Computing Parallel-Jaw Grips*

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ABSTRACT

In this paper we propose a new model for gripping parts with an industrial parallel-jaw gripper. In contrast to many previous models, we use two grip points and define candidate grips that are resistant to slipping and torque about the part's center of mass. These grips must also be accessible and robust to perturbations in part position. We give an $O(n^3)$ algorithm for computing and ranking $O(n^2)$ such grips on an n-sided polygonal slice through the part. The algorithm will be part of a design and simulation system that can rapidly provide feedback to designers; thus it must run quickly and reliably. We have also implemented the algorithm in a Java applet with a graphical user interface that allows Internet users to define a part; the applet computes, ranks, and displays the set of computed grips. To try the applet, please visit:

http://ford.ieor.berkeley.edu/grip/

0 INTRODUCTION

Parallel-jaw gripping is an important aspect of automated assembly. Our aim is to develop an efficient algorithm that will directly compute grips for picking up a polyhedral part with a parallel-jaw robotic gripper. In particular we consider the common case where the gripper is attached to a SCARA-type robot arm with 4 degrees of freedom.

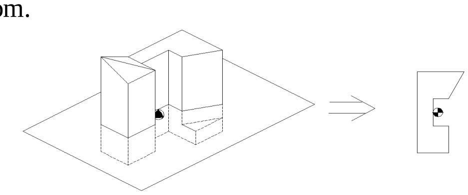

FIG 1. A 3D part being sliced through the com and the resulting 2D polygonal part.

We define a *grip* by two points on the boundary of the part; we refer to the line connecting those points as the *grip axis*. SCARA robot kinematics requires the grip axis to lie in a horizontal plane. This reduces to gripping a 2-dimensional, polygonal part formed by the intersection of the 3D part with a horizontal plane through the part's center of mass (com) as shown in Figure 1.

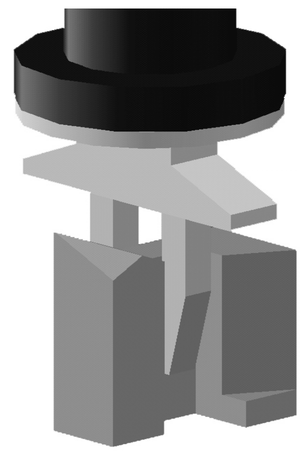

FIG 2. Computed Grip for Parallel-Jaw Gripper.

We consider all pairs of polygon edges and find the best grip, if one exists, for each pair. The algorithm runs in $O(n^3)$ time for an *n*-sided polygonal part.

The algorithm has been implemented in a Java applet available on the WWW. The applet, shown in Figure 3, allows users to design parts and provides a graphical user interface to adjust parameters such as center of mass, friction coefficient, vertex uncertainty, and grip ranking criteria.

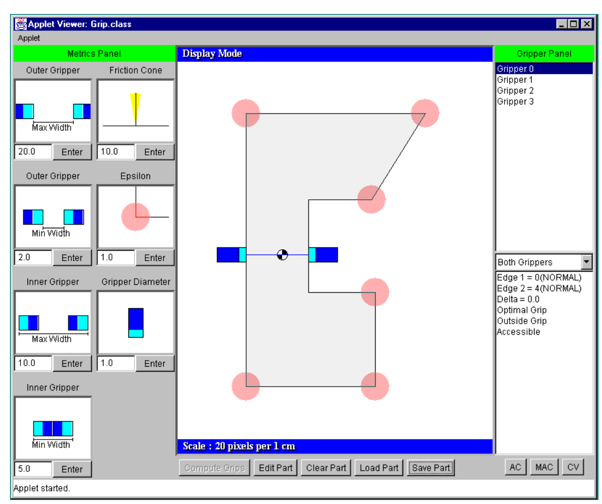

FIG 3. Screenshot of Grip Planner Applet

* This work was supported by the National Science Foundation under Presidential Faculty Fellow Award IRI-9553197, and Postdoctoral Research Associate Award CDA-9705022. Research funding was also provided by Adept Technology. Inquiries should be addressed to: goldberg@ieor.berkeley.edu.

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1.1 Problem Definition

We make the following assumptions. Contacts between the part and gripper are modeled as two point-like "soft fingers" as developed by Nguyen [1]. These contacts can exert force within the friction cones at both points of contact and provide torque about the grip axis as the part is lifted. Candidate grips on each pair of edges are identified using the following:

Input:

a. Part geometry: *n* vertices and center of mass.
b. Coefficient of friction between gripper and part: μ.
c. Radius of an uncertainty ball around each convex vertex: ε.

Grip Criteria:

1. Contacts must be at least a distance ε from any convex vertex to insure robustness to small uncertainties in part or gripper configuration.
2. The grip axis must lie within the friction cones at each contact edge so that the tips do not slip with respect to the part. The friction cone is defined by the halfangle $\alpha=\arctan(\mu)$, where μ is the coefficient of friction.
3. The grip should have minimal dependence on friction, therefore the grip axis is aligned as close as possible to the normal of each part edge.
4. The grip should also minimize the amount of torque required as the part is lifted, which is proportional to its weight times the moment arm from the grip axis to the com. Therefore we minimize the length of this moment arm.
5. Both contacts must be *accessible* to the gripper. A point is accessible if it is possible to reach the point from infinity along the grip axis.

Output:

All candidate grips ranked by frictional dependence or torque and the values that quantify those criteria.

Contacts at concave vertices are desirable because they are resistant to slip. Concave vertices are treated in a manner similar to that developed in Markenskoff, Ni, and Papadimitriou [2] and Brost [3]. A friction cone is constructed at the concave vertex that is the Minkowski sum of the friction cones of its neighboring edges as shown in Figure 4. Each concave vertex is treated as an infinitesimal edge.

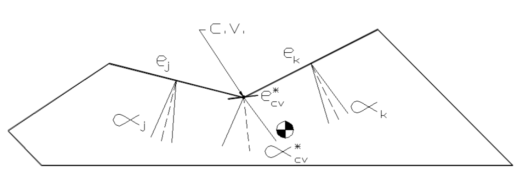


FIG 4. Edge e^*_{cv} and the friction cone at a convex vertex.

1.2 Related Work

Space does not permit a thorough review of the robot gripping literature; for thorough reviews please see [4], [5], [6], [7], [8], [9], [10].

Markenskoff and Papadimitrio [11] give algorithms to achieve form closure on a polygon with 4 frictionless fingers. Blake and Taylor [12] consider frictional grips on 2D smooth curves and show that locally optimum grips are orthogonal to local curvature. They do not consider polygonal parts, center of mass or accessibility.

Mirtich and Canny [13] consider grasping polygonal parts with friction and assume rounded fingertips so that polygonal parts can be treated as smooth curves. Their decoupled wrench criterion lexicographically maximizes resistance to adversarial forces in the grip plane and to moments perpendicular to this plane. For two-finger grips, they show that the optimal grip is one that maximizes the distance between contacts, which puts the grip axis along the maximum diameter, independent of the part's center of mass. For polygonal parts, this criterion produces contacts at convex vertices. Montana [14] suggests that such grips are not resistant to perturbations in part position.

Rao, Kriegman, and Goldberg [15] consider a related problem: determining grips with a pivoting gripper such that the part passively rotates to a new orientation when lifted. Our grip model avoids pivoting and takes into account robustness and accessibility.

2 FIVE GRIP CRITERIA

Consider two part edges, e_j and e_k. Let I be the point where these edges would intersect if they were extended as shown in Figure 5. Construct a weighted bisector of the angle, $\beta_{j,k}$, between edges e_j and e_k such that the angle between edge e_j and the bisector is defined:

$$\gamma_j = \left(\frac{\alpha_j}{\alpha_j + \alpha_k}\right)\beta_{j,k}, \qquad (1)$$

and similarly for γ_k.

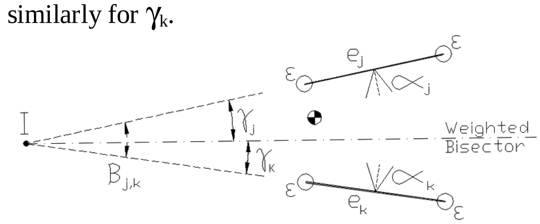


FIG 5. Construction of the weighted bisector of e_j and e_k.

We can parameterize the grip points by their distance from I along the lines formed by the edges. Let s_j be the distance along e_j, and s_k along e_k.

2.1 ε, Radius of Uncertainty Ball

Let ε be the radius of a circular "stayout" zone around each convex. If an edge consists of all points from $s_{j,start}$ to $s_{j,end}$ then any grip point must satisfy

$$(s_{j,start} + \varepsilon) \le s_{j,grip} \le (s_{j,end} - \varepsilon).$$

This insures that the gripper will not miss the specified edge due to small uncertainty in part or gripper configuration.

2.2 Friction Cones

All grips must have a grip axis that lies within the friction cone of each contact edge. If ϕ_j is the angle between the normal to edge e_j and the grip axis, and ϕ_k is similarly defined, then

$$|\phi_j| \le \alpha_j \quad \text{and} \quad |\phi_k| \le \alpha_k$$

must be satisfied. This will prevent the part from slipping during gripping.

2.3 Dependence on Friction

A grip should have minimum dependence on friction. This is done by minimizing both ϕ_j and ϕ_k. In order to strike a balance between minimizing ϕ_j and ϕ_k we now define ϕ as the angle between the grip axis and the normal to the weighted bisector. We note that when ϕ=0, as shown in Figure 6, the following is true:

$$\frac{\phi_j}{\alpha_j} = \frac{\phi_k}{\alpha_k}.$$

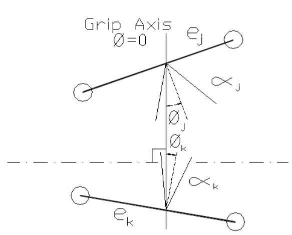


FIG 6. *When ϕ =0 frictional dependence between the two edges is balanced.*

In other words, the grip axis is the same fraction of the friction cone away from the normal for both edges. The intuitive case is two edges with the same friction cone angle and the grip should have the same angle at both faces. Also, when ϕ = 0 it is not possible to decrease ϕ_j without increasing ϕ_k. Therefore the third criteria is to minimize ϕ.

2.4 Torque

We also want to minimize the amount of torque that the tips must provide when the part is lifted. Let δ be the moment arm from the grip axis to the com. The torque is simply weight times δ, therefore minimum torque is ensured by minimizing δ.

2.5 Accessibility

Finally, we need to consider whether a pair of grip points can be reached the gripper. We use a critical point analysis for edges within the convex hull to determine accessibility cones. This is covered in more detail in Section 3.3.

2.6 Graphical Representation

For any pair of edges we can consider the 2D space of grips defined by (s_j,s_k). Associated with each grip is a value of δ and a value of ϕ. It is therefore possible to determine $\phi=\phi(s_j,s_k)$ and $\delta=\delta(s_j,s_k)$. We omit the derivations for lack of space but give the equations below

$$\phi = \arctan\left[\frac{\left|s_j \cos(\gamma_j) - s_k \cos(\gamma_k)\right|}{s_j \sin(\gamma_j) + s_k \sin(\gamma_k)}\right].$$

The graph of this function has contour lines, or iso-ϕ lines, that are radial with slopes

$$m = \frac{\cos(\gamma_k) - \sin(\gamma_k)\tan\phi}{\cos(\gamma_j) + \sin(\gamma_j)\tan\phi}$$

except for the case of two parallel edges in which case the contour lines are parallel. Note that the ϕ = 0 line has a slope of

$$m = \frac{\cos(\gamma_k)}{\cos(\gamma_j)}.$$

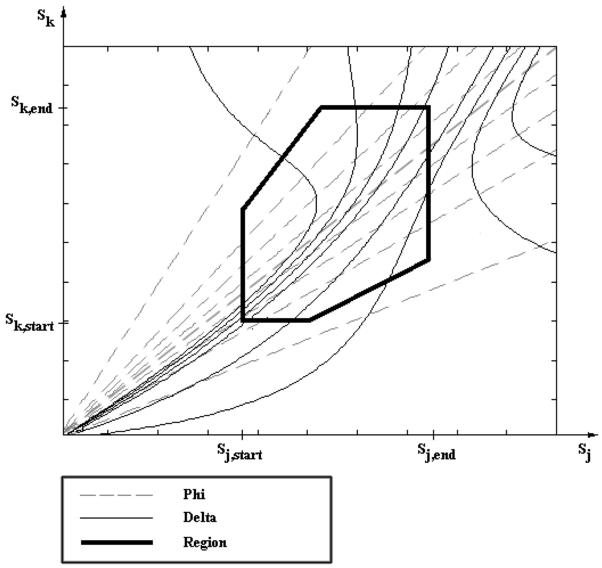
For calculating δ it is necessary to know the location of the com. We will parameterize its location by D and L. D is the perpendicular distance from the weighted bisector and L is the distance along the weighted bisector.

$$\delta = \frac{\left|(s_j\cos\gamma_j - L)(s_k\sin\gamma_k - D) + (s_k\cos\gamma_k - L)(s_j\sin\gamma_j + D)\right|}{\left[(s_j\cos\gamma_j - s_k\cos\gamma_i)^2 + (s_j\sin\gamma_j + s_k\sin\gamma_k)^2\right]^{1/2}}$$

We can overlay the contour graphs of the two functions to allow us to take both friction and torque into consideration simultaneously. The first two criteria, ε and the friction cones, bound a region of grips that are possible. Only the rectangular region satisfying $(s_{j,start}+\varepsilon) \le s_j \le (s_{j,end}-\varepsilon)$ and $(s_{k,start}+\varepsilon) \le s_k \le (s_{k,end}-\varepsilon)$ is included. This region may be truncated by either or both of two ϕ lines that represent the edges of the friction cones. Those lines are defined by $\phi = \min(\gamma_j+\alpha_j , \gamma_k+\alpha_k)$ and $\phi = \min(\gamma_j-\alpha_j , \gamma_k-\alpha_k)$. A region of possible grips in this space is shown in Figure 7.

The dependence on friction criteria and torque criteria can indicate an optimum grip from the region of possible grips. However, these two criteria are not always in agreement therefore two separate optimizations can occur. One branch emphasizes friction, while the other emphasizes torque. When they do not yield the same result our algorithm performs both optimizations.

FIG 7. The region of possible grips on the ϕ and δ contour graphs.



3 ALGORITHM

Our algorithm uses basic geometry and simple conditional tests to quickly determine grip points according to the criteria explained above. The algorithm runs in $O(n^3)$ time. The friction test runs in $O(n^2)$. Then the grip section runs in O(1) time for each of the $O(n^2)$ pair of edges. Finally, accessibility runs in $O(n^3)$. Figure 8 outlines the basic operation.

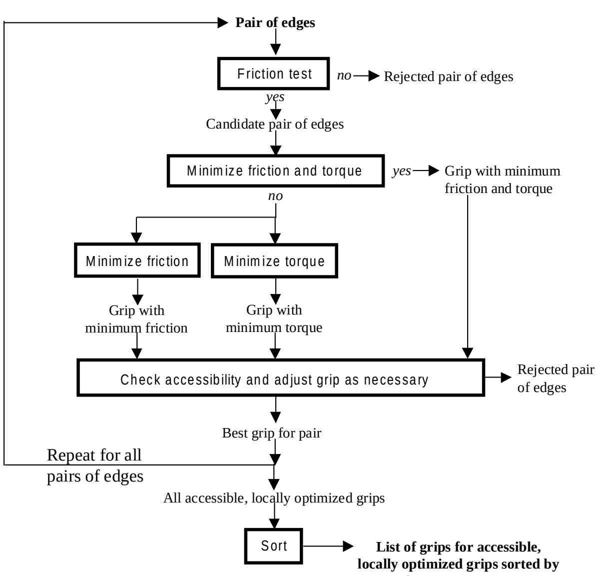


FIG 8. Flowchart showing the steps of our algorithm.

3.1 Friction Test

For each edge pair we first run a short test to reject pairs that could not contain a grip. If the angle, $\beta_{j,k}$, between the edges e_j and e_k is greater than the sum of the friction cone angles for those edges, $\beta_{j,k} > (\alpha_j + \alpha_k)$, then no grip on this pair of edges can meet the friction criteria. Figure 9 demonstrates that in practice this significantly reduces the number of candidate pairs of edges.

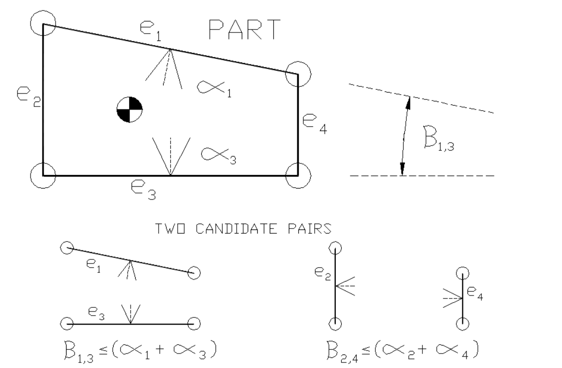


FIG 9. A part with n=4 has 6 pairs of edges but only 2 candidate pairs to be considered in the main algorithm.

3.2 Grips

For each edge our objective is to determine the best grips. First, it is helpful to shrink e_j and e_k by ε at each end so that the entire length of the remaining edges meets the ε criteria. We use the term "endpoints" to refer to the new beginning and end to distinguish them from the vertices.

3.2.1 Minimizing Friction and Torque

We first check whether there is a grip that minimizes both friction and torque or equivalently has $\delta = 0$ and $\phi = 0$ such as the one shown in Figure 10. Construct a line that goes through the com and is perpendicular to the weighted bisector. If this line intersects both edges then the intersections define grip points for that grip.

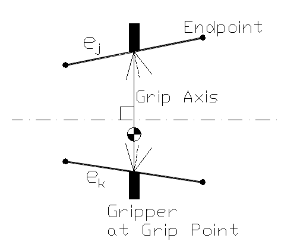


FIG 10. Grip with $\phi = 0$ and $\delta = 0$.

When the line fails to hit one or both edges then we must adjust this first attempt to meet all criteria. In general, at this point, it is possible to minimize either frictional dependence or torque but not both at the same time. We therefore determine two grips for the pair of edges. One grip has minimum friction and the other has minimum torque. Note that the minimum friction grip also has the least torque of all grips with that friction and similarly, the minimum torque grip has the least friction of all grip with that torque.

3.2.2 Minimizing Friction

We first determine the grip that minimizes the dependence on friction. Define a "critical edge" as an edge not intersected by the line perpendicular to the bisector through the com. Then define a "critical point" as the end point of the critical edge nearest to that line. These are shown in Figure 11.

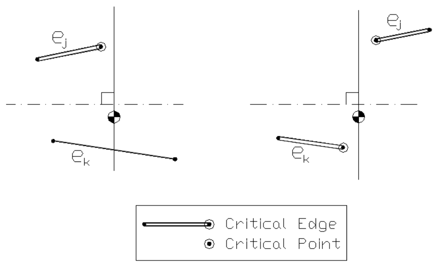


FIG 11. Critical Edges and Critical Points.

We check for a grip with $\phi = 0$ through each critical point. If none exists then the minimum ϕ grip has one grip point at a critical point and the other at one of the endpoints of the other edge. We check those four possibilities to find the one with minimum ϕ, then check that the criteria $\phi_j \leq \alpha_j$ and $\phi_k \leq \alpha_k$ are met. If so we have the minimum ϕ grip as shown in Figure 12; if not then there is no grip for this pair of edges.

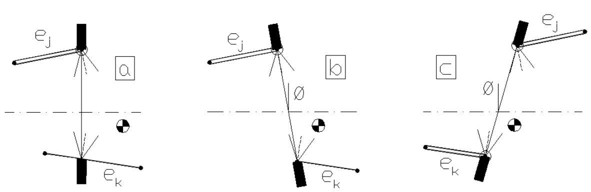


FIG 12. Some examples of minimum ϕ grips. a has ϕ =0 while b and c have ϕ = min.

3.2.3 Minimizing Torque

We now determine the grip that minimizes the necessary torque provided by the gripper tips. First note that if there is not a friction minimizing grip then there is also not a torque minimizing grip. We will use the same critical edges and critical points defined earlier.

We check for a grip with a grip axis through both the com and a critical point. If none exists then the minimum δ grip has grip points at a critical point and an endpoint of the opposite edge. We test these few cases to find the minimum δ grip and check that $\phi_j \leq \alpha_j$ and $\phi_k \leq \alpha_k$ are satisfied. If not, we perturb the grip axis to have a direction that satisfies the friction criteria and check grips with that direction at each endpoint. One will be the minimum δ grip. Example minimum δ grips are shown in Figure 13.

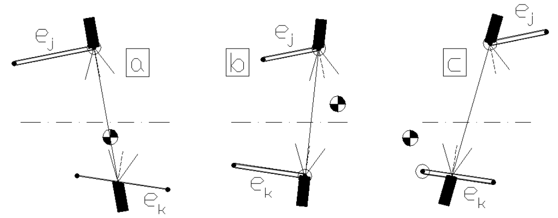


*FIG 13. Some examples of mini*mum δ grips. *a has δ = 0, b has δ = min, and c has δ = min after being perturbed to be within the friction cones.*

3.3 Accessibility

Finally we check if a grip is accessible. We call a grip accessible if both grip points are reachable from infinity in the approaching direction of the grip axis. Extend the grip axis and check for intersections with edges of the part besides the candidate pair. This can be done in O(n) time for each grip.

At this point it may seem simplest to eliminate any grip that is not accessible. We note, however, that it may be possible to perturb a grip to be sub-optimal but accessible. We therefore run an algorithm that perturbs the grip to the best accessible grip. The procedure runs at $O(n^3)$ for the part as a whole and in fact only needs to run when an optimal grip has failed the accessibility test.

3.3.1 Accessibility Cones

First, find the convex hull of the part. Any grip failing the accessibility test must have one or both contact edges within a concavity. Within that concavity, find the set of all convex vertices and add the vertices of both edges of the candidate pair. Connect every pair of points in the set with a line. This operation yields at most one line from every vertex to every vertex in the part even when run for all candidate pairs and is therefore $O(n^2)$. Eliminate any line that does not intersect both edges in the candidate pair. Extend the lines and eliminate any line that intersects any edge besides the edges of the candidate pair. This is an O(n) operation for each line totaling $O(n^3)$ for the part. The remaining pairs of lines form "accessibility cones". A critical vertex and a range of angles characterize each accessibility cone.

We then truncate the range of angles of each accessibility cone to include only those angles within the friction cones of both edges to form "modified accessibility cones". If there are no remaining cones then there is no accessible grip for that pair of edges. This procedure of checking critical points to generate accessibility and modified accessibility cones is shown in Figure 14. If the gripper is to reach the contact edge at an angle that is within the range of angles of a particular modified accessibility cone then the critical vertex of that cone is the point that it must clear.

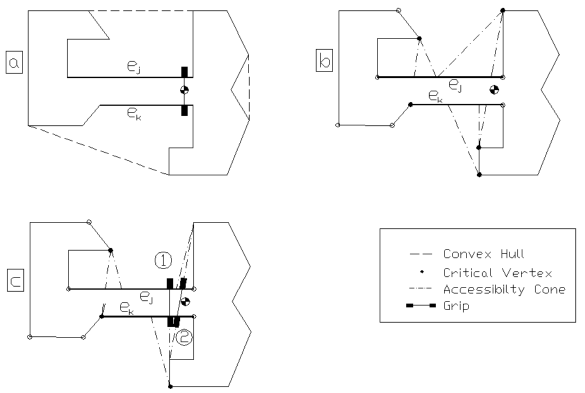


FIG 14. The steps for generating accessibility cones and modifying grips. a: Finding the convex hull of the part when a grip is inaccessible; b: Determining the critical vertices and accessibility cones; c: Creating the modified accessibility cones, and modifying the grips to [1]accessible minimum ϕ grip, and [2]accessible minimum δ) grip.

We must again divide our consideration to both friction minimizing and torque minimizing grips. If the grip that failed the accessibility test was torque minimizing then only the torque optimization is run. Similarly for the friction minimizing grip.

3.3.2 Accessible and Friction Minimizing Grip

Find the modified accessibility cones that include the angle $\phi = 0$. There will be either two or zero. If there are two then the line with $\phi = 0$ through the critical vertex of one of those cones is the grip axis of the accessible, friction minimizing grip. If there are zero then the edge of the accessibility cone with the minimum ϕ is the grip axis.

3.3.3 Accessible and Torque Minimizing Grip

The com either lies within zero or two modified accessibility cones. If it lies within two then the grip axis of the accessible, torque minimizing grip is through the critical vertex of one of those accessibility cones and the com. If the com is in none then the nearest side of the nearest accessibility cone is the grip axis of the accessible, torque minimizing grip. Note that it is possible that the algorithm finds no grips.

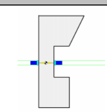
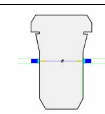
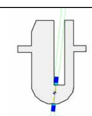
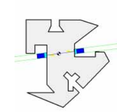
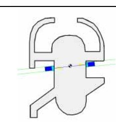
4 IMPLEMENTATION

The implementation lets users define a part and visualize the grips generated. Java's Abstract Windowing Toolkit (AWT) package provides various GUI components, such as buttons, menus, etc. Anyone with a Java-enabled browser can run and test the algorithm. The following sections describe the user interface and the runtime performance of the algorithm.

4.1 User Interface

The user interface is designed so that parts and parameters can be intuitively defined and grips can be easily viewed. The Java applet is divided into three columns. The left column contains the control panel where the user can change the friction angle, ε, and the gripper's width. These parameters take effect once the user clicks on the button labeled "Compute Grip" located at the bottom of the applet. The center column of the applet shows the polygonal slice of the part. It is the drawing area where the user can define new parts. It also gives a graphical representation of the grips generated by the algorithm. The right column contains a list of all the generated grips. The user can view each grip by clicking on an item in the list. Below that list is where relevant information about the current grip is given.

Table 1. Sample applet running times.

Part Description	# of Vertices	Running time (ms)	# of Grips Generated
	8	220	7
	19	490	11
	23	1270	22
	35	2970	7
	71	3890	14

5 DISCUSSION

This work was motivated by the desire to rapidly generate "reasonable" grips to facilitate design and visualization of automated industrial assembly lines. Our model differs from previous models of gripping by considering the parts center of mass to minimize out-of plane-torque, accessibility, and robustness to position uncertainties. The current implementation also finds grips that push outward in opposite directions along the grip axis. We invite readers to try the applet:

http://ford.ieor.berkeley.edu/grip/

After six months of testing on the Internet, the algorithm is now being ported to C++ so that it can be included in an integrated simulation package.

Table 1 gives running time of the applet on an Intel Pentium Pro/200 running Symantec Visual Café's JVM. Since the algorithm finds at most two candidate grips for each pair of edges, and the running time for a single pair is O(n), the total running time for the algorithm is $O(n^3)$. Depending on the given coefficient of friction, it is also possible that candidate grips do not exist: for example an isosceles triangle with a friction cone angle less than 30^o cannot be gripped. In such cases the algorithm will return a negative report. Although the Java virtual machine is not known for its efficiency, Table 1 shows that a part with 71 vertices can be processed in less than 4 seconds on a garden variety Pentium 200.

The next step is to develop a practical model of part perturbation during grasping in order to calculate capture regions for each grip. To extend the algorithm to 3D we will consider choosing the best grip plane and checking for out-of-plane barriers to accessibility.

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