount by which the maximum voltage requirement at bus n is violated
v^r_n	Real component of voltage at bus n
v^j_n	Imaginary component of voltage at bus n

Parameters:

f_n	Cost coefficient on the square of p_n^G in MATPOWER
e_n	Cost coefficient on p_n^G in MATPOWER
$c_{n,pen}$	Penalty cost of violating constraints at bus n
$c_{n,t}$	Step-cost of real power at bus n at step t
$b_{k(n)}$	Electrical susceptance of transmission line k at bus n
$g_{k(n)}$	Electrical conductance of transmission line k at bus n
$i_{k(n,m)}^{max}$	Maximum allowed current on line $k(n, m)$
p_n^D	Real power demand at bus n
q_n^D	Reactive power demand at bus n
p_n^{min}	Minimum required real power at bus n
p_n^{max}	Maximum allowed real power at bus n
$p_n^{G,step}$	Step size for real power; equals $\frac{p_n^{max}-p_n^{min}}{T}$
q_n^{min}	Minimum required reactive power at bus n
q_n^{max}	Maximum allowed reactive power at bus n
v_n^{min}	Minimum required voltage magnitude at bus n
v_n^{max}	Maximum allowed voltage magnitude at bus n
T	Total number of steps for the stepwise approximation
t	Index of the step; t =1, 2, …,T

1 Introduction

The amount of current that can flow through power system transmission lines is limited by thermal restrictions. The thermal ratings of the transmission lines depend on the materials that compose them and environmental conditions. Heat loss on the line is proportional to the magnitude of the line's current squared. If the line current exceeds the recommended limit for too long, the excessive heat caused by the line current can deform and degrade transmission lines and cause them to sag. Limiting current is the most direct measurement to limit the temperature of the line; however, this thermal limit is often converted to a limit on apparent or real power as it is easier to represent in traditional optimal power flow (OPF) formulations that consider the variables of voltage and power but not current. Additionally, current and other line limits may be imposed to enforce stability.

The IEEE test systems [5] and Polish systems [20] are commonly used to test new algorithms for solving power flow problems. However, no IEEE test problems include current magnitude limits on the transmission lines, and many of the test systems include no thermal limits at all, including the 14-, 57-, 118-, and 300-bus systems. If a test problem does not include these limits, reasonable constraints may need to be created for testing purposes. Solving an OPF problem is often difficult due to the line limits; testing algorithms on cases without line limits does not provide a good representation on how the algorithm will work in practice. In MATPOWER [20], the 30-bus system contains apparent power limits on lines; these constraints do not cause much congestion and do not give much insight into a stressed system. The larger test cases do contain line limits; however, these limits need to be converted from being a limit on apparent power to a limit on current.

In the absence of thermal constraints, such as in the 14- to 300-bus systems, one approach is to create constraints based on the physical characteristics and expected environmental conditions. Often, there is little information available about the lines. It takes considerable time to develop constraints based on physical characteristics, and the result may not be binding constraints.

The purpose of this paper is to develop a methodology for creating constraints on the line current magnitude using a set of the IEEE test problems and to establish standard limits that can be used in the testing of power flow algorithms. Binding constraints based on maximum current magnitude are created rather than on constraints on apparent power on the lines or on voltage angle differences.

For systems without limits, the proposed approach is to create constraints from the optimal solution (with no line limits imposed). Subsequent testing explores how modifying the line limits, created from the solution of the non-constrained problem, affects the resulting power flow solution. For the Polish systems containing thermal limits, the apparent power limits are converted to current limits. Two types of line constraints are proposed for the test systems. The "tight" constraint level restricts current on lines to a lower amount than the "loose" constraint level. This paper has essentially created new power system test cases that provide current limits on the lines.

The rest of the paper is organized as follows: the background of the ACOPF problem and how line constraints work is discussed in Sect. 2. Section 3 describes the approach to the formulation used and how the line constraints are constructed. Numerical results and the recommended line limits are given in Sect. 4. The paper is summarized in Sect. 5.

2 Background

This section discusses the history of the power flow problem, the standard for establishing current limits, and the relationship between the different line limits.

2.1 AC power flow problem

The power flow problem provides network solutions, such as bus voltage magnitudes and angles, for a given set of conditions. The optimal power flow (OPF) problem involves executing these requirements at maximum market surplus. If demand is considered inelastic, maximizing market surplus is equivalent to minimizing power procurement costs. Power networks have buses that are connected by lines with both resistance and reactance, so these power flow problems are generally designated as alternating current optimal power flow (ACOPF) problems. Independent System Operators (ISOs) and Regional Transmission Organizations (RTOs) in the United States and Canada run power markets based on optimal power flow over large regions. Since these markets are so large and the dispatch time is typically 5 min, these organizations generally run the direct current (DC) approximation of the ACOPF with some modifications in order to find a solution within the time window.

While the alternating current optimal power flow problem has been studied for over 50 years [4], no single approach has been permanently settled on. The ACOPF is nonlinear and nonconvex, which makes it hard to find a feasible solution and even harder to find one that is the global solution. There are many approaches to solving this problem. These include Newton methods [16–18], interior point algorithms [10–17], semidefinite relaxations [11,12,14], conic approaches [9], and direct current approximations. As many theoretically good algorithms may run into trouble during implementation, the different methods and approximations used to solve the OPF are extensively tested on sample data.

2.2 Current line limits standards

Transmission line limits used in solving power flow problems include limits on real power, voltage angle difference, apparent power, and current. The IEEE standard [1] suggests calculating a thermal limit on current based on environmental conditions and material properties. Transmission lines generally have maximum temperature ratings. These include a normal and emergency rating, and these ratings typically depend on the characteristics of the conductor material. Resistance of a line is dependent on temperature; increasing the temperature increases the resistance. Since the IEEE test systems included only steady-state data, the current limits are also derived at steady-state. While there are many different ways to calculate maximum current, the IEEE Standard 738 , shown in (1), provides one well-accepted method.

$$i^{max} = \sqrt{\frac{q_c + q_r - q_s}{R(T_C)}} \tag{1}$$

In the IEEE Standard 738, q_c is the convected heat loss, q_r is the radiated heat loss, q_s is the solar heat gain, and $R(T_C)$ is the resistance of the line per length at temperature T_C. The physical properties that increase the current line limit include a higher maximum temperature rating, larger diameter, and higher emissivity. The environmental factors that increase this limit include a higher wind speed, a lower ambient temperature, and less direct sun. The IEEE test systems present part of the Midwest electric grid. While Bočkarjova and Andersson [3] derived current line constraints on the IEEE 14-bus system using the IEEE 738 Standard; however, they had to make extensive assumptions about the network characteristics and environmental conditions in order to use the IEEE standard. The data needed to compute line limits from the IEEE standards is not available. One would need the diameter of the lines, their maximum temperature ratings, ambient temperature, line length, and many more parameters that are not given. Therefore, this paper examines creating line limits based on finding limits that bind rather than trying to derive them from physical properties.

2.3 Converting apparent power limits to current limits

The limits on real power and voltage angle differences are most often used when solving a DC approximation of the ACOPF as reactive power is neglected in the

DCOPF. Limits on apparent power, as shown in (5)–(6), or current (7) are often used when solving the AC power flow formulation. A limit on apparent power or real power is often used for line limits for the power-voltage formulation of the ACOPF, and a limit on current is often used for the current-voltage formulation.

While all of the different limits have the impact of preventing temperature on the lines from becoming too large, the different limits are not equivalent.

$$p_{k(n,m)} = i^r_{k(n,m)} \left(v^r_n - v^r_m\right) + i^j_{k(n,m)} \left(v^j_n - v^j_m\right) \tag{2}$$

$$q_{k(n,m)} = i^r_{k(n,m)} \left(v^j_n - v^j_m\right) + i^j_{k(n,m)} \left(-v^r_n + v^r_m\right) \tag{3}$$

$$s_{k(n,m)} = p_{k(n,m)} + j q_{k(n,m)} \tag{4}$$

$$|s_{k(n,m)}| = \sqrt{p^2_{k(n,m)} + q^2_{k(n,m)}} \tag{5}$$

$$s^2_{k(n,m)} = v^2_n i^2_{nmk} \tag{6}$$

Given a limit on the apparent power on the line, the equivalent limit on the current flow on the line can be bounded, as shown in (7).

$$\frac{s^{max}_{k(n,m)}}{v^{max}_n} \le i^{max}_{k(n,m)} \le \frac{s^{max}_{k(n,m)}}{v^{min}_n} \tag{7}$$

3 Methodology

For the IEEE problems, the problems are solved without line constraints, and tight and loose limits are created based on restricting current below the highest current when the OPF was solved. Section 3.1 gives the formulation that is used to solve for the optimal power flow, and Sect. 3.2 describes how the IEEE line limits are set. Section 3.3 shows how the apparent power limits of the Polish systems are converted to current limits using the bounds on current flow given in (7).

3.1 Formulation

Here, the current-voltage formulation of the ACOPF, first discussed in [6] and used to solve power flow in [7] is used to consider the different current constraints. Alternatively, the current constraints could be added to other formulations of the ACOPF, such as the polar or rectangular voltage-based formulations.

The cost function used by MATPOWER [20] is a quadratic function.

$$Cost = \sum_n f_n \left(p^G_n\right)^2 + e_n \left(p^G_n\right) \tag{8}$$

However, most ISOs and RTOs accept power bids as step functions rather than the smooth cost. Therefore, the objective considered is the piecewise linear function that approximates the quadratic function. To find this approximation, the amount of real power supplied is broken up into T equal pieces. Since the real power is bounded above

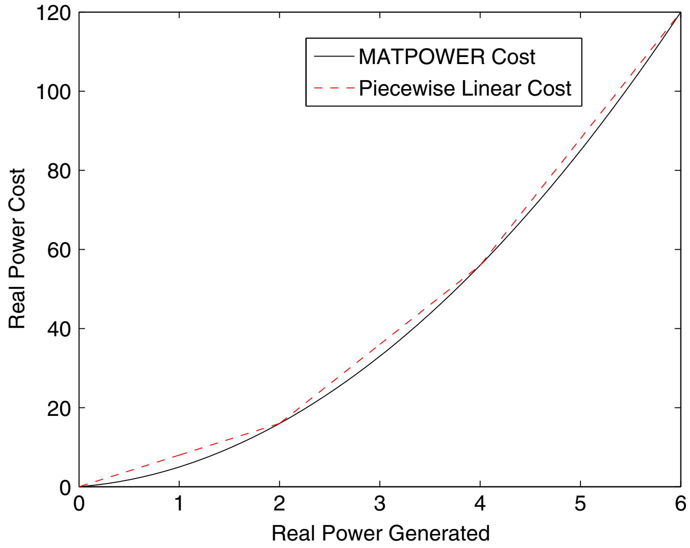

Fig. 1 MATPOWER cost versus piecewise linear approximation cost

and below ($p_n^{G,min} \leq p_n^G \leq p_n^{G,max}$) and must take on the value of greater than or equal to $p_n^{G,min}$, each segment $p_{n,t}$ is bounded by $\frac{p_n^{G,max} - p_n^{G,min}}{T}$ and the sum of the segments plus the minimum generation will equal the total generation, $p_n^G = \sum_t p_{n,t} + p_n^{G,min}$. If all cost coefficients are positive, which is true for all of the systems tested, the real power cost is convex as a function of real power, so we know that $p_{n,t}$ takes on a non-zero value only if $p_{n,t-1} = \frac{p_n^{G,max} - p_n^{G,min}}{T}$.

$$c_{n,t} = e_n + f_n \left(2p_n^{min} + (2t-1) \left(\frac{p_n^{G,max} - p_n^{G,min}}{T} \right) \right) \tag{9}$$

Figure 1 displays the quadratic MATPOWER cost versus the piecewise linear approximation. The costs match perfectly at the beginning and end of each step with the estimated cost being slightly higher than the quadratic cost.

Additionally, reactive power is priced at a fraction of real power to account for the reactive power support [2]. Often, better convergence results from allowing deviations from the limits (for real power, reactive power, and voltage) with penalties for these violations [15]. Therefore, small deviations from limits were allowed but priced very highly in the objective function, with $c_{n,pen} = 100{,}000$.

$$\begin{aligned} \text{Min Cost} = \sum_{n,t} c_{n,t} p_{n,t}^G &+ \frac{1}{10}(f_n + e_n) q_n^G \\ &+ c_{n,pen} \left(v_n^{viol,-} + p_n^{viol,-} + q_n^{viol,-} \right) \\ &+ c_{n,pen} \left(v_n^{viol,+} + p_n^{viol,+} + q_n^{viol,+} \right) \end{aligned} \tag{10}$$

such that:

$$i^r_{k(n,)} = g_{k(n)} v^r_n - g_{k(m)} v^r_m - b_{k(n)} v^j_n + b_{k(m)} v^j_m \tag{11}$$

$$i^j_{k(n,m)} = b_{k(n)} v^r_n - b_{k(m)} v^r_m + g_{k(n)} v^j_n - g_{k(m)} v^j_m \tag{12}$$

$$i^r_n = \sum_{m \in N(n)} i^r_{k(n,m)} \tag{13}$$

$$i^j_n = \sum_{m \in N(n)} i^j_{k(n,m)} \tag{14}$$

$$- v^{viol,-}_n + \left(v^{min}_n\right)^2 \leq \left(v^r_n\right)^2 + \left(v^j_n\right)^2 \leq \left(v^{max}_n\right)^2 + v^{viol,+}_n \tag{15}$$

$$\left(i^r_{k(n,m)}\right)^2 + \left(i^j_{k(n,m)}\right)^2 \leq \left(i^{max}_{k(n,m)}\right)^2 \tag{16}$$

$$p^G_n = v^r_n i^r_n + v^j_n i^j_n + p^D_n \tag{17}$$

$$p^G_n = p^{min}_n + \sum_t p^G_{n,t} \tag{18}$$

$$0 \leq p_{n,t} \leq p^{G,step}_n \tag{19}$$

$$q^G_n = v^j_n i^r_n - v^r_n i^j_n + q^D_n \tag{20}$$

$$- p^{viol,-}_n + p^{G,min}_n \leq p^G_n \leq p^{G,max}_n + p^{viol,+}_n \tag{21}$$

$$- p^{viol,-}_n + p^{D,min}_n \leq p^D_n \leq p^{D,max}_n + p^{viol,+}_n \tag{22}$$

$$- q^{viol,-}_n + q^{G,min}_n \leq q^G_n \leq q^{G,max}_n + q^{viol,+}_n \tag{23}$$

$$- q^{viol,-}_n + q^{D,min}_n \leq q^D_n \leq q^{D,max}_n + q^{viol,+}_n \tag{24}$$

3.2 IEEE systems: create one single limit

The IEEE systems do not contain thermal constraints. Here, the approach to creating current constraints is to set the same limit on current on every line. This approach would be closest to the physical system if it is assumed that all lines experience similar environmental conditions and are of similar length and material.

$$i^{max}_{k(n,m)} = i^{max} \forall k \tag{25}$$

As constraining the lines at a level greater than the highest current in the unconstrained system will not change the optimal solution, the maximum current should be set no greater than this level. Denoting the highest current in the unconstrained system as i^*, the sensitivity of the system to the current limit is analyzed by studying different fractions of i^* as shown in Eq. (26).

$$i^{max}_{k(n,m)} = \left(\frac{t}{100}\right) i^*, \quad t = 1, 2, 3, \ldots, 100 \tag{26}$$

We create two different current limits for each IEEE system—one is a strict limit ("tight") if one wants to examine a highly stressed system; the less-strict limit ("loose")

Table 1 IEEE test system data

Case (# buses)	Lines	Number of generators	Total generation capacity	Demand
14	20	5	7.72	2.59
30	41	6	326.80	42.42
57	80	7	326.78	235.26
118	186	54	99.66	42.42
300	411	69	235.25	234.79

aids in the analysis of a less stressed system. The tight limit is obtained when the penalty cost is close to nonzero; the loose limit is set roughly halfway between this point and the highest unconstrained current. As shown in Sect. 4.3, these limits occur at different restrictions of the maximum current for the different networks.

3.3 Polish systems: convert apparent power limits to current limits

In these problems, limits on apparent power are given, and so these are converted to current constraints. As shown in Eq. (27), the maximum current on a line is bounded by the maximum apparent power.

$$\frac{s_{k(n,m)}^{max}}{v_n^{max}} \leq i_{k(n,m)}^{max} \leq \frac{s_{k(n,m)}^{max}}{v_n^{min}} \tag{27}$$

The tight and loose limits can then be set as the lowest (28) and highest (29) bounds.

$$\text{Tight limit} \qquad \frac{s_{k(n,m)}^{max}}{v_n^{max}} \tag{28}$$

$$\text{Loose limit} \qquad \frac{s_{k(n,m)}^{max}}{v_n^{min}} \tag{29}$$

4 Numerical results

In this section, the results for the different current constraints on both the IEEE and Polish systems are given.

4.1 Test case characteristics

The characteristics of the IEEE test systems are detailed in Table 1 and those of the Polish test systems are detailed in Table 2.

In these IEEE cases, the networks are fairly sparse, with the number of transmission lines being less than twice the number of buses. Except for the 300-bus system, the demand is significantly lower than the generation capacity in the network.

Table 2 Polish test system data

Case (# buses)	Lines	Number of generators	Total generation capacity	Demand
2383wp	2896	327	295.94	245.58
2736sp	3504	420	199.97	180.75
2737sop	3506	399	140.28	112.67
2746wop	3514	514	236.39	189.62
2746wp	3514	520	275.39	248.73
3012wp	3572	502	300.08	271.70
3120sp	3693	505	253.18	211.81
3375wp	3161	596	660.8	483.63

In the Polish system, the suffix at the end of the number of buses signifies the season, with w for winter and s for spring, and whether the case is from a peak time, p, or an off-peak time, op. For example, the 2383wp case has 2383 buses and is a winter peak case. The Polish systems generally have adequate generation capacity and are even sparser networks than the IEEE cases.

4.2 Test cases without constraints on current

The ACOPF-IV was formulated in Pyomo [8] and solved using IPOPT version 3.11 [19] with linear solver ma57. Multiple generators at one bus were aggregated into one generator. For the base test cases without any line limits, the currents on the lines take on the values shown in Table 3. In this table, one sees that the average current is well below the maximum current. Additionally, some lines have near-zero current flowing through them; some show zero current due to the sensitivity of the current levels being to two decimal places.

4.3 IEEE system limits on current

In Table 4, the line current infeasibility level is defined as where there is no solution to the power flow when current is lowered any more than the line current infeasibility level. Since we relax hard constraints and penalize violations in our formulation, this line current infeasibility level is the lowest current where the penalty cost is zero and where lowering the current limit any farther would result in a positive penalty cost. If we let any solution with a nonzero penalty designate an infeasible solution, we can see that the IEEE systems do not give a feasible power flow solution using IPOPT when the current limit on the line is lowered too far. That is, there is no solution for the system to satisfy the power demand requirements given the very low line capacity.

As the current limit is lowered, the different IEEE systems behave differently. The current limit can be lowered to less than 30 % of the maximum unconstrained value on the 14- and 118-bus systems before the penalty cost is nonzero, while lowering the limit by 11 % for the 30-bus system makes the penalty cost nonzero. The limit on the

Table 3 Unconstrained current levels, in p.u.

System (# buses)	Min. current	Avg. current	Max. current
14	0.02	0.28	1.14
30	0.00	0.12	0.35
57	0.01	0.22	1.77
118	0.00	0.44	4.02
300	0.01	1.316	10.820
2383wp	0.00	0.36	8.83
2736sp	0.00	0.26	4.81
2737sop	0.00	0.17	3.66
2746wop	0.00	0.27	6.44
2746wp	0.00	0.32	6.22
3012wp	0.00	0.35	8.66
3120sp	0.00	0.31	9.07
3375wp	0.00	0.52	10.00

Table 4 Current level below which the penalty cost is >0 for this limit on all lines

No. of buses	Line current infeasibility level (p.u.)	Percent of i^*
14	0.227	20
30	0.309	89
57	1.408	80
118	1.136	28
300	6.780	63

300-bus system can be lowered to 63 % of its original level before there are problems finding a feasible solution.

Figure 2 plots the ratio of the constrained system's objective function (including penalty costs) to the unconstrained system's objective function on the y axis and the fraction of the maximum unconstrained current that is set as the current maximum on the x-axis (the value of t in (26)). The point where the curve becomes an asymptote is where the system becomes infeasible. Reducing the current limit has different effects on the different systems. Examining the ratio of the constrained objective function to the unconstrained objective function, the highest objective function for the 14-bus system with a zero-penalty solution is 26 % higher than the unconstrained objective value; for the 30-bus system, it is only 1.7 % higher. The objective function for the 57-bus system is only 0.004 % higher; the 118-bus system has more difference with the objective function being 5.3 % higher than the unconstrained function. For the 300-bus system, the objective function at the lowest current limit before the system is infeasible is 0.8 % higher than with no limits on current.

Two levels of current limits are suggested—one is a strict limit ("tight") if one wants to examine a highly stressed system; the non-strict limit ("loose") aids in the analysis

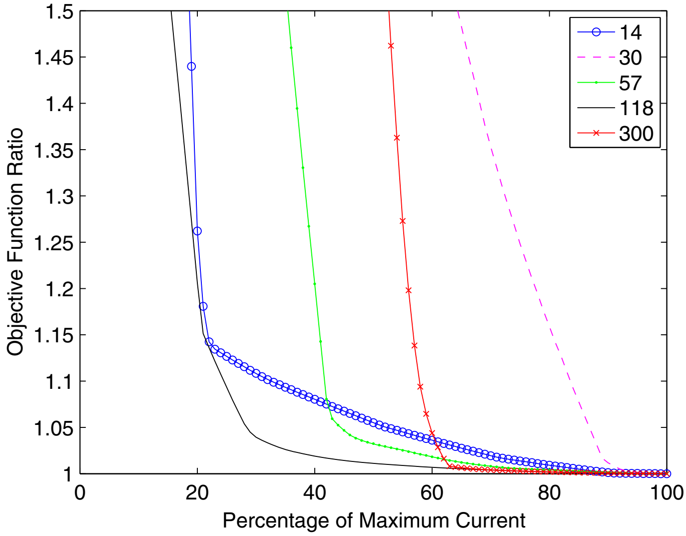

Fig. 2 One single limit results

Table 5 Recommended curret limits, one level for all lines

No. of buses	Tight limit (p.u.)	Loose limit (p.u.)
14	0.2675	0.7100
30	0.3125	0.3300
57	1.4275	1.6000
118	1.1400	2.3500
300	6.8200	8.8200

of a less stressed system (Table 5). The tight limit is obtained when the penalty cost is close to nonzero; the loose limit is set roughly halfway between this point and the highest unconstrained current.

Figures 3 through 7 show the impacts of the different limits on each individual line. If the current on the line is different at the from and to nodes, the larger value is graphed. As the current magnitude constraint is decreased, the effect on other lines is shown in Fig. 3. In the 14-bus system when current is not constrained, one line has nearly twice the current magnitude as the line with the next highest current. The current magnitudes that change the least under restriction are the lines with comparatively lower current magnitudes. For the loose constraint, only one line is congested. For the tight constraint, four line limit constraints are binding or near binding.

For the 30-bus system, the current level could not be restricted much lower than the maximum optimal current level (0.35) before the system became infeasible. In this system, there were two lines that both had high current magnitudes in the unrestricted problem, as seen in Fig. 4. When restricting the current level, there are many lines where current actually increases on the line versus under no restriction. While the difference between the tight (0.3125) and loose (0.3300) constraints is small, these

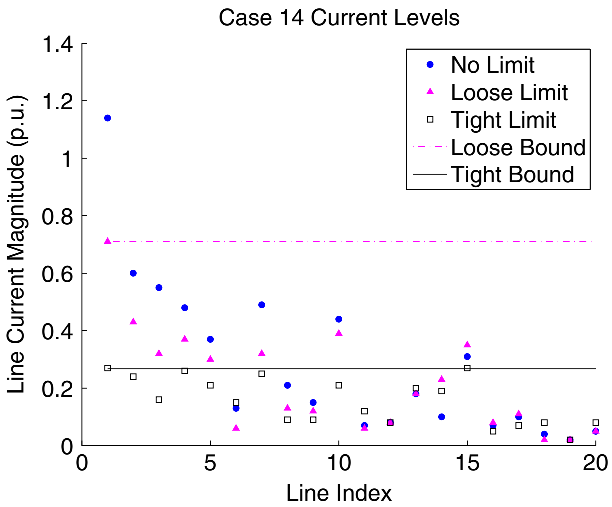

Fig. 3 14-bus line currents at no, tight, and loose limits

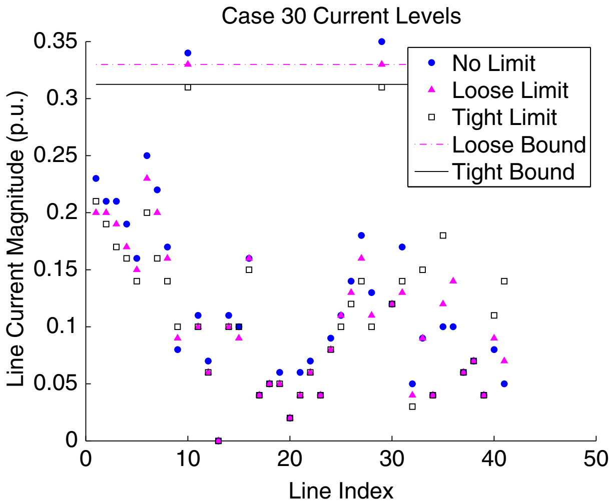

Fig. 4 30-bus line currents at no, tight, and loose limits

restrictions do have a large impact on the resulting current through some of the lines. In both the tight and loose limit cases, two lines have binding constraints. In each of these cases, these line connects generation (either directly or indirectly) to a demand node where the other connections are not directly to generators.

For the 57-bus system, shown in Fig. 5, the optimal solution has most of current magnitudes of half or less the highest current magnitude, similar to the 14-bus problem. Most lines have very low current levels compared to the highest line. When the current is restricted, most of the lines exhibit minimal change, while about one third of them have currents that change significantly with the restriction. Like the 30-bus problem,

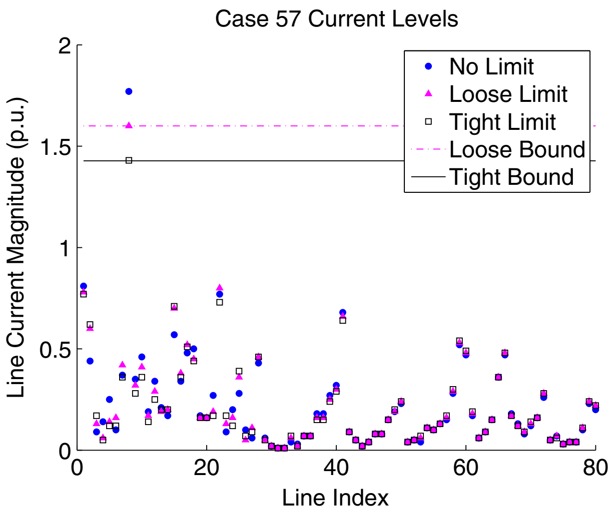

Fig. 5 57-bus line currents at no, tight, and loose limits

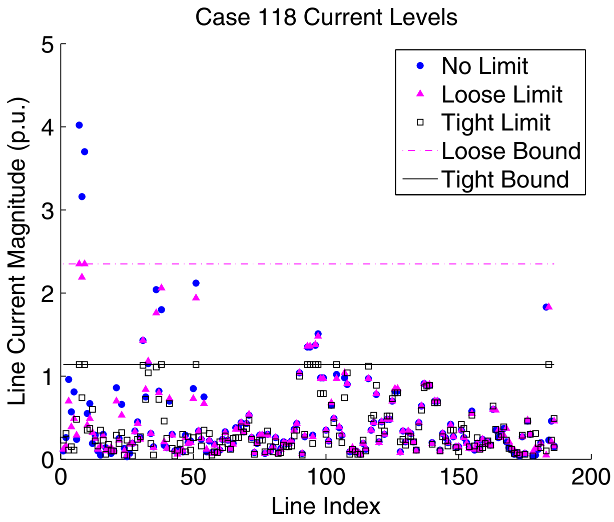

Fig. 6 118-bus line currents at no, tight, and loose limits

the two current restrictions are close together, with 1.4275 for the tight limit and 1.6000 for the loose limit. For both limits, only two lines are congested. However, even this small difference in limits greatly impacts the current on some lines.

The 118-bus system, shown in Fig. 6, has several lines with higher currents, similar to the 30- and 57-bus systems. The current can be restricted greatly before the problem becomes infeasible. Enforcing the tight limit has the most impact on the highest currents. Constraining the current with the loose limit impacts the currents slightly; however, constraining the current with the tight limit has a major impact on the cur-

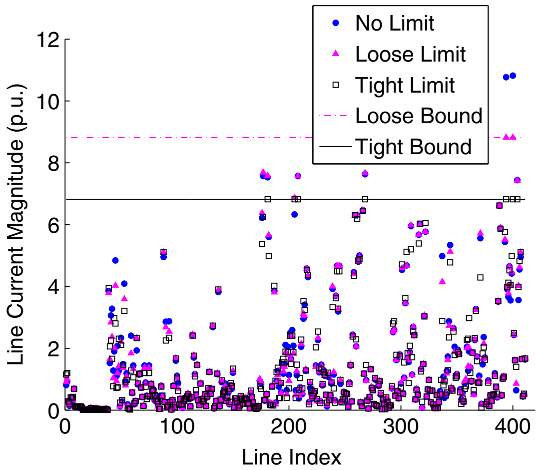

Fig. 7 300-bus line currents at no, tight, and loose limits

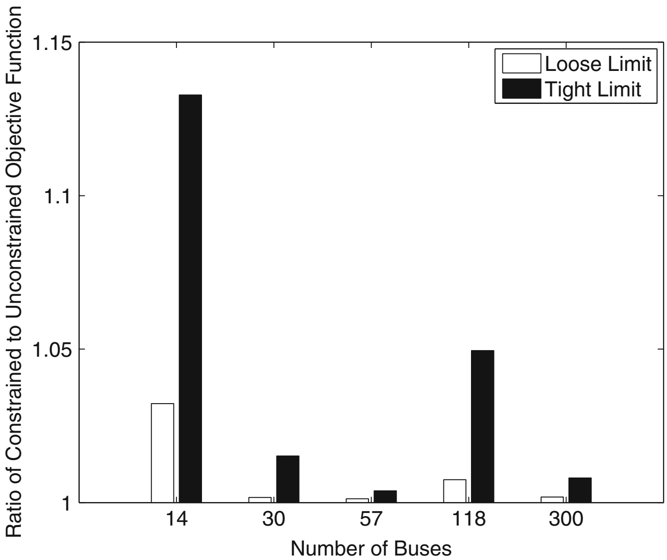

Fig. 8 Objective function ratio at no, tight, and loose limits for 14-, 30-, 57-, 118-, and 300-bus cases

rent magnitudes. With the loose constraint, only two lines are congested. Under the tight current magnitudes, 10 of the lines have the highest permissible current under this restriction, and even lines with lower currents exhibit big differences from the unrestricted and loose current cases, both higher and lower than before.

As seen in Fig. 7, under no restrictions, the 300-bus system has two lines with very high current magnitudes (10.82 and 10.77 p.u.), the remaining lines have current magnitudes of about 70 % of that value of current or lower (all less than 7.63 p.u.). Reducing the maximum current level reduces the line current of about 2/3 of the lines

Table 6 Objective function at different current limits

Case	Objective function ratio	
	Tight limit	Loose limit
2383wp	1.0213	1.0034
2736sp	1.0088	1.0027
2737sop	1.0000	1.0019
2746wp	1.0087	1.0050
2746wop	1.0002	0.9965
3012wp	1.0088	1.0027
3120sp	1.0034	1.0033
3375wp	1.0038	0.9991

and increases it on about 1/3 of the lines. The per unit value of current on lines in the 300-bus case is much higher than in the other 4 IEEE test cases.

As shown in Fig. 8, the current magnitude constraints increase the objective value to different degrees in the different systems. The 14-bus case has the highest increase in the objective value from adding current limits, then 118, then 30 and 300, and finally 57. The 14- and 118-bus cases could have currents reduced greatly before active penalties are required to solve the problem, while 30- and 57-bus could not.

4.4 Polish system limits on current

Including the current limits only raises the objective function slightly in most cases, as given in Table 6. The largest impact of including line limits is on the Polish system 2383wp, where the tight limit increases system costs by 2 %. There is some odd behavior; the cases with loose limits for the networks 2746wop and 3375wp find a cheaper solution than having no limits. This may be due to the interior point solver not finding the global optimum for the case with no limits, or having too large of a convergence tolerance.

5 Conclusion

This paper has examined the impact of different current limit levels on the IEEE test systems. It has recommended standard current limit settings for testing algorithms on IEEE test systems. These limits are binding under the nominal IEEE test system settings.

For each IEEE problem, one single limit is applied to all lines that makes the optimal solution without these limits infeasible. The resulting problem is solved using the IV-ACOPF formulation. For each problem a tight and a loose limit are determined. The tight limit highly constrains the system while the loose limit loosely constrains the system.

For the 14-, 30-, 57-, 118-, and 300-bus problems, creating line current magnitude constraints for the ACOPF problem can result in problems that may be infeasible. As one tightens the current magnitude constraints, the objective function increases gradually at first, then increases exponentially once penalties are required to solve the problem. Different test problems exhibit different characteristics in the line current magnitude distribution and at what current magnitude level constraint the problem becomes infeasible. The Polish systems exhibit small changes between the constrained and unconstrained systems.

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